Bifurcation Analysis of Endogenous Growth Models

Abstract

It is important to recognize that bifurcation boundaries do not necessarily separate stable from unstable solution domains. Bifurcation boundaries can separate one kind of unstable dynamics domain from another kind of unstable dynamics domain, or one kind of stable dynamics domain from another kind, such as monotonic stability from damped periodic stability or from damped multiperiodic stability. There are not only an infinite number of kinds of unstable dynamics, some very close to stability in appearance, but also an infinite number of kinds of stable dynamics. Hence subjective prior views on whether the economy is or is not stable provide little guidance without mathematical analysis of model dynamics.

The thesis analyzed, within its feasible parameter space, the dynamics of the Uzawa-Lucas endogenous growth model. We examine the stability properties of both centralized and decentralized versions of the model and locate Hopf and transcritical bifurcation boundaries. In an extended analysis, we investigate the existence of Andronov-Hopf bifurcation, branch point bifurcation, limit point cycle bifurcation, and period doubling bifurcations. While these all are local bifurcations, the presence of global bifurcation is confirmed as well. We find evidence that the model could produce chaotic dynamics, but our analysis cannot confirm that conjecture.

Further this thesis analyses the dynamics of a variant of Jones semi-endogenous growth model "Sources of US Economics growth in a World of Ideas" The American Economics Review, March 2002, Vol. 92 No. 1, pp 220-239. A detailed bifurcation analysis is done within the feasible parameter space of the models. We showed the existence of codimension-1 bifurcations (Hopf, Branch Point, Limit Point of Cycles, and Period Doubling). In addition some codimension-2 (Bogdanov-Takens and Generalized Hopf) bifurcations are detected in the modified Jones model. While the aforementioned are all local bifurcations, the Uzawa-Lucas model also shows the presence of global bifurcation.