How credible is the Federal Reserve? A structural estimation of policy re-optimizations *

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April 2014

Abstract

Using a Markov-switching Bayesian likelihood approach, the paper proposes a new measure of the degree of credibility of the Federal Reserve. We estimate a medium-scale macroeconomic model, where the central bank has access to a commitment technology, but where a regime-switching process governs occasional re-optimizations of announced plans. The framework nests the commonly used discretion and commitment cases, while allowing for a continuum of intermediate cases. Our estimates reject both full-commitment and discretion. We instead identify occasional re-optimization episodes that are consistent with changes in Federal Reserve policymakers and operating procedures. Finally, through counterfactual analyses we assess the role of credibility over the past four decades.

JEL *classification*: C32, E58, E61.
Keywords: Commitment, Regime-Switching Bayesian Estimation, DSGE models

*We thank Andrea Tambalotti for his discussion, and Jordi Galí, Jim Hamilton, Cosmin Ilut, Tatiana Kirsanova, Kristoffer Nimark, Ricardo Nunes, Valerie Ramey, Frank Schorfheide and seminar participants at UCSD, Michigan State, SED annual meeting (Seoul), Federal Reserve Board, the mid-year meeting of the NBER "Methods and Applications for DSGE Models" group, Barcelona GSE Winter Forum, Applied Econometrics Workshop at St. Louis Fed and Lancaster for their useful comments. Computational work in support of this research was performed at Michigan State University's High Performance Computing Facility. All remaining errors are our own. Contact emails: davide.debortoli@upf.edu, aeimit@msu.edu.

"Whether we have the credibility to persuade markets that we'll follow through is an empirical question."

Ben Bernanke, Federal Reserve Chairman, September 13^{th} 2012.

1 Introduction

Both academics and policymakers agree on the importance of central bank credibility in conducting monetary policy. Over the past few decades significant effort has been devoted to enhance credibility in monetary policy, through the creation of independent central banks, the adoption of clear policy objectives, improved transparency and communication strategies, among other measures. Whether central banks are indeed credible, however, remains a largely open question. This paper proposes a novel measure of central bank credibility, and provides new evidence about the credibility of the Federal Reserve over the past few decades.

The term credibility is used in practice to refer to a multiplicity of different concepts.[1] Our definition of credibility coincides with the notion of "commitment", as in the seminal works of Kydland and Prescott (1977) and Barro and Gordon (1983). The presence of a policy trade-off (e.g. stabilizing inflation vs. output), combined with the forward looking nature of economic agents, makes it desirable for the central bank to commit to a policy plan. By committing to a plan, the central bank can shape agents' expectations in a way that improves the short-run policy tradeoffs. However, once those short-run benefits have been reaped, there is an ex-post temptation to deviate from the original plan, and to re-optimize. Credibility is then defined as the ability to resist the temptation to re-optimize. This definition is widely accepted in the monetary policy literature, and is also consistent with the central bank having a "a history of doing what it says it will do", which both academics and policymakers selected as the most important factor in building central bank credibility in the survey by Blinder (2000).

The monetary policy literature has typically considered two alternative (and extreme) scenarios about the ability of the central bank to commit. It has either assumed that the central bank always follows its announced plans (commitment case), or that it always deviates (discretion case). Following Roberds (1987), Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010), this paper adopts a more flexible approach that nests

[1] As surveyed by Blinder (2000), academics and policymakers identify the term "credibility" with various different measures, such as "transparency", "independence", "aversion to inflation", etc.

commitment and discretion as special cases, while allowing for a continuum of intermediate cases – i.e. the so-called *loose commitment* setting.[2] The central bank has the ability to commit to its future plans, but it may occasionally give in to the temptation to revise its plans. Both the central bank and the private sector are aware of the possibility of policy re-optimizations, and take it into account when forming expectations. This setting is meant to capture the fact that central bankers understand the benefits of credibility, but at the same time there could be situations when a central banks disregards its commitments. These situations may arise because of changes in the dominating views within a central bank due to time-varying composition of its decision-making committee or outside pressures by politicians and the financial industry.[3]

In particular, we consider a model where the behavior of the central bank is described by a two-state regime-switching process. In each period, with probability γ the central bank follows its previous plan, while with probability $1-\gamma$ it makes a new plan. The probability $\gamma \in [0,1]$ can then be interpreted as a measure of credibility, in between the commitment ($\gamma = 1$) and discretion ($\gamma = 0$) extremes.[4] Using a regime-switching likelihood approach, we obtain an estimate of the (unconditional) probability of commitment, and identify specific episodes where the Federal reserve has likely abandoned its commitments.

The empirical analysis is conducted within the medium-scale model for the US economy of Smets and Wouters (2007) (henceforth SW). That model can be viewed as the backbone of the estimated models developed at central banks in recent years, and used for monetary policy analysis and forecasting. We depart from that model in two important ways. First, monetary policies are chosen optimally by a central bank operating under *loose commitment*, rather than being described by a simple (Taylor-type) rule. Second, we deal with a version of the SW model with regime-switching. In addition to the regime-switching process driving policy re-optimizations described earlier, we also allow the variance of the shock processes

[2]Roberds (1987) used the term "stochastic replanning" while Schaumburg and Tambalotti (2007) used the term "quasi-commitment".

[3]In the case of the United States, the reserve bank presidents serve one-year terms as voting members of the FOMC on a rotating basis, except for the president of the New York Fed. Furthermore, substantial turnover among the reserve bank presidents and the members of the Board of Governors arises due to retirement and outside options. With the (up to) seven members of the Board of Governors being nominated by the U.S. President and confirmed by the U.S. Senate, the composition of views in the FOMC may be affected by the views of the political party in power at the time of the appointment. Chappell et al. (1993) and Berger and Woitek (2005) find evidence of such effects in the U.S. and Germany, respectively. Also, the book by Havrilesky (1995) provides evidence on when politicians tried to influence monetary policy, and when the Federal Reserve did and did not respond.

[4]Equivalently, that probability can be thought of as a continuous variable measuring the durability of the Federal Reserve's promises, where longer durability corresponds to higher levels of credibility.

to shift over time to control for additional potential sources of time variation. Estimation is carried out using a Bayesian Markov Chain Monte Carlo (MCMC) algorithm.

Two main results emerge from the estimates. First, the model supports the idea that the Federal Reserve is to some extent credible, but that credibility is not perfect. This result differs from the existing literature, as it signals that both the commonly used assumptions of commitment and discretion are rejected by the data. Within a variety of different exercises, the posterior mode of the unconditional probability of commitment is estimated to be about 0.80, with fairly narrow confidence intervals. Such a value could be viewed as closer to either commitment or discretion, depending on the metric used. In order to provide a clearer interpretation of our result, we perform counterfactual simulations in which the central bank is assumed to operate either under commitment or under discretion throughout the entire sample. We find that during the 1970s that actual data is close to the counterfactual path under discretion. But the fall in inflation in the early 1980s and the subsequent low levels are closer to the counterfactual path under commitment. Overall, these issues highlight the importance of using our general framework, where sometimes the dynamics of the economy are better described by the case of commitment, while at other times the case of discretion is better.

The second contribution of the article is the identification of historical episodes when the Federal Reserve likely abandoned its commitments, as measured by the (smoothed) probability of re-optimization. We find that policy re-optimizations likely occurred with the appointments of Arthur Burns and Paul Volcker but not with the appointments of Alan Greenspan and Ben Bernanke. Re-optimization episodes were also likely around changes in operating procedures of the Federal Reserve, specifically during the reserves targeting experiment conducted under Volcker in the early 1980s and the FOMC policy to start announcing the target for the Federal Funds rate around 1994. Additionally, we find a re-optimization episode in 2008, around the start of the quantitative easing policy under Ben Bernanke. An alternative interpretation of our re-optimization episodes is to view them as a source of monetary policy shocks. According to this perspective, we find that typically the deviations from commitment during the 1970s have implied policies that are relatively more expansionary, while deviations in the 1990s and 2000s imply policies that are relatively more contractionary.

Alternative approaches to measure central banks' credibility have been proposed in the literature. For instance, through an index-based aggregation of information contained in by-laws and questionnaires, Cukierman (1992) develops some indicators of independence, transparency and aversion to inflation. Also, as initially proposed in Svensson (1994), several studies have inferred a measure of "inflation-target" credibility by looking at the deviations of long-run inflation expectations from the central bank's inflation target. Here we study instead the role of credibility as a device to improve the policy responses to short-run economic disturbances.[5] As a result of these disturbances, temporary deviations of inflation expectations from target do not necessarily signal a credibility problem. In this respect, the advantage of our structural estimation approach is the possibility to disentangle commitment problems from other factors affecting agents' expectations.

Our work is also related to the empirical literature on optimal monetary policy. For the most part, that literature has abstracted from assessing the empirical plausibility of alternative commitment settings, by focusing either on commitment or discretion.[6] Few exceptions are the recent works of Givens (2012) and Coroneo et al. (2013), who compare estimates of models with commitment and discretion, and conclude that the data favor the specification under discretion. Kara (2007) obtains a structural estimate of the degree of commitment through a least-squares estimation of a monetary policy rule obtained within the framework of Schaumburg and Tambalotti (2007), and provides evidence against the cases of commitment and discretion. To the best of our knowledge, ours is the first empirical study that considers a framework where the central bank's behavior regarding its previous commitment may change over time, with occasional switches between re-optimizations and continuations of previous plans. In contemporary and independent work Chen et al. (2013) perform a similar Bayesian regime-switching estimation of a simple monetary policy model, and study switches between active and passive regimes. We instead focus on the commitment problem, and perform our analysis within a state-of-the-art DSGE model.

From a methodological viewpoint, our setting is closely related to recent empirical studies in the DSGE regime-switching literature [see Liu et al. (2011) and Bianchi (2012)] that analyze regime-switches in the inflation target or in the coefficients of a monetary policy rule, while allowing the variances of the shocks to switch over time. The main difference with respect to those studies is that in our model the central bank formulates an optimal plan, rather than following a simple interest rate rule. The restrictions implied by optimal policy under loose commitment allow us to distinguish policy re-optimization episodes from

[5]As discussed in Clarida et al. (1999), a central bank without commitment is not able to smooth over time the costs of economic fluctuations, thus giving rise to the so-called "stabilization bias". That concept needs to be distinguished from the "inflation bias" that arises when the central bank, because of long-run inefficiencies, wishes to push output above its natural level.

[6]Some examples are the works of Dennis (2004), Söderström et al. (2005), Salemi (2006), Ilbas (2010) and Adolfson et al. (2011).

other types of regime-switches.

The rest of the paper is organized as follows. Section 2 describes the baseline model, while Section 3 discusses the formulation of optimal policy in the loose commitment framework. Section 4 describes the estimation procedure, and Section 5 outlines the main results. Section 6 provides some concluding remarks. Additional details regarding the estimation procedure and robustness exercises are contained in an appendix.

2 The model

As discussed in the introduction, the distinctive feature of our model concerns the way monetary policy is designed. The underlying economy is instead described by a standard system of linearized equations

$$A_{-1}x_{t-1} + A_0 x_t + A_1 E_t x_{t+1} + B v_t = 0 \tag{1}$$

where x_t denotes a vector of endogenous variables, v_t is a vector of zero-mean, serially uncorrelated, normally distributed exogenous disturbances, and A_{-1}, A_0, A_1 and B are matrices whose entries depend (non-linearly) on the model's structural parameters. The term E_t denotes the rational expectations operator, conditional on the information up to time t.

The analysis is conducted within the model of Smets and Wouters (2007). The model, based on the earlier work by Christiano et al. (2005), includes monopolistic competition in the goods and labor market, nominal frictions in the form of sticky price and wage settings, allowing for dynamic inflation indexation.[7] It also features several real rigidities – habit formation in consumption, investment adjustment costs, variable capital utilization, and fixed costs in production. The model describes the behavior of 14 endogenous variables: output (y_t), consumption (c_t), investment (i_t), labor (l_t), the capital stock (k_t), with variable utilization rate (z_t) and associated capital services (k_t^s), the wage rate (w_t), the rental rate of capital (r_t^k), the nominal interest rate (r_t), the value of capital (q_t), price inflation (π_t), and measures of price-markups (μ_t^p) and wage-markups (μ_t^w). The model dynamics are driven by six structural shocks: two shocks – a price-markup (e_t^p) and wage-markup (e_t^w) shock – follow an ARMA(1,1) process, while the remaining four shocks – total factor productivity (e_t^a), risk-premium (e_t^b), investment-specific technology shock (e_t^i) and government spending shock (e_t^g)

[7] Monopolistic competition is modeled following Kimball (1995), while the formulations of price and wage stickiness follow Yun (1996) and Erceg et al. (2000).

– follow an AR(1) process. All the shocks are uncorrelated, with the exception of a positive correlation between government spending and productivity shocks, i.e. $Corr(e_t^g, e_t^a) = \rho_{ag} > 0$.[8] The model can be cast into eq. (1) defining x_t as a 22x1 vector containing all the variables described above (i.e. endogenous variables, structural shocks and corresponding MA components), and v_t as a vector containing the i.i.d. innovations to the structural shocks.

We depart from the original SW formulation in two fundamental ways. First, we account for changes in the volatility of the exogenous shocks. Recent studies (see Primiceri (2005), Sims and Zha (2006) and Cogley and Sargent (2006) among others) find that exogenous shocks have displayed a high degree of heteroskedasticity. For our purposes, ignoring this heteroskedasticity would potentially lead to inaccurate inference: the time variation in the volatility of the shocks could be mistakenly attributed to policy re-optimization episodes, thus biasing our measure of credibility. To deal with this issue, we model heteroskedasticity as a Markov-switching process

$$v_t \sim N(0, Q_{s_t^{vo}}),$$

where the variance-covariance matrix $Q_{s_t^{vo}}$ depends on an unobservable state $s_t^{vo} \in \{h, l\}$, that differentiates between high (h) and low (l) volatility regimes. While in principle one could consider a process with more states, a specification with two states has been found to fit the data best in estimated regime-switching DSGE models [see Liu et al. (2011) and Bianchi (2012)]. The Markov-switching process for volatilities (s_t^{vo}) evolves independently from the regime-switching process that governs re-optimizations (s_t, described in detail in the next section). The transition matrix for s_t^{vo} is given by

$$P^{vo} = \begin{bmatrix} p_h & (1-p_h) \\ (1-p_l) & p_l \end{bmatrix}$$

The second and more important departure from the original SW model concerns the behavior of the central bank. Rather than including a (Taylor-type) interest rate rule, and the associated monetary policy shock, we explicitly solve the central bank's decision problem. As discussed in the next section, this allows us to describe the central bank's commitment problem, and to characterize the nature of policy re-optimizations. Throughout our analysis, it is assumed that the central bank's objectives are described by a (period) quadratic loss function

$$x_t' W x_t \equiv \pi_t^2 + w_y \widetilde{y}_t^2 + w_r (r_t - r_{t-1})^2 \tag{2}$$

[8] All the variables are expressed in deviations from their steady state. For a complete description of the model, the reader is referred to the original Smets and Wouters (2007) paper.

Without loss of generality, the weight on inflation (π_t) is normalized to one so that w_y and w_r represent the weights on output gap ($\widetilde{y}_t$) and the nominal interest rate (r_t), relative to inflation. Those weights will be estimated from the data. According to equation (2), the central bank's inflation target coincides with the steady state level of inflation $\bar{\pi}$. The target for output is instead its "natural" counterpart, defined as the level of output that would prevail in the absence of nominal rigidities and mark-up shocks. This formulation is consistent with the natural rate hypothesis, i.e. that monetary policy cannot systematically affect average output. It is also consistent with the original SW specification, where because of price and wage indexation, the steady state inflation does not produce any real effect. As a result, the central bank's credibility problems do not lead to an average inflation-bias, but only to a stabilization-bias in response to economic shocks, as illustrated in Clarida et al. (1999).[9] The last term in the loss function ($w_r(r_t - r_{t-1})^2$) indicates the central bank's preference for interest rate smoothing, as supported by the recent evidence of Coibion and Gorodnichenko (2012).

A common approach in the literature is to describe the central bank behavior through simple rules, that are known for their good empirical properties. Here we adopt a similar approach, and adopt a simple loss function that has been shown to realistically describe the behavior of the Federal Reserve (see e.g. Rudebusch and Svensson (1999), or more recently Ilbas (2010) and Adolfson et al. (2011)). We then investigate to what extent the central bank was credible in implementing such empirically plausible objectives. One alternative would be to consider a theoretical loss function, consistent with the representative agent's preferences [see e.g. Benigno and Woodford (2012)]. However, there are several reasons why the central bank's objectives may not reflect the preferences of the underlying society. For instance, for all practical purposes it would be infeasible to specify the central bank's goals in terms of a utility-based welfare criterion, as it would include a very high number of targets in terms of variances and covariances of different variables.[10] Also, prominent scholars like Svensson (1999) argue that a simple mandate is more robust to model and parameter uncertainty than a complicated theoretical loss function.

[9] Notice however that since the markup-up shocks are allowed to follow an ARMA(1,1), the stabilization-bias could potentially be very persistent, and closely resemble an average inflation bias.

[10] For instance, the utility-based welfare criterion in the SW model contains more than 90 target variables. We verified that a version of the model with such a welfare criterion provides a much poorer empirical fit. Also, we verified that our simple loss function approximates well the preferences of the representative agents, leading to minimal losses in comparison to the ideal Ramsey plan. Results are available upon request to the authors.

3 The Loose Commitment Framework

The system of equations (1) implies that current variables (x_t) depend on expectations about future variables ($E_t x_{t+1}$). This gives rise to the time-inconsistency problem at the core of our analysis. The central bank's plans about the future course of policy could indeed have an immediate effect on the economy, as long as those plans are embedded into the private sector expectations. Having reaped the gains from affecting expectations, the central bank has an ex-post incentive to disregard previous plans, and freely set its policy instruments. The literature has typically considered one of two dichotomous cases to deal with the time-inconsistency problem: commitment or discretion. In this paper we use a more general setting that nests both these frameworks. Following Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010), it is assumed that the central bank has access to a *loose commitment* technology. In particular, the central bank is able to commit, but it occasionally succumbs to the temptation to revise its plans. Both the central bank and private agents are aware of the possibility of policy re-optimizations and take it into account when forming their expectations.

More formally, at any point in time monetary policy can switch between two alternative scenarios, governed by the unobservable state $s_t \in \{0; 1\}$. If $s_t = 1$, previous commitments are honored. Instead, if $s_t = 0$, the central bank makes a new (state-contingent) plan over the infinite future, disregarding all the commitments made in the past. The variable s_t evolves according to a two-state stochastic process, with transition matrix

$$P = \begin{bmatrix} Pr(s_t = 1|s_{t-1} = 1) & Pr(s_t = 0|s_{t-1} = 1) \\ Pr(s_t = 1|s_{t-1} = 0) & Pr(s_t = 0|s_{t-1} = 0) \end{bmatrix} = \begin{bmatrix} \gamma & 1-\gamma \\ \gamma & 1-\gamma \end{bmatrix}$$

and where $\gamma \in [0, 1]$. The limiting case where previous promises are always honored (i.e. $\gamma = 1$) coincides with the canonical commitment setting. Instead, if $\gamma = 0$ the central bank operates under discretion.

Notice that s_t constitutes an independent switching process, where $Pr(s_t = j|s_{t-1} = 1) = Pr(s_t = j|s_{t-1} = 0)$. In other words, honoring commitments in a given period does not make a policy re-optimization (or continuing plans) more or less likely in the future.[11] As a result, there is a direct and intuitive mapping between a single parameter of the model – the probability of commitment γ – and the degree of central bank's credibility: the higher

[11] In standard monetary regime-switching models, a process like s_t displays instead some degree of persistence, capturing the fact that once a monetary regime (e.g. *Dovish* or *Hawkish*) takes office, it is likely to remain in power for a prolonged period of time.

is γ, the more credible is the central bank.[12]

As is common in the DSGE regime-switching literature, we maintain the assumption that s_t is an exogenous process. Accordingly, we are interpreting policy re-optimizations as exogenous shocks influencing the behavior of the central bank, in a similar fashion to common monetary policy shocks. The validity of this assumption could be questioned on the grounds that central banks may deliberately choose to abandon their commitments in specific situations, e.g. when unusually large shocks hit the economy.[13] That criticism would be especially valid if the central bank had to commit to strict targets for its variables of interest: it would be very costly, if not impossible, to achieve those targets in turbulent times. In our setting, however, the central bank has more flexibility. This is because the responses to the shocks v_t are always part of the central bank's state-contingent plan.[14] In that case, it is not obvious that deviations from the original plan should depend on the occurrence of particular shocks. In section 5.3 we formally check this conjecture. Granger causality tests show the model state variables do not have a statistically significant predictive power for the estimated (smoothed) probability of re-optimization, thus supporting the validity of our assumption.

3.1 The central bank's problem and policy re-optimizations

The problem of the central bank when making a new plan can be written as

$$x'_{-1}Vx_{-1} + d = \min_{\{x_t\}_{t=0}^{\infty}} E_{-1} \sum_{t=0}^{\infty} (\beta\gamma)^t [x'_t W x_t + \beta(1-\gamma)(x'_t V x_t + d)] \tag{3}$$

$$s.t.\ \ A_{-1}x_{t-1} + A_0 x_t + \gamma A_1 E_t x_{t+1} + (1-\gamma) A_1 E_t x_{t+1}^{reop} + Bv_t = 0 \ \ \forall t \tag{4}$$

The terms $x'_{t-1}Vx_{t-1} + d$ summarize the value function at time t. Since the problem is linear-quadratic, the value function is given by a quadratic term in the state variables x_{t-1}, and a constant term d reflecting the stochastic nature of the problem. The objective function

[12]Such a mapping would be less straightforward if we were to adopt a more general Markov-Switching process. In that case, it would indeed be necessary to distinguish between conditional and unconditional measures of credibility, that would depend on two regime-switching probabilities. Also, following that approach would significantly complicate the solution to the central bank problem.

[13]Admittedly, it would be ideal to let policy re-optimizations to depend on the model's state variables, as in Debortoli and Nunes (2010). That specification, however, requires adopting a non-linear solution method, that would make our estimation exercise infeasible.

[14]As a result, in our model the central bank does not face a tradeoff between credibility and flexibility, as considered e.g. in Lohmann (1992).

is given by an infinite sum discounted at the rate $\beta\gamma$ summarizing the history in which re-optimizations never occur. Each term in the summation is composed of two parts. The first part $x_t'Wx_t$ is the period loss function. The second part $\beta(1-\gamma)(x_t'Vx_t+d)$ indicates the value the policymaker obtains if a re-optimization occurs in the next period. The sequence of constraints (4) corresponds to the structural equations (1), with the only exception that expectations of future variables are expressed as the weighted average between two terms: the allocations prevailing when previous plans are honored (x_{t+1}), and those prevailing when a re-optimization occurs (x_{t+1}^{reop}). This reflects the fact that private agents are aware of the possibility of policy re-optimizations, and take this possibility into account when forming their expectations.[15]

We solve for the Markov-Perfect equilibrium of the above economy, where the equilibrium choices x_{t+1}^{reop} only depend on natural state-variables. We can thus express the expectations related to the re-optimizations state as $E_t x_{t+1}^{reop} = \tilde{F}x_t$, where the matrix $\tilde{F}$ is a matrix of coefficients to be determined, and is taken as given by the central bank.[16]

The presence of the (unknown) matrix $\tilde{F}$ complicates the solution of the central bank problem. For any given $\tilde{F}$, the solution to the central bank's problem can be derived using the recursive techniques described in Kydland and Prescott (1980) and Marcet and Marimon (2011). The associated system of first-order conditions could then be solved using a standard solution algorithm for rational expectations models [e.g. Sims (2002)]. However, a Markov-Perfect equilibrium additionally requires the matrix $\tilde{F}$ to be consistent with the policies actually implemented by the central bank. This involves the solution of a fixed point problem.[17]

The solution to the central bank's problem takes the form:

$$\begin{bmatrix} x_t \\ \lambda_t \end{bmatrix} = F_{s_t} \begin{bmatrix} x_{t-1} \\ \lambda_{t-1} \end{bmatrix} + Gv_t \tag{5}$$

where λ_t is a vector of Lagrange multipliers attached to the constraints (4), with initial

[15]To simplify the notation, we have dropped regime dependence and replaced $x_{t+1}|s_t=0$ with the more compact term x_{t+1}^{reop}.

[16]We are therefore ruling out the possibility of reputation and coordination mechanism as those described for instance in Walsh (1995).

[17]Methods to solve for Markov-Perfect equilibria are described in Backus and Driffill (1985), Söderlind (1999), and Dennis (2007). Debortoli et al. (2012) extended those methodologies to analyze loose commitment problems in large-scale models. The algorithm makes use of the fact that in equilibrium it must be that $x_t^{reop} = F^{xx}x_{t-1} + G_x v_t$. Rational expectations then implies that $E_t x_{t+1}^{reop} = F_{xx}x_t$. Thus, one must solve the fixed point problem such that $F_{xx} = \tilde{F}$.

condition $\lambda_{-1} = 0$. In particular, the Lagrange multipliers λ_{t-1} contain a linear combination of past shocks $\{v_{t-1}, v_{t-2}, \ldots, v_0\}$, summarizing the commitments made by the central bank before period t.[18] A policy re-optimization implies that previous commitments are disregarded, so that the current variables are not affected by λ_{t-1} – or equivalently as if λ_{t-1} were reset to zero. Therefore, the effects of policy re-optimizations can be described by the state dependent matrices

$$F_{(s_t=1)} = \begin{bmatrix} F^{xx} & F^{x\lambda} \\ F^{\lambda x} & F^{\lambda\lambda} \end{bmatrix} \qquad F_{(s_t=0)} = \begin{bmatrix} F^{xx} & 0 \\ F^{\lambda x} & 0 \end{bmatrix}. \tag{6}$$

In particular, notice that the unobservable state s_t only affects the columns of the matrices F_{s_t} describing the responses to λ_{t-1}. On the contrary, the policy responses to the state variables x_{t-1} and to the shocks v_t remain the same, regardless of whether the central bank re-optimizes or not.

The above formulation highlights the nature of policy re-optimizations, and provides an intuition for how re-optimizations can be identified in in the data. From a reduced-form perspective, a policy re-optimization implies that macroeconomic variables cease to depend on a subset of the historical data – summarized in our model by the vector λ_{t-1} – and thus display a lower degree of persistence. From a more structural perspective, policy re-optimizations could instead be viewed as a particular type of monetary policy shock defined as

$$e_t^{reop} \equiv x_t^{reop} - x_t = -F^{x\lambda}\lambda_{t-1}. \tag{7}$$

Notice, however, that while the timing of these "re-optimization shocks" is exogenous – as for standard monetary policy shocks – the sign and magnitude of their impact are instead endogenous, and depend on the history of past shocks summarized by λ_{t-1}. For example, if a re-optimization shock occurs when λ_{t-1} is large (small) the shock will have a large (small) impact on the economy. Thus, the effects of policy re-optimizations change over time. As discussed in section 5.3, this has implications for how the specific re-optimization episodes are identified in the data.

Our setting bears many similarities to some of the recent monetary regime-switching models [see e.g. Davig and Leeper (2007), Farmer et al. (2009), Liu et al. (2011) and Bianchi (2012)]. As in those models, an exogenous shock governs switches from one regime to another, where the conduct of monetary policy is different. And as in those models,

[18] For this reason the Lagrange multipliers λ_t are often referred to as co-state variables.

because of the forward-looking nature of economic agents, what happens under a certain regime depends on what the agents expect is going to happen under alternative regimes, and on the probability of switching to a new regime. This can be noticed from the fact that probability of commitment γ not only enters the transition matrix P, but it also affects the matrices F_{s_t} and G.

An important difference with respect to the existing regime-switching model is that our regimes are described by the same structural parameters.[19] In other words, modeling policy re-optimization does not require introducing any additional parameters, besides the switching probability γ. As indicated by equation (6), policy re-optimizations only impose specific zero-restrictions on the model's law of motion. These restrictions differentiate our policy re-optimizations from other types of regime-switches, such as switches in Taylor rule parameters or changes in the inflation target that are typically considered in the literature. In fact, commitment problems could be viewed as one of the causes of the monetary regime switches typically found in these studies. Other possible candidates are changes in the central bank's preferences – e.g. between a "Hawkish" to a "Dovish" monetary regime – or changes in other structural features of the economy.[20] In this respect, our approach could be extended to incorporate these features and analyze the deep sources of regime switches. While our analysis abstracts from these considerations, our inferred re-optimizations episodes could be related to changes in members of the FOMC, or to changes in the operating procedure of the Federal Reserve, as discussed in section 5.2.

4 Estimation

For estimation purposes, we combine the law of motion (5) with a system of observation equations

$$x_t^{obs} = A + Hx_t \tag{8}$$

where x_t^{obs} denotes the observable variables, the matrix H maps the state variables into the observables, and A is a vector of constants. For comparability with SW, the model is estimated using the same seven quarterly US time series as observable variables: the log

[19]Note that since the same type of central bank is in power, our regime-switching framework does not display an indeterminacy problem as described in Farmer et al. (2009). There would need to be an additional layer of uncertainty or mismeasurement to give rise to the possibility of indeterminacy.

[20]See for instance, Lakdawala (2013) for a continuous time-varying preference approach and the discussion in Debortoli and Nunes (2013), comparing the implications of switches in central banks' preferences with changes in Taylor-rule parameters within a baseline New Keynesian model.

difference of real GDP, real consumption, real investment, the real wage, log hours worked, the log difference of the GDP deflator and the federal funds rate. The monetary policy shock in SW is replace by an i.i.d. measurement error, so that the number of shocks is the same as the number of observable variables. This is required to ensure that we have enough shocks to avoid the stochastic singularity problem in evaluating the likelihood.

The estimation is carried out using a Bayesian likelihood approach. The likelihood function for a standard DSGE model can be evaluated using the standard Kalman Filter. Given the regime-switching nature of our model, the standard Kalman filter needs to be augmented with the Hamilton (1989) filter, following the procedure described in Kim and Nelson (1999). The likelihood function is then combined with the prior to obtain the posterior distribution. The detailed steps in evaluating the likelihood function, together with the outline of the Bayesian estimation algorithm are provided in the appendix.

We estimate a total 42 parameters, while fixing 6 parameters.[21] Table 1-3 summarize the priors used for the estimated parameters. For the common structural parameters as well as for the shock processes we use the same priors used in SW.[22] Regarding the three new parameters describing the central bank behavior, we proceed as follows. For the probability of commitment γ we use a uniform prior on the interval [0,1], as we do not want to impose any restrictive prior beliefs about whether the optimal policy is conducted in a setting that is closer to commitment or discretion. Thus the posterior of γ will be entirely determined by the data. For the loss function parameters w_y and w_r we instead choose fairly loose Gamma priors.

The data sample in the baseline estimation runs from 1966:Q1-2012:Q2. There may be concern about using the data from 2007 onwards that includes the financial crisis and periods where the zero lower bound was binding. It is important to note that our specification remains agnostic about the specific monetary policy instrument used by the central bank to implement its policy. Thus we can use data that goes through the financial crisis, which would be a challenge if monetary policy were described by a rule for the nominal interest rate. As a robustness check, we estimate the posterior mode of the model where the data sample does not include the financial crisis and find very similar results. Additionally we

[21]As in SW, the depreciation rate δ is fixed at .025, spending-GDP ratio g_y at 18%, steady-state markup in the labor market at 1.5 and curvature parameters in the goods and labor markets at 10. We additionally fix the real wage elasticity of labor supply at $\sigma_l = 1$, as that parameter is estimated imprecisely in the original SW paper, and fixing it greatly improves the convergence of our estimation algorithm. In Appendix A-1 we show that our results are robust to adopting different values.

[22]In our model the regime switching variance specification introduces two values for the standard deviation of each shock, as well as two parameters of the transition matrix (P^{vo}).

estimate the model using long-term interest rates (instead of the fed funds rate) which did not face the zero lower bound constraint. All these results are discussed below.

5 Results

5.1 Parameter Estimates

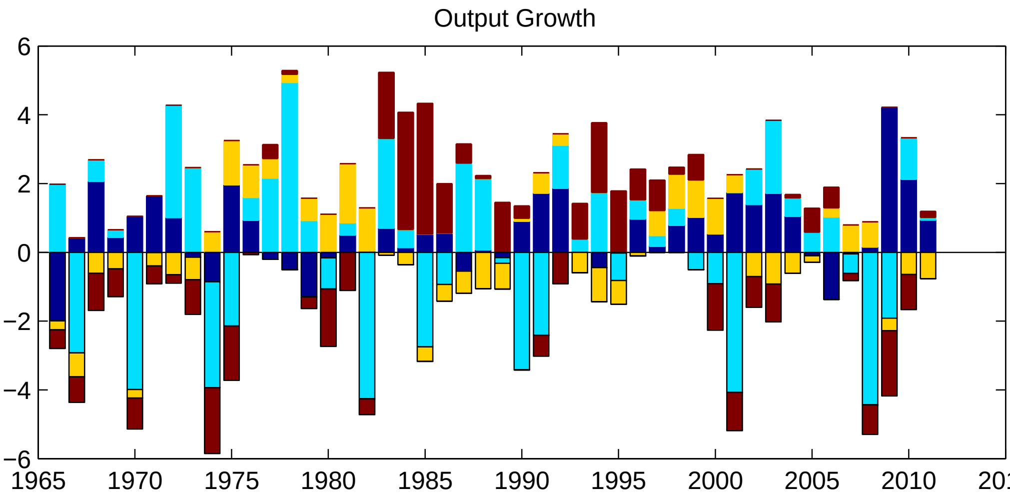
Table 1 reports the priors and the posterior mode, mean, 5^{th} and 95^{th} percentiles for the structural parameters. Despite the different modeling choices and sample data, our estimates are very similar to those obtained in SW. Similar considerations hold for the parameters of the shock processes, as summarized in Table 2. The standard deviations are not directly comparable to SW, since we allow them to switch over time. But the weighted average of our estimated standard deviations across the two regimes is very similar to the SW estimates. The parameters related to the price-markup shock process are somewhat different, since both the autoregressive parameter ρ_p and the MA parameter μ_p are estimated to be larger than in SW. In this respect our findings are closer to the results of Justiniano and Primiceri (2008), who also find different persistence of price-markup shocks, and ultimately adopt an i.i.d. specification.[23] In general, and as shown in Figure 1, the contributions of the different shocks to historical fluctuations is consistent with the original SW findings. Markup shocks play a major role in explaining the historical behavior of inflation, while demand-type shocks are the most important drivers of output fluctuations. Additionally, as shown in Appendix A-2 our model is comparable with the original SW model in fitting the data as measured by the marginal likelihood.

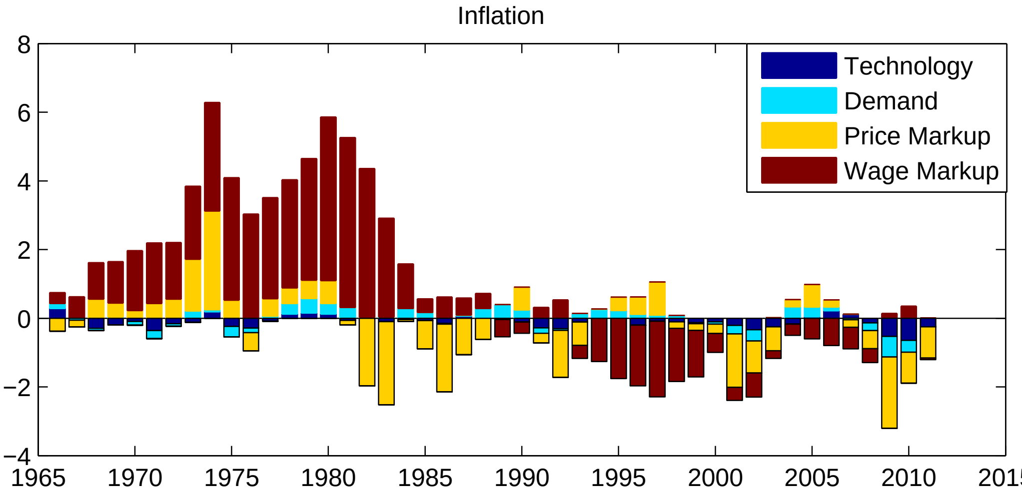
Let's now turn our attention to the parameters describing the central bank's behavior, summarized in Table 3. Our estimates for the two weight parameters in the central bank's loss function fall in the estimated range in the literature. Available estimates of the weight tend to be very sensitive with respect to the particular model and data sample. For instance, estimates range from values of $[w_r = 0.005, w_y = 0.002]$ in Favero and Rovelli (2003) to the values of $[w_r = 4.517, w_y = 2.941]$ in Dennis (2006). The posterior mode of our estimates $[w_r = 1.824, w_y = 0.015]$ falls in the middle of this range.[24]

[23] As discussed in Appendix A-1, when we estimated the posterior mode of the model using an AR(1) specification we found a very low coefficient on the auto-regressive term similar to the i.i.d. setup of Justiniano and Primiceri (2008), while all the other estimates remain very similar to the benchmark case.

[24] Allowing for an additional term in the loss function that involves interest rate variability tends to reduce the estimate of w_r, see e.g. Ilbas (2010).

Figure 1: Historical Decomposition





Note: The figure reports the contribution of different shocks to the historical fluctuations of output-growth and inflation. Demand shocks denote the combination of risk-premium, investment-specific and government expenditure shocks.

Table 1: Prior and Posterior Distribution of Structural Parameters

		Distr.	Prior Mean	St. Dev	Mode	Mean	Posterior 5%	95%
$\bar{l}$	St. State Labor	Normal	0.000	2.000	0.243	0.234	0.213	0.253
$\bar{\pi}$	St. State Inflation	Gamma	0.620	0.100	0.742	0.754	0.647	0.874
$\bar{\gamma}$	Growth Rate	Normal	0.400	0.100	0.182	0.184	0.148	0.221
$\bar{\beta}$	Discount Factor	Gamma	0.250	0.100	0.223	0.241	0.129	0.365
α	Capital Income Share	Beta	0.300	0.050	0.192	0.192	0.166	0.219
ψ	Capital Cap. Utilization	Normal	0.500	0.150	0.697	0.686	0.525	0.831
φ	Capital Adj. Cost	Normal	4.000	1.500	6.316	6.532	5.084	8.125
σ_c	Risk Aversion	Normal	1.500	0.370	1.771	1.766	1.488	2.091
h	Habit Persistence	Beta	0.700	0.100	0.765	0.765	0.700	0.821
Φ	Fixed Cost	Normal	1.250	0.120	1.614	1.600	1.490	1.714
ι_w	Wage Indexation	Beta	0.500	0.150	0.500	0.527	0.316	0.734
ι_p	Price Indexation	Beta	0.500	0.150	0.809	0.809	0.689	0.908
ξ_p	Price Stickiness	Beta	0.500	0.100	0.783	0.775	0.727	0.822
ξ_w	Wage Stickiness	Beta	0.500	0.100	0.626	0.619	0.539	0.695

Note: The table reports the prior and the estimated posterior mean, mode, 5^{th} and 95^{th} percentiles for the model structural parameters

Most interestingly, the last row of Table 3 shows that the posterior mode for the probability of commitment γ equals 0.81. Figure 2 which plots the entire marginal posterior distribution shows that it is quite precisely estimated. The main implication of our result is that the data clearly rejects both the commonly used setups of commitment ($\gamma = 1$) and discretion ($\gamma = 0$). This result is robust to a variety of different specifications. For instance, and as discussed more extensively in Appendix A-1, we considered different subsamples (e.g. the pre-financial crisis sample) and different measures of the interest rates (e.g. the Long-Term interest rate) to address potential concerns related to a binding zero-lower bound on the Federal Funds rate. We also used different priors for γ (Beta prior rather than Uniform) to have an uninformative prior even if credibility is measured according to a different metric. In all the cases considered, the estimated values γ remains close to 0.8, thus rejecting both commitment and discretion. The same conclusion holds when we separately estimate the alternative commitment settings, and compare the corresponding marginal likelihoods – see Appendix A-2.

The specific value of γ *per se* is not indicative of whether the central bank has a high or low level of credibility. On the one hand, a probability of commitment of 80% could be viewed as

Table 2: Prior and Posterior Distribution of shock processes

		Prior			Posterior		
	Distr.	Mean	Std. Dev	Mode	Mean	5%	95%
Standard deviations in high and low regimes							
σ_a^l	Inv Gamma	0.1	2	0.343	0.352	0.307	0.403
σ_b^l	Inv Gamma	0.1	2	0.158	0.158	0.123	0.194
σ_g^l	Inv Gamma	0.1	2	0.356	0.359	0.303	0.418
σ_I^l	Inv Gamma	0.1	2	0.412	0.415	0.350	0.488
σ_p^l	Inv Gamma	0.1	2	0.146	0.149	0.129	0.173
σ_w^l	Inv Gamma	0.1	2	0.274	0.279	0.243	0.318
σ_m^l	Inv Gamma	0.1	2	0.064	0.066	0.055	0.079
σ_a^h	Inv Gamma	0.1	2	0.643	0.652	0.562	0.759
σ_b^h	Inv Gamma	0.1	2	0.292	0.296	0.228	0.372
σ_g^h	Inv Gamma	0.1	2	0.650	0.660	0.568	0.766
σ_I^h	Inv Gamma	0.1	2	0.569	0.573	0.472	0.689
σ_p^h	Inv Gamma	0.1	2	0.221	0.227	0.194	0.264
σ_w^h	Inv Gamma	0.1	2	0.346	0.355	0.293	0.428
σ_m^h	Inv Gamma	0.1	2	0.315	0.322	0.279	0.378
$diag(P^{vo,l})$	Beta	0.8	0.16	0.934	0.916	0.858	0.961
$diag(P^{vo,h})$	Beta	0.8	0.16	0.883	0.857	0.759	0.934
MA parameters (μ) and AR parameters (ρ)							
μ_w	Beta	0.5	0.2	0.894	0.874	0.789	0.936
μ_p	Beta	0.5	0.2	0.986	0.977	0.947	0.995
ρ_{ga}	Beta	0.5	0.2	0.425	0.430	0.295	0.567
ρ_a	Beta	0.5	0.2	0.999	0.999	0.997	1.000
ρ_b	Beta	0.5	0.2	0.447	0.456	0.308	0.617
ρ_g	Beta	0.5	0.2	0.940	0.940	0.908	0.968
ρ_I	Beta	0.5	0.2	0.769	0.776	0.710	0.841
ρ_p	Beta	0.5	0.2	0.947	0.927	0.880	0.958
ρ_w	Beta	0.1	0.2	0.996	0.991	0.978	0.999

Note: The table reports the prior, and the estimated posterior mean, mode, 5^{th} and 95^{th} percentiles for the parameters describing the shock processes and the diagonal elements of transition matrix for the volatility regimes. The superscripts l and h refer to the low volatility and high volatility regimes.

Table 3: Prior and Posterior Distribution of Monetary Policy Parameters

		Prior			Posterior			
		Distr.	Mean	St. Dev	Mode	Mean	5%	95%
w_y	Output-Gap weight	Gamma	1.000	1.000	0.015	0.017	0.010	0.026
w_r	Interest rate weight	Gamma	1.000	1.000	1.824	2.248	1.403	3.288
γ	Prob. of Commitment	Uniform	0.500	0.290	0.811	0.815	0.777	0.851

Note: The table reports the prior, and the estimated posterior mean, mode, 5^{th} and 95^{th} percentiles for the parameters describing the parameters describing the central bank behavior.

sufficiently close to the ideal commitment case. On the other hand, the use of quarterly data implies that the Federal Reserve is expected to re-optimize on average once every 5-quarters, arguably a relatively short commitment horizon. Fortunately, counterfactual exercises can shed light on the actual role of commitment in our estimated model, as discussed in next section.

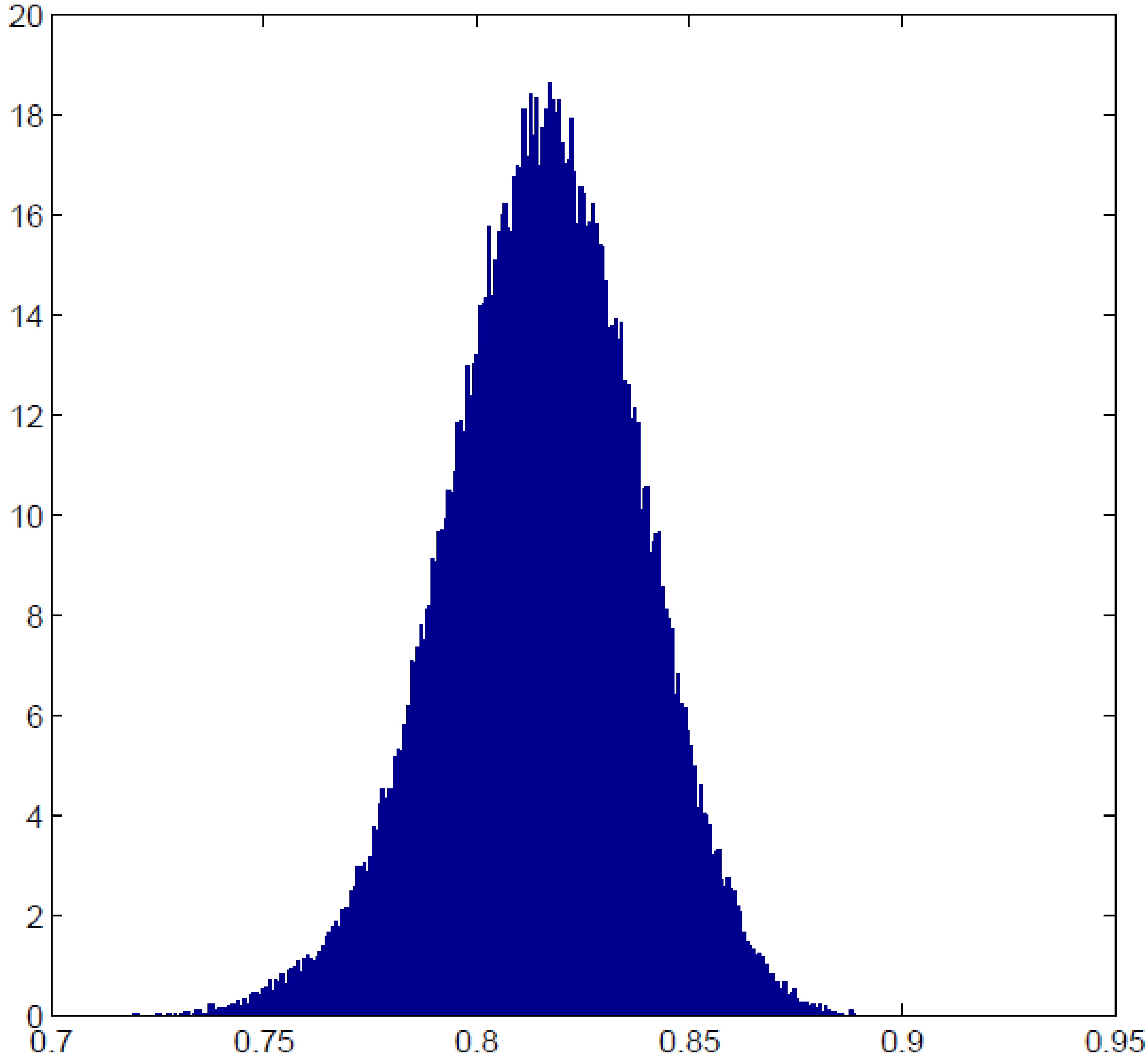
5.2 Counterfactual analysis

The main question we address in this section is what would happen under alternative commitment scenarios. To that end we perform counterfactual simulations of the model assuming that the central bank operates either under commitment ($\gamma = 1$) or under discretion ($\gamma = 0$). The remaining parameters of the model are left unchanged.

Table 4 shows how commitment affects the unconditional second moments for some relevant variables. In general, the relative standard deviations and the cross-correlations with output in a model with $\gamma = .81$ are closer to the discretion than to the commitment case. The last line of the table also reports the implied welfare losses with respect to the commitment case, measured in terms of equivalent permanent increase in the inflation rate.[25] According to that measure, the total gains of passing from discretion to commitment are equivalent to a permanent decrease in the inflation rate of 1.2% per year. Most of those gains – corresponding to a 1% permanent reduction in inflation – could still be achieved if increasing credibility from .081 to 1. We can thus conclude that the economy would behave quite differently if the central bank had perfect commitment, and thus there is still some scope to improve credibility.

[25] Such a measure is often used to gauge losses for the objective functions employed by central banks and is described, for instance, in Jensen (2002).

Figure 2: Posterior distribution of γ



Note: The figure shows the marginal posterior distribution of γ, the probability of commitment.

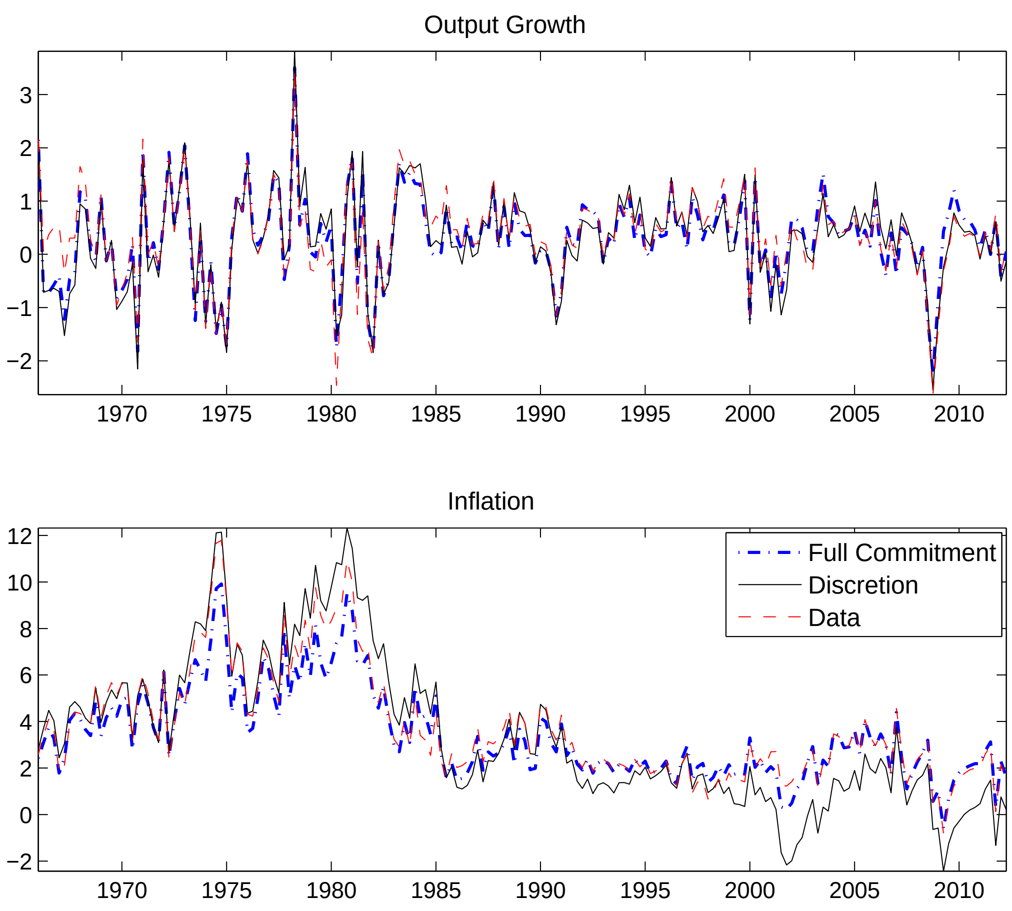
Table 4: Second Moments and Welfare

	Model			US Data
	Commitment	$\gamma = .81$	Discretion	1966-2012
Standard deviation (relative to output)				
Fed Fund Rate	0.051	0.085	0.095	0.074
Price Inflation	0.035	0.070	0.081	0.049
Wage Inflation	0.068	0.095	0.101	0.065
Hours	0.534	0.541	0.540	0.286
Cross-correlations (w.r.t. output)				
Fed Fund Rate	0.052	-0.478	-0.471	-0.571
Price Inflation	-0.046	-0.551	-0.637	-0.641
Wage Inflation	0.071	-0.319	-0.423	-0.501
Hours	0.397	0.417	0.404	0.102
Welfare Loss (% permanent change in inflation)	0.000	1.090	1.209	-

Note: The first three columns report the model unconditional moments of selected variables under alternative commitment settings. Parameters are set at the estimated posteriod mode. The last column reports the corresponding values for the US data 1966-2012.

Next we look at counterfactual paths of inflation and output growth within our sample period under the assumption of discretion and commitment. The structural shocks are fixed at the values estimated under the loose commitment setting (i.e., the baseline model). Figure 3 displays these counterfactual paths. For output growth, both the counterfactuals under commitment and discretion do not display big differences compared to the data. For inflation, in the period from the mid-1970s to early-1980s the counterfactual under discretion is closer to the data. Inflation under commitment is lower during this period, but not low enough to conclude that the Federal Reserve acting under commitment could have avoided the "Great Inflation" of the 1970s. On the contrary, since the early-1980s we see that the commitment counterfactual path tracks the inflation data almost perfectly. Under discretion, inflation would have been more volatile, especially so in the past decade. Our exercise thus suggests that starting from the early 1980s commitment largely contributed to reduce inflation fluctuations. More generally, these counterfactual exercises further confirm the idea that the dynamics of the macro variables are not easily captured by either discretion or commitment. The data is sometimes closer to commitment, sometimes closer to discretion,

Figure 3: Counterfactual analysis



Note: The figure reports counterfactual simulations under commitment and discretion, in response to the estimated structural shocks.

and sometimes does not even lie in between the commitment and discretion extremes.

5.3 Policy re-optimizations episodes

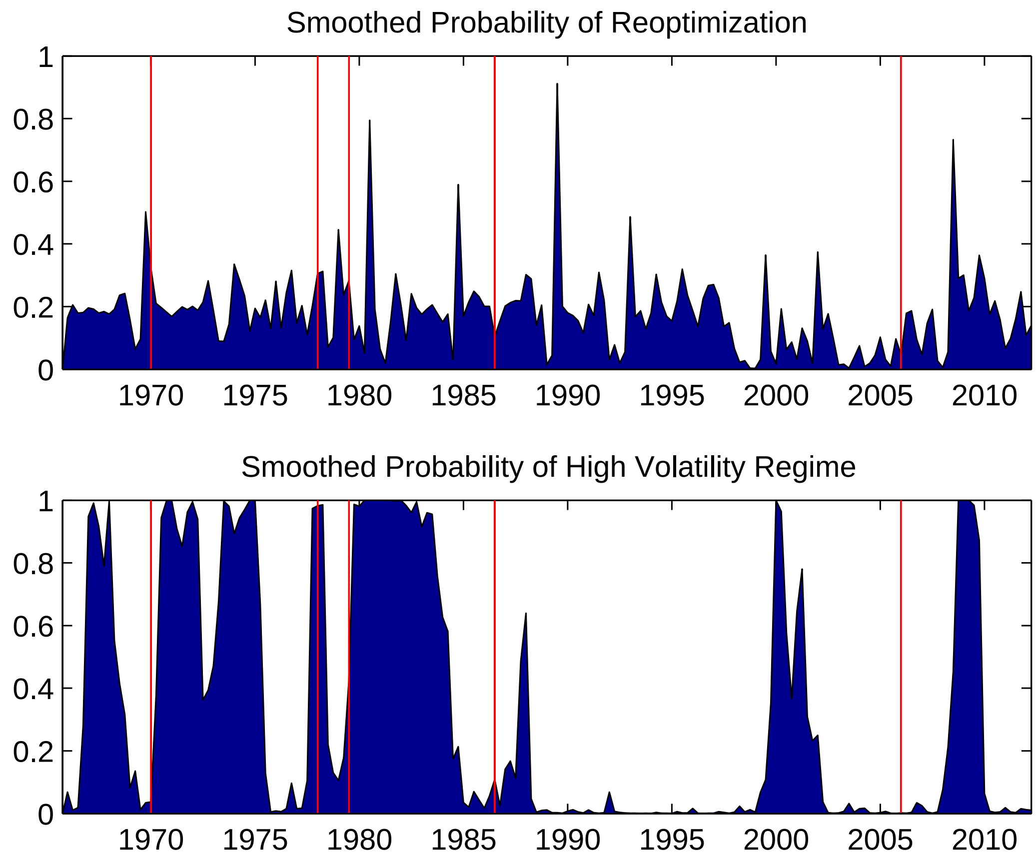
In addition to the measure of credibility discussed above, our estimates also give us an indication of specific historical episodes when the Federal Reserve likely abandoned its commitments. This can be done by looking at the evolution of the (smoothed) probabilities of re-optimization, as reported in the top panel of Figure 4. Such smoothed probabilities refer to inference about which regime was prevalent based on using all available information in the sample.

The probability of re-optimization mostly hovers around 0.2 with only a handful of spikes. At a first glance, this may appear inconsistent with our estimated value of $\gamma = 0.81$, as one would expect that the probability of re-optimization should be close to one 20% of the time, and close to zero the remaining 80%. However, and as illustrated in Section 4, our model identifies re-optimization episodes when there are large differences in the path of variables with or without re-optimizations $x_t - x_t^{reop}$.[26] When such differences are small, it is nearly impossible to distinguish re-optimizations from continuations of past plans, so that the smoothed probability remains at the unconditional average – i.e. at a level of about 0.2. One possible interpretation of our results is that during prolonged periods with moderate fluctuations, commitment plays a minor role, and it is therefore hard to find evidence in favor or against central bank's deviations from commitment.

We can isolate five dates when re-optimizations were more likely than continuations of previous plans – i.e. the probability of re-optimization exceeds 50%. Those dates are (i) 1969:Q4, (ii) 1980:Q3, (iii) 1984:Q4, (iv) 1989:3 and (v) 2008:Q3. If we lower the cutoff threshold to 40% then we get two additional dates (vi) 1979:Q1 and (vii) 1993:Q1. A natural test for the validity of our results is to contrast these dates with existing narrative accounts of the US monetary policy history. The first two episodes coincide with the appointment of new Federal Reserve Chairmen: Arthur Burns in early 1970 and Paul Volcker in mid 1979. In late 1980 there is another re-optimization, corresponding to a view that there has been a policy-reversal during 1980, and that the Volcker policy was credible and effective only from late 1980 or early 1981 (see e.g. Goodfriend and King (2005)). We see another re-optimization in 1984 which could potentially correspond to the end of the experiment of targeting non-borrowed reserves that was undertaken in the first few years under Volcker. Only two episodes

[26] For instance, Figure 8 shows those differences for output growth, inflation and the Fed Funds rate, while the estimation procedure entails accounting for this difference for all the seven observables.

Figure 4: Smoothed Probabilities: Re-optimization and High Volatility Regime



Note: The figure shows the smoothed probability of being in a re-optimization state (upper panel), and of being in the high-volatility regime (lower panel) for the posterior mode estimates. Shaded areas correspond to the NBER recessions, and the vertical solid lines indicate the appointment of a new Federal Reserve Chairman.

are identified over the 20-year long Greenspan tenure. A first re-optimization occurred in 1989, close to the "Romer and Romer" date of December 1988 (see Romer and Romer (1989)). A second re-optimization is identified in 1993. Arguably, this could be related with the major policy change of February 1994 when the Federal Reserve began explicitly releasing its target for the federal funds rate, along with statements of the committee's opinion on the direction of the economy. Those announcements were intended to convey information about future policies, as an additional tool to influence current economic outcomes. The last re-optimization is identified in 2008, when the Federal Reserve started adopting unusual policy decisions like the purchases of mortgage-backed securities and other long-term financial assets. Thus overall it appears that some of our dates align with changes in Federal Reserve chairmen while others correspond to changes in operating procedures of the Federal Reserve.

Moreover, there does not seem to be any systematic correspondence between re-optimizations and recessions, or switches in the volatility regime. This can be seen in the bottom panel of figure 4, showing the smoothed probabilities of being in a high volatility regime. The identified periods of high-volatility are consistent with canonical analyses of US business-cycles, but are not aligned with our re-optimization episodes. The 70s and the early 80s are characterized by long and recurrent episodes of high-volatility. The probability of high-volatility surges in correspondence with well-known oil shock episodes: the OPEC oil embargo of 1973-1974, the Iranian revolution of 1978-1979 and Iran-Iraq war initiated in 1980.[27] From 1984 onwards, the economy entered a long period with low-volatility – the Great-Moderation – interrupted by the bursting of the dot-com bubble in 2000, and by the events in the aftermath of September 11, 2001. Finally, periods with high-volatility are clearly identified in correspondence with the Great-Recession and financial crisis of 2008-2009.

There may also be a concern that the switching is actually driven by the state of the economy. We address this concern showing that the observable variables of the model do not help forecast changes in the probability of re-optimization. More specifically, we run Granger Causality tests regressing the smoothed probability on four of its lags, and four lags of the seven macro variables. Results are reported in Table 5. The first row labeled "All" considers the restriction that jointly all the observed variables have no forecasting power for the probability of commitment. The p value shows that we cannot reject this at the 10% level. The next rows show the tests of whether individually each of the variables can forecast the probability of re-optimization. At the 10% level we fail to reject for all the variables except for investment where the p value is extremely close to 0.1. These tests suggest that

[27] See for example the recent historical survey of Hamilton (2011).

our assumption about exogenous switching appears to be a reasonable one.

Table 5: Granger Causality Test

	F statistic	p value
All	1.3993	0.1038
GDP	1.1258	0.3466
Consumption	0.2326	0.9197
Investment	2.0064	0.0965
Wage	1.4927	0.2072
Hours	0.9282	0.4493
Inflation	0.6068	0.6583
Fed Funds	1.1415	0.3393

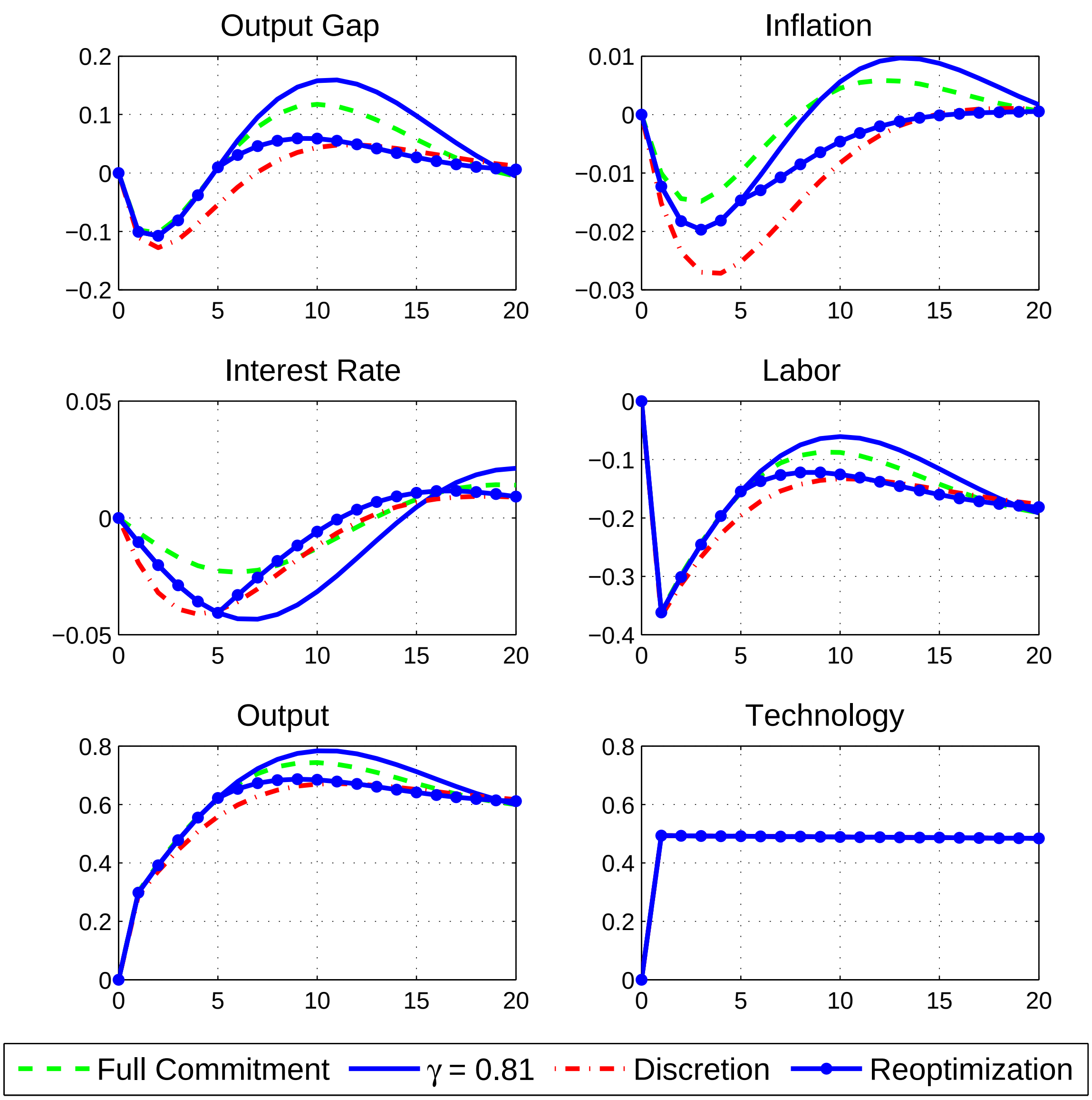
Note: The table reports the F-values and p-values of Granger causality tests. The unrestricted regression involves regressing the smoothed probability on four of its lags and four lags of all the seven macro variables, and the restricted regression imposes zeros on the coefficients of those macro variables.

5.4 What are the effects of policy re-optimizations?

As discussed earlier, the effects of a re-optimization are state-dependent, in the sense that they crucially depend on the history of economic shocks preceding the re-optimization period. Figures 5 - 7 illustrate this phenomenon showing the impulse responses to a technology, government spending and wage markup shock. The solid line shows the path under the assumption that a re-optimization never occurs (even though agents expect it to occur with probability $1-\gamma$). The line with 'dots' refers instead to the scenario where a re-optimization occurs after 5 quarters, but not after that. The difference between the two lines thus measures the effects of a policy re-optimization that occurs in period $t = 5$. A few interesting observations stand out. A re-optimization that occurs after a technology shock involves a reduction in inflation relative to continuing past plans. Whereas a re-optimization occurring after a markup shock involves an increase in inflation relative to continuing past plans. Similarly, the response of the interest rate and output gap to re-optimization shocks is also dependent on which shock has occurred previously.

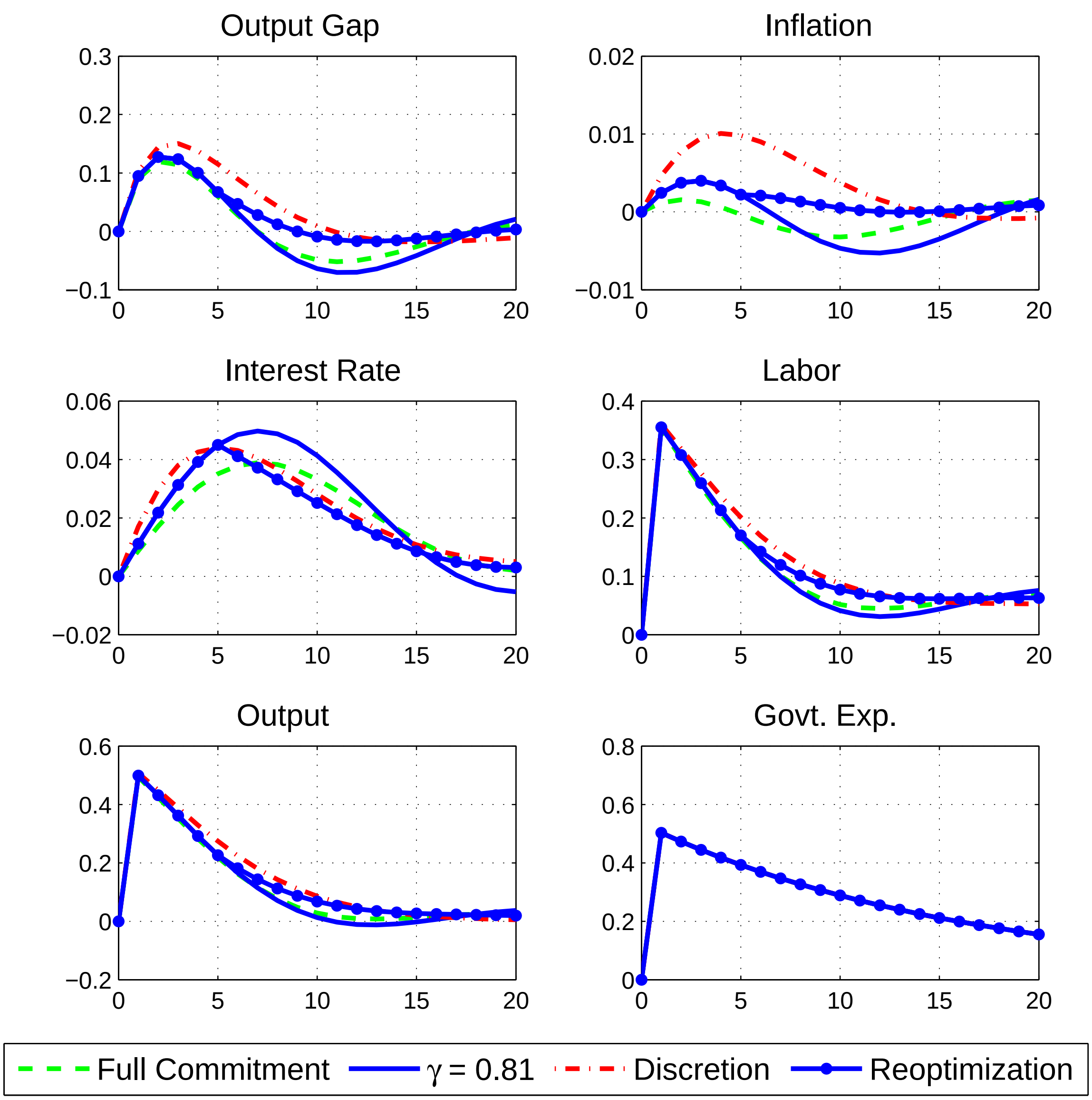
For comparability purposes, Figures 5 - 7 also report the commitment (dashed green line) and discretion (dashed - dotted red line) cases. It can be noticed that the responses under loose commitment (blue lines) do not always lie in between the discretion and commitment

Figure 5: Impulse responses to a Technology Shock



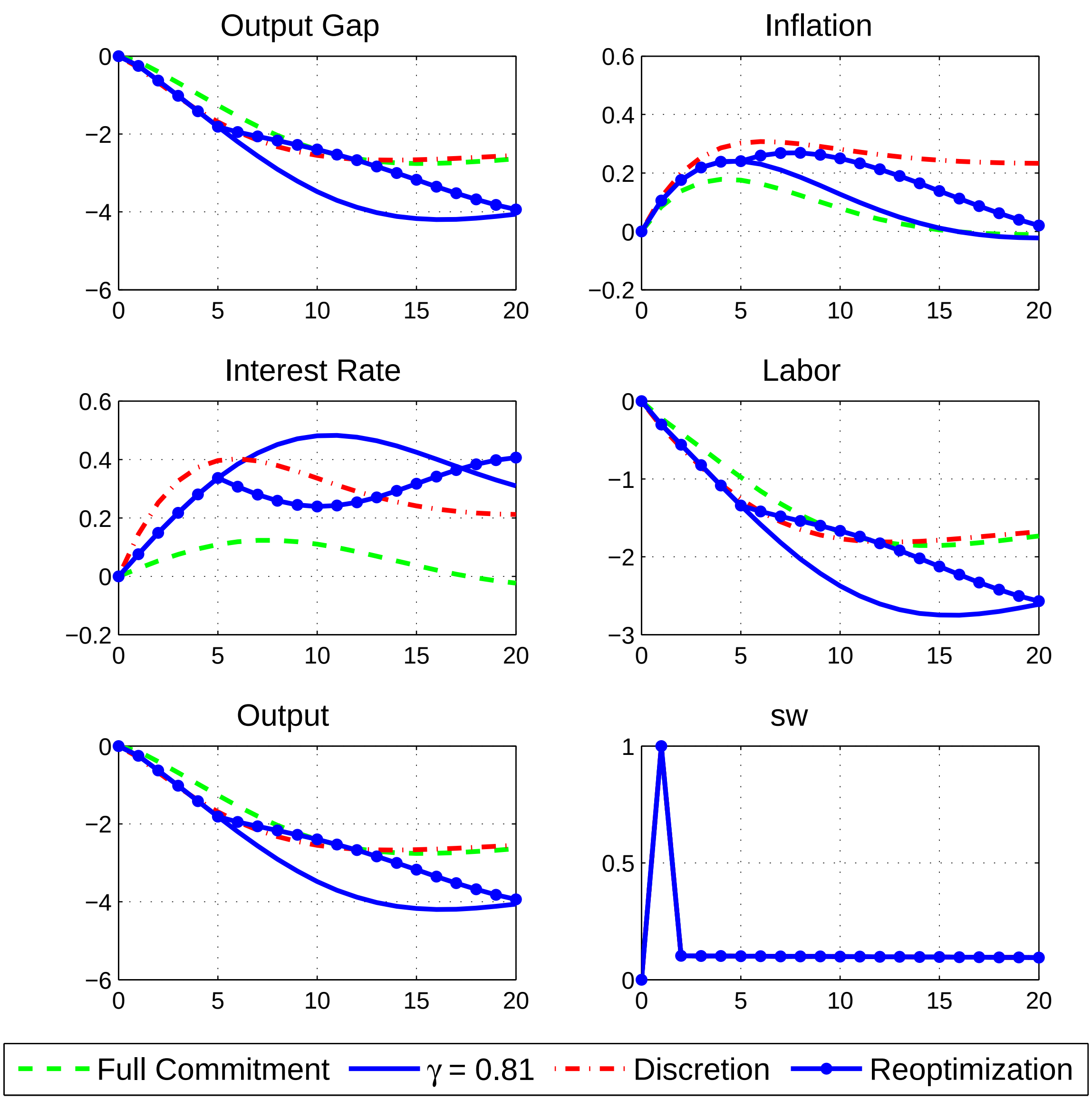
Note: Impulse responses to a 1 standard deviation shock under alternative commitment settings. The line with "dots" indicates the responses under "loose commitment", assuming that a policy re-optimization occurs after 5 quarters, and there is no policy re-optimization thereafter.

Figure 6: Impulse responses to a Demand Shock



Note: Impulse responses to a 1 standard deviation government expenditure shock under alternative commitment settings. The line with "dots" indicates the responses under "loose commitment", assuming that a policy re-optimization occurs after 5 quarters, and there is no policy re-optimization thereafter.

Figure 7: Impulse responses to a Wage-Markup Shock



Note: Impulse responses to a 1 standard deviation wage markup shock under alternative commitment settings. The line with "dots" indicates the responses under "loose commitment", assuming that a policy re-optimization occurs after 5 quarters, and there is no policy re-optimization thereafter.

cases. This is because there is uncertainty about the timing of future re-optimizations, a feature that is unique to our framework.

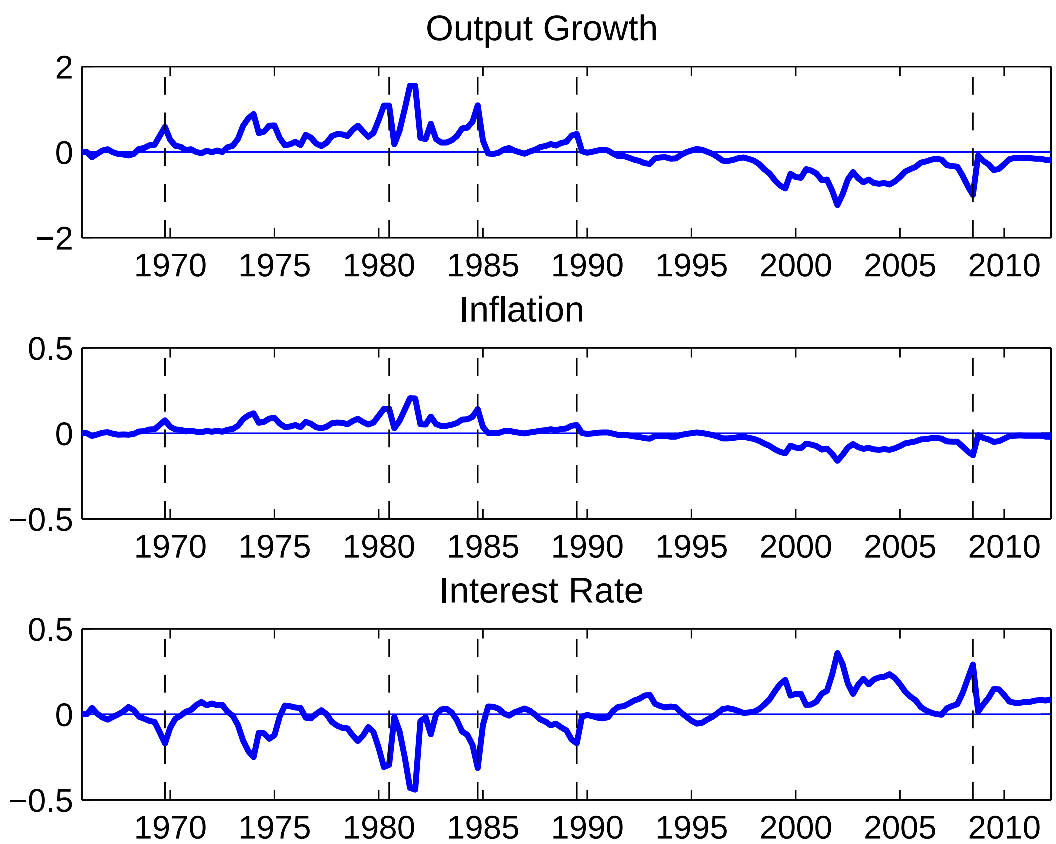
Our re-optimizations could also be viewed as a particular class of monetary policy shocks. Within our model, a deviation from previous commitment, like a generic monetary policy shock, constitutes an exogenous and unanticipated change in the course of policy. But there is an important difference between policy re-optimizations and generic monetary shocks. For example, suppose the economy was hit by a sequence of increases in oil prices, and that the Federal Reserve had committed to keep the interest rate high over a certain horizon. In that case, a policy re-optimization would bring about a more expansionary policy than expected. On the contrary, in an economy affected by negative demand shocks, the central bank would commit to keep the interest rate low, and and a re-optimization would then be associated with an unanticipated contractionary policy. Thus, whether a re-optimization has a positive or a negative effect depends on the entire sequence of shocks previously experienced by the economy.

It then seems useful to analyze the effects of re-optimizations over our sample period. To that end, figure 8 illustrates the effects of deviating from a promised plan on a given date. Specifically it shows the difference, $[x_t|s_t = 0, x_{t-1}] - [x_t|s_t = 1, x_{t-1}]$ for output growth, inflation and the nominal interest rate. The thought experiment is the following: if a re-optimization were to occur at each period in our sample, how would the values of output, inflation and interest rates be different relative to the case where the previous commitment is honored?[28] Policy re-optimizations would have made output and inflation higher until the early 1980s, but would have had a negligible effect (or lowered them) during the Great Moderation. This is because in periods with high volatility the central bank needs to make significant commitments to stabilize the economy, with regards to its future actions. These commitments constitute a relevant burden in subsequent periods, and abandoning them would lead to a radically different outcome. Instead, in a low volatility economy, the central bank carries over less relevant commitments, and there is less need to stabilize the economy. As a consequence, the effects of abandoning past commitments are relatively small.

Regarding the episodes discussed above, Figure 8 shows that the re-optimizations of the '70s and '80s are all associated with an increase in the level of inflation and output growth. In other words, those re-optimizations implied a "looser" policy than under the commitment plan. The two re-optimizations of 1993 and 2009 are instead associated with

[28]Keep in mind that this exercise is conducted conditioning on the estimated parameters being consistent with an unconditional probability of commitment being equal to our estimated value of 0.81.

Figure 8: Historical Effects of Policy Re-optimizations



Note: The figure shows the effects of re-optimizations over time, measured as the difference between the value conditional on re-optimization and the value conditional on continuation of previous commitment, i.e. $[x_t|s_t = 0, x_{t-1}] - [x_t|s_t = 1, x_{t-1}]$. Vertical lines indicate the episodes where $Prob(s_t = 0) > 50\%$.

a more contractionary policy. This suggests that Quantitative Easing does constitute a deviation from a commitment plan, but in the sense that monetary policy should have been more expansionary than it actually was. This conforms with the common view that as the economy hit the zero-lower bound, quantitative easing was a necessary but insufficient tool.

6 Conclusion

This paper proposes a structural econometric approach to measure the degree of the Federal Reserve's credibility, within a standard medium-scale macroeconomic model. Monetary policy choices are modeled according to a *loose commitment* setting, where deviations from commitment plans are governed by a regime-switching process.

The conventional approach to think about central banks' credibility is to distinguish between two polar cases: commitment and discretion. Our results depict a very different picture regarding the actual behavior of the Federal Reserve. Over the past four decades, the Fed displayed a certain ability to commit, but its credibility was not perfect. There have been periods where the Fed honored its commitments, but also episodes when it did not. The re-optimization episodes sometimes line up closely with changes of Fed chairmen, and at other times with changes in the operational procedures of the Federal Reserve.

Moreover, counterfactual exercises provide new insights regarding the behavior of the Federal Reserve over the past decades. During the '70s, the actual inflation dynamics resemble a discretionary behavior. Instead, starting from the mid 1980s inflation dynamics are more consistent with a commitment behavior. But it would be misleading to conclude that there has been a one-time change from discretion to commitment. The Federal Reserve has occasionally deviated from its commitment plans throughout the entire sample. If anything, the main difference is that while the deviations in the 70's implied more expansionary policies, the deviations in the 90's and 2000's has been more contractionary. In this respect, our results are consistent with earlier studies in the monetary policy literature arguing that the Fed moved from a passive to an active regime.

According to our model, there is still some scope to increase the credibility of the Federal Reserve. Credibility gains would reduce the fluctuations in inflation and economic activity, and thus enhance welfare. Our study, however, remains silent about the specific sources of credibility problems, and on the possible ways to correct them. For instance, imperfect information and model uncertainty may give rise to a trade-off between credibility and flexibility, where occasional deviations from commitment could be desirable. Also, under the helm of chairman Ben Bernanke, the Federal Reserve has taken several measures to better communicate with the public about current and future policy actions. In 2012 the Federal Reserve announced an official inflation target of 2%. Additionally it started releasing individual forecasts of the FOMC members' relating to economic activity. Looking forward, our approach could be used to assess the effectiveness of this type of policies.

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Appendix

A-1 Robustness Checks

In this section we discuss a variety of robustness checks. Results are summarized in Table 6, where the first column describes how each specification differs from the benchmark case considered in the main text. For practical purposes, the table only reports the posterior mode estimates of the policy parameters w_y, w_r and γ. The complete results are available upon request to the authors.

To facilitate the comparison, row (a) reports again the benchmark results. Row (b) contains the estimates excluding data since the beginning of the financial crisis, so that the data sample is restricted to end at Q2:2007. This is because, we want to make sure that our main results are not affected by features of the financial crisis that are not explicitly included in our model, like unconventional policies and the zero-lower bound constraint. The estimates of γ and w_y remain very close to the benchmark case, whereas the estimate of w_r is lower at 0.87. This lower estimate is most likely due to the fact that the Federal Reserve has kept the Fed funds rate constant (around zero) from late 2008 onwards. Similarly, Row (c) shows the estimates under a specification where the Fed Funds rate is replaced with the interest rate on a 10-year Treasury note. This allows us to estimate the model using the entire sample without worrying about the zero lower bound. The resulting estimate of the probability of commitment is 0.8.

Table 6: Estimates under various specifications

	Specification	Posterior mode		
		w_y	w_r	γ
(a)	Benchmark	0.015	1.824	0.811
(b)	Pre-fin crisis sample	0.031	0.874	0.781
(c)	10-year interest rate	0.019	1.244	0.803
(d)	Beta(0.5,0.5) prior for γ	0.015	1.829	0.812
(e)	Gamma(2,4) prior for w_r and w_y	0.016	1.950	0.812
(f)	AR(1) price-markup shocks	0.012	2.009	0.729
(g)	$\sigma_l = 0.5$	0.022	0.879	0.801
(h)	$\sigma_l = 1.92$	0.014	1.804	0.812
(i)	$\sigma_l = 3$	0.022	0.879	0.801

Note: The table shows the posterior mode estimates of the loss function weight on output w_y, the loss function weight on interest rate smoothing w_r and the probability of commitment γ for various specifications.

We also consider different prior specifications for the monetary policy parameters. Row (d) considers a Beta prior for the probability of commitment γ, with both shape parameters set to 0.5. This prior distribution gives roughly the same weight to values in the [0.2,0.8] interval while putting more weight on values near the end points 0 and 1. Such specification confirms that even with a higher prior probability weight on discretion (0) and commitment (1), the data chooses a value of γ close to 0.8. Row (e) considers Gamma distribution for w_y and w_r with a higher variance. Specifically we use the Gamma distribution with mean 2 and variance 4, but the resulting estimates are very similar to the benchmark case.

We then estimate the model allowing for a different specification of the price-markup process. In the benchmark estimation the price-markup disturbance is modeled as an ARMA(1,1), $\varepsilon_t^p = \rho_p \varepsilon_{t-1} + \eta_t^p - \mu_p \eta_{t-1}^p$. The estimates from Table 2 show that we have an issue of near-cancelation of the roots, and potential weak identification. A similar problem is reported in Justiniano and Primiceri (2008), who end up using an i.i.d. specification. To ensure the our estimate of the probability of commitment is not affected by this issue, we consider an additional specification of the price-markup process, where ε_t^p is AR(1). The results are presented in row (f) of table 6. In this specification the estimate of ρ_p is very close to zero, making this very similar to the i.i.d. case of Justiniano and Primiceri (2008). Importantly, the estimates of γ, w_y and w_r are not affected much by the specification of the price-markup process.

Finally, as opposed to the original SW model, in our benchmark estimation we fix the elasticity of labor supply with respect to the real wage, $\sigma_l = 1$. That parameter was not estimated precisely in the SW estimates, and fixing it improves the convergence properties of our estimation algorithm. We then consider three alternative values of σ_l: a value of 0.5, that is close to what is typically estimated in micro studies; a value of 1.92 corresponding to the posterior mode SW; and a value of 3 that is close to the estimates in Bianchi (2012) (who estimates a regime-switching model similar to ours). The estimates of w_y and γ (shown in the last 3 rows) are very similar to the benchmark case, while the estimate of w_r does change somewhat (see section 5.1 for a more in-depth discussion of the sensitivity of this parameter). In conclusion, the estimate of the probability of commitment γ is stable for all the different specifications considered.

A-2 Comparison of fit of the model

Our model nests both the cases of commitment and discretion, thus the estimated results imply that the data gives a higher weight to the posterior at $\gamma = .81$ as compared to $\gamma = 0$

or $\gamma = 1$. However a potentially different way to estimate the model at the end points, $\gamma = 0$ or $\gamma = 1$, is to use an estimation strategy that does not involve regime-switching in the coefficient matrices. What we are worried about is the following. Even though the posterior (and also the likelihood given our diffuse priors) puts a higher weight on $\gamma = .81$, once we take into account the estimation uncertainty involved with the regime-switching, is it possible that the fit of the discretion or commitment model can be better than the loose commitment model? To explore this, we separately estimate the commitment and discretion versions of the model using an estimation strategy that does not involve regime-switching in the coefficients. There is still regime-switching in the variance of the shocks.

Table 7 shows the marginal likelihoods. These are based on the harmonic mean method of Gelfand and Dey (1994) and specific one we use is the modified harmonic mean estimator of Geweke (1999). It is reassuring to see that the fit of the loose commitment model is significantly better than the discretion case. With commitment, we had difficulty getting the Bayesian MCMC algorithm to converge. But the posterior mode for commitment is much lower and we are confident that discretion and loose commitment fit the data much better than for commitment. See Givens (2012) among others for supporting evidence that commitment tends to fit the data much worse than discretion.

Finally, we compare our model with the original SW model to compare the fit. For this exercise the models are estimated stopping the sample before the start of the financial crisis. We use the original priors used in the SW model, and the baseline priors for the loose commitment model. The marginal likelihood for our model is higher but not by much. These marginal likelihood calculations do depend to a small extent on the priors and so we conclude that our model fits the data at least as well as the SW model, which is the workhorse model in the literature.

Table 7: Comparison of Marginal Likelihhod

Model	Data Sample	Marginal Likelihood
Discretion	Full	-1127.69
Benchmark	Full	-1020.36
Smets & Wouters (2007)	Pre-fin crisis	-880.25
Benchmark	Pre-fin crisis	-874.45

A-3 Details of the Estimation Algorithm

A-3.1 Evaluating the Likelihood Function

Consider the state-space form that can be obtained combining equations (5) and (8)

$$x_t^{obs} = A + H\beta_t$$
$$\beta_t = F_{s_t}\beta_{t-1} + Gv_t$$

where $\beta_t \equiv [x_t, \lambda_t]$ and $v_t \sim N(0, Q_{s_t^{vo}})$. s_t is regime-switching variable that governs reoptimizations, while s_t^{vo} governs the switching variances, with corresponding transition matrices P and P^{vo}. In our case we have 2 states for s_t, and 2 states for s_t^{vo}. We define a new Markov-switching process S_t that combines the two switching processes, s_t and s_t^{vo}, and which can take on $M = 4$ possible values. The transition matrix for S_t is a function of P and P^{vo}.

We will use the notation ψ_t to denote information available upto time t. We initialize the continuous latent variables β_t at their unconditional mean and the regime switching variable S_t using the steady-state probabilities from the corresponding transition matrix. The likelihood function is then evaluated using the following four steps.

Step 1: Perform the Kalman Filter for $i = 1, ..M, j = 1, ..M$

$$\begin{aligned}
\beta_{t|t-1}^{i,j} &= F_j \beta_{t-1|t-1}^{i} \\
P_{t|t-1}^{i,j} &= F_j P_{t-1|t-1}^{i} F_j' + GQ_jG' \\
\eta_{t|t-1}^{i,j} &= x_t^{obs} - H\beta_{t|t-1}^{i,j} - A \\
f_{t|t-1}^{i,j} &= HP_{t|t-1}^{i,j}H' \\
\beta_{t|t}^{i,j} &= \beta_{t|t-1}^{i,j} + P_{t|t-1}^{i,j}H'[f_{t|t-1}^{i,j}]^{-1}\eta_{t|t-1}^{i,j} \\
P_{t|t}^{i,j} &= (I - P_{t|t-1}^{i,j}H'[f_{t|t-1}^{i,j}]^{-1}H)P_{t|t-1}^{i,j}
\end{aligned}$$

Step 2: Perform the Hamilton Filter

$$\begin{aligned}
p(S_t, S_{t-1}|\psi_{t-1}) &= p(S_t|S_{t-1})p(S_{t-1}|\psi_{t-1}) \\
f(x_t^{obs}|\psi_{t-1}) &= \sum_{S_t}\sum_{S_{t-1}} f(x_t^{obs}|S_t, S_{t-1}, \psi_{t-1})p(S_t, S_{t-1}|\psi_{t-1}) \\
p(S_t|\psi_t) &= \sum_{S_{t-1}} p(S_t, S_{t-1}|\psi_t)
\end{aligned}$$

Note that the conditional density is normal and given by

$$f(x_t^{obs}|S_{t-1}=i,S_{t=j},\psi_{t-1})=(2\pi)^{-\frac{N}{2}}\,|f_{t|t-1}^{i,j}|^{-\frac{1}{2}}exp\{-\frac{1}{2}\eta_{t|t-1}^{i,j\prime}(f_{t|t-1}^{i,j})^{-1}\eta_{t|t-1}^{i,j}\}$$

Step 3: Perform the Kim & Nelson approximations to collapse the M^2 unobservable $\beta_{t|t}^{i,j}$ into M ones. For each j calculate the following

$$\beta_{t|t}^{j}=\frac{\sum_{i=1}^{M}p(S_t=i,S_t=j|\psi_t)\beta_{t|t}^{i,j}}{p(S_t=j|\psi_t)}$$

$$P_{t|t}^{j}=\frac{\sum_{i=1}^{M}p(S_t=j,S_{t-1}=i|\psi_t)[P_{t|t}^{i,j}+(\beta_{t|t}^{j}-\beta_{t|t}^{i,j})(\beta_{t|t}^{j}-\beta_{t|t}^{i,j})']}{p(S_t=j|\psi_t)}$$

Step 4: After performing steps 1-3 $\forall t$ we can evaluate the log likelihood function

$$l(\theta)=\sum_{t=1}^{T}ln(f(x_t^{obs}|\psi_{t-1}))$$

A-3.2 Metropolis-Hastings Algorithm

This section explains the Bayesian estimation procedure that uses the Metropolis-Hastings algorithm and will be used to estimate all the parameters of the model jointly. Our estimation method follows the Metropolis-Hastings algorithm used in SW, specifically a single-block Random-Walk Metropolis-Hastings algorithm. The main difference is that the evaluation of the likelihood has to be modified to deal with the addition of regime-switching, as outlined in the previous section. The estimation follows a two step procedure. In the first step we numerically maximize the log posterior distribution to get an estimate of the posterior mode. In the second step, using the posterior mode calculated in the first step as a starting value, we use the Metropolis-Hastings algorithm to completely characterize the posterior distribution.

Let θ be the parameters to be estimated. The M-H algorithm involves generating a draw from a candidate generating density, $q(.)$. Given a draw $\theta^{(g)}$, let the candidate draw be called $\theta^{(g+1)}$. Then this new draw is accepted with the following probability.

$$\alpha(\theta^{(g+1)},\theta^{(g)})=min\left(\frac{p(\theta^{(g+1)}|Y).q(\theta^{(g)})}{p(\theta^{(g)}|Y).q(\theta^{(g+1)})},1\right)$$

Following SW(2007) we use the inverse of the Hessian at the posterior mode (that comes out of the numerical optimization procedure) in the candidate generating density which is centered around the current draw $\delta^{(g)}$.

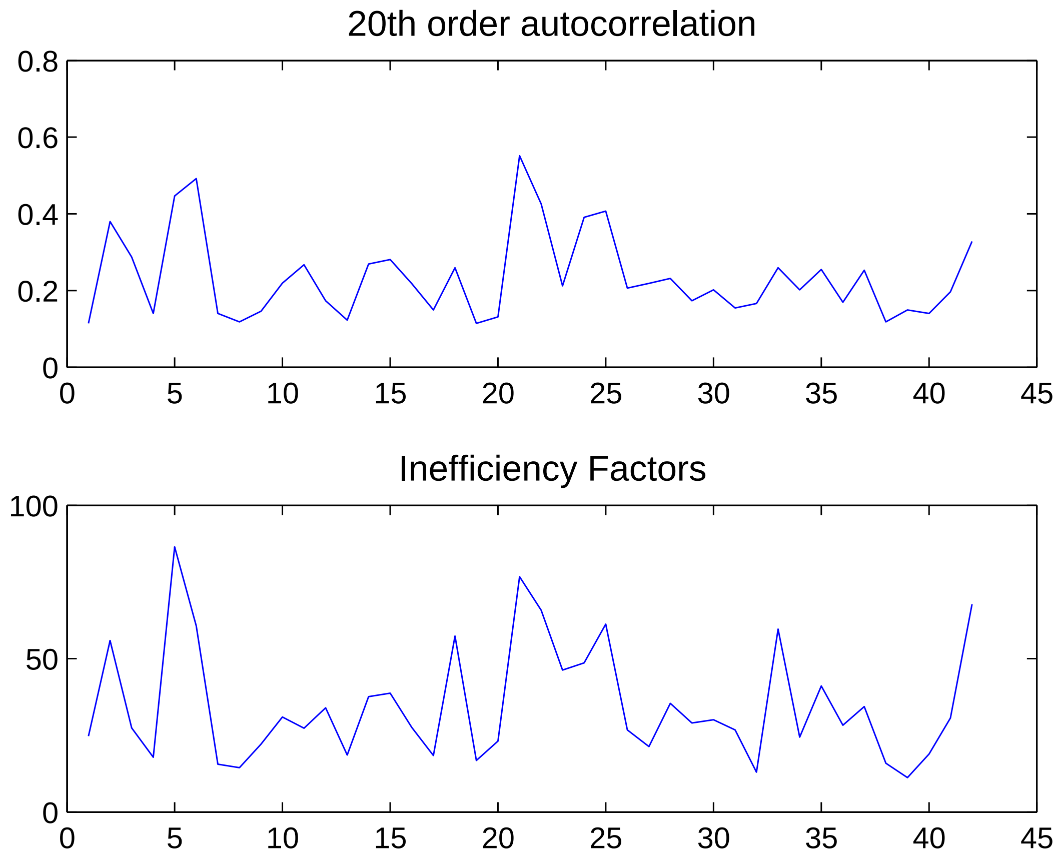
$$\delta^{(g+1)} = \delta^{(g)} + c\tilde{H}^{-1}$$

where c is a scale factor and $\tilde{H}$ is the Hessian at the posterior mode. This is known as a Random-Walk Metropolis-Hastings step. We then tune the parameter c to get an acceptance rate of between 25% and 35% as recommended by Gamerman and Lopes (2006). The full parameter vector θ is sampled in one block. We have also tried blocking by splitting the parameter vector θ into 2 or more blocks but found that the Metropolis-Hastings algorithm ran most efficiently with one block and had good convergence properties as discussed below.

A-3.3 Convergence Diagnostics

The Metropolis-Hastings algorithm is run for 500,000 draws where the first 25,000 draws are discarded. Note since the chain is initialized at the posterior mode, a large number of burn-in draws is not required. From the remaining 475,000 draws, one of out of every ten draws is saved resulting in an effective sample of 47,500. The trace plots of the draws suggest that the Metropolis-Hastings algorithm mixing well. To get a better idea of the correlation of the draws the top panel of figure 9 plots the 20th order autocorrelation for each of the estimated parameters. These autocorrelations are all below 0.6 and most them lower than 0.3, suggesting that the dependence in the draws diminishes fairly rapidly. The lower panel shows the inefficiency factors, this is the inverse of the relative numerical efficiency of Geweke (1992). These numbers are mostly below 40 while some of them are a bit higher. Note these are much lower as compared to the SW M-H algorithm as reported by Chib and Ramamurthy (2010). We have also conducted other checks and overall the convergence diagnostics are satisfactory.

Figure 9: 20th Order Autocorrelations



Note: The top panel of this figure shows the 20th order autocorrelations (y-axis) of the MCMC draws for all the estimated parameters (x-axis) while the bottom panel shows inefficiency factors.