Tax and Monetary Policy Rules in a Small Open Economy with Dissaggregate Government Purchases

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Abstract

This paper aims to evaluate the impact of tax and monetary policy rules with disaggregate government purchases on welfare, real exchange rate and business cycle in a small open economy using a new-Keynesian dynamic stochastic general equilibrium framework. The model predicts that the government consumption has more impact than government investment on both private consumption and investment, but less impact on the real GDP. Besides, the government purchases-real exchange rate puzzle is generated by the model. In this sense, the government consumption contributes more on generating the puzzle than the government investment. Moreover, both government consumption and investment have positive impact on welfare for any policy rules. The optimized policy rules have a pronounced anti-inflation stance and entail significant nominal and real exchange rate volatility for monetary policy. For tax policy rules, the public debt stance is the optimized rules.

JEL classification : E32; E43; E52; E62; F31; F41

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1 Introduction

This paper aims to evaluate the impact of tax and monetary policy rules with dissaggregate government purchases (consumption and investment) on welfare, real exchange rate and business cycle for an small open economy using a dynamic stochastic general equilibrium (DSGE) model with nominal rigidities: monopoly competition and sticky prices à la Calvo for intermediate producers. The model also takes into account seven chocks due to: (i) the domestic productivity, (ii) the world interest rate, (iii) the uncovered interest parity condition (UIP) (Fama 1984), (iv) the world output, (v) the government consumption, (vi) government investment, and (vii) the world inflation. In fact, this paper adds the government sector to the model developed by Kollmann 2002. Many papers have analyzed the impact of government purchases on private consumption, output, and real exchange rate (Ravn, Schmitt-Grohé, and Uribe, 2007; Monacelli and Perotti, 2010; Kollmann, 2010; Basu and Kollmann, 2013). Basu and Kollmann 2013 finds that a rise in government purchases not only increases the private consumption, but also depreciates the real exchange rate due to the introduction of productive government investment if these purchases increase domestic private sector, and labor supply is highly elastic. These main results raise two important puzzles: (i) the government purchases-consumption puzzle (Sanchez, 2001; Linnemann and Schabert, 2003; Blanchard and Perotti 2002; Linnemann, 2008; Gali et Al. 2007; Escolani, 2007; Mountford and Uhlig, 2009; Woodford, 2010) and (ii) government purchases-real exchange rate puzzles (Sanchez, 2001; Forni and Pisani, 2010; Monacelli and Perotti, 2010; Kollmann, 2010; Basu and Kollmann, 2013). This paper aims to analyze the effect of decomposed the government purchases (consumption and investment)on a small open economy. Which government purchases contribute more to the depreciation of the real exchange rate, the household’s welfare and the business cycle? This question will help to understand the mechanism by which the decomposed government purchases affect an small open economy with respect to optimized tax and monetary policy rules.
The puzzles are due to the fact that the main results mentioned above contradict the predictions of the standard neoclassic models which find that the rise in government purchases (i) decreases private consumption (Baxter and King, 1993; Backus et al. 1994; Ramey and Shapiro 1998; Ramey, 2008), and (ii) appreciates the real exchange due to the wealth effect (Kollmann, 1991, 1995; Backus et al. 1993, 1994) which lowers private consumption and increases hours of work and output. Therefore, if the consumption risk is internationally shared through complete financial markets, the rise in the home marginal utility of consumption is accompanied by an appreciation of the home real exchange rate. However, the model developed in this paper generates the government purchases-real exchange rate puzzle through marginal product of both labor and capital.

The introduction of the government purchases in both utility and production functions follows the seminal work of Baxter and King (1993) in which the private and government consumption in the utility function of the representative agent are separated. However, this paper will consider a non-separate formulation between the private and government consumption which should sustain their complementary. Besides, most of macroeconomic models have neglected to analyze the effect of the decomposed government purchases in the entire economy and the literature related to the question is not abundant. This paper aims to enrich the literature by analyzing the effect of the decomposed government purchases in a small opened economy through the optimized tax and monetary policy rules.

As the model deals with many relative prices, the paper takes into account the departure from the law of one price (LOP) which implies limited exchange rate pass-through and price to market (PTM) behavior of producers (Knetter, 1993; Betts and Devereux, 2000; Kollmann, 2002; Devereux and Engel, 1998, 2002; Devereux and Yetman, 2014). In fact, the price to market assumption means that the producers will charge price in the currencies of their costumers. Therefore, the monetary authority set a rule which links the nominal interest rate to gross domestic producer price index (PPI) inflation.
This rule will be compared to alternative measures of inflation such as the producer currency pricing and exchange rate peg.

Furthermore, the financial market for bonds is assumed to be incomplete because the world households do not buy bonds issued in small open economy currency. Only the households in small open economy hold both bonds in domestic and foreign currencies. Therefore, the consumption risk is not efficiently shared internationally (e.g., Kollmann 2010). Any rise in the government purchases will depreciate the real exchange rate.

I assume here that the government consumption is complement to the private consumption (e.g., Bouakez and Rebei, 2007; Gali et al., 2007; Monacelli and Perotti, 2010). This channel works perfectly through the international risk sharing of the consumption growth between the domestic and the foreign economies to explain the depreciation of real exchange. Besides, government consumption contributes to increase the aggregate demand which has positive impact on final goods. However, the government investment is productive in the sense that it increases the productivity of both labor and the effective capital stock through public capital stock. Therefore, the government investment increases output and depreciates the real exchange rate (the second channel). So far in the literature, the role of the government consumption to generate the depreciation of real exchange rate has never been studied. This paper investigates the impact of decomposed government purchases not only on real exchange rate, but also on welfare-optimized tax and monetary policy rules and business cycle.

The paper finds that the government consumption has more impact than government investment on both private consumption and investment, but less impact on the real GDP. Besides, the government purchases-real exchange rate puzzle is generated by the model. In this sense, the government consumption contributes more on the generate the puzzle than the investment. Moreover, both government consumption and investment have positive impact on welfare for any policy rules. The optimized policy rules have not only
a pronounced anti-inflation stance and entail significant nominal and real exchange rate volatility for monetary policy, but also a public debt stance for tax policy rules.

The mechanism by which the decomposed government purchases-exchange rate puzzle is generated by the model is different for each component of the government purchases. An increase in the government investment shock improves both marginal productivity of labor and capital stock which have a positive impact on labor supply, private capital stock, and output. This supply effect boosts the private consumption and wealth which depreciate the real exchange rate (first channel). The second and the third channels through which the government consumption deteriorates the real exchange rate is the international risk sharing. If the real exchange rate is expressed in terms relative marginal utility of domestic and foreign consumption, an increase in government consumption will depreciate the real exchange rate through two channels. The first channel is a direct effect of the government consumption on real exchange rate. And, the second channel is the complement relationship between private consumption and government consumption. Any increase in government consumption has a positive impact on private consumption which boost the effective consumption in aggregate level and therefore increase the marginal productivity of domestic consumption relative to foreign one. Thus, the real exchange rate depreciates.

The rest paper will be organized as follow. I will present the model in details in section 2. The results are presented in section 3 and will conclude the paper in section 4.
2 The Model

The model describes here is a New-Keynesian DSGE model for a small open economy with a representative household, firms, a government, a central bank, and rest of the world. The model adds the government agent to the model developed by Kollmann (2002).

The representative household maximizes his utility under the budget constraint and the law of motion of the private capital. Also, he holds all means of production (labor and private capital stock) that are rent/supply to firms in a perfect competitive market. From the revenues that he earns, the representative household has three options:

- He may consume by purchasing the final goods,
- He may invest in capital stock, and
- He may save by buying bonds (domestic and foreign).

There are two types of firms: the firms which produce the final goods and the ones which produce the intermediate goods. I assume that the final composite good is not tradable in the world market. Only the intermediate goods are tradable between the small open economy and the rest of the world. The composite final good is produced by combining the intermediate goods from the domestic and import foreign firms. There is a continuum of intermediate goods indexed by $s \in [0, 1]$.

The government sector collects taxes, issues the debt through the central bank and finances its consumption and investment purchases. The latter is used to produce the domestic intermediate goods and the former to be consumed by the representative household. This is one the contribution of this paper.

The central bank issues bonds for domestic government and set the nominal interest according to the Taylor rules.
Finally, the rest of the world export the intermediate goods which are used to produce the composite final domestic goods. Besides, it import the intermediate goods from the small open economy that are used to produce its final composite goods. Moreover, it sells bonds to the representative household of small open economy.

2.1 The representative household

The representative household maximizes his preferences through

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, GC_t, L_t).$$

(1)

where $E_0$ represents the mathematical expectation conditional upon the complete information pertaining to period t and earlier. Also, $0 < \beta < 1$ is the subjective discount factor. $U(\cdot)$ is the utility function which is monotone and continuous. Finally, $C_t$, $L_t$, and $GC_t$ represent respectively the private consumption, labor effort, and government consumption at time period t. Let the utility function take the following form:

$$U(C_t, GC_t, L_t) = \ln(C_t + \gamma GC_t) - \psi L_t.$$

(2)

I assume here that the government consumption is complement to the private one. This assumption is important to generate the government purchases-real exchange rate puzzle. The government consumption can be understood as positive externality that the representative household receives directly from the government or indirectly from its public services in terms of goods and services like passports, security, driver-license, stumps from post-service, public light...

Since the household holds all domestic producers and accumulates physical
capital. The law of motion of the capital stock is given by

\[ PK_t = (1 - \delta)PK_{t-1} + Ip_t \]  

where \( PK_t \), \( PK_{t-1} \), \( Ip_t \), and \( 0 < \delta < 1 \) represent respectively the private capital stock at period \( t \) , the private capital stock at time \( t - 1 \), the gross investment at time \( t \), and the depreciation rate of the capital stock.

Besides, the household’s budget constraint at period \( t \) is defined as

\[
\begin{bmatrix}
A^d_{t+1} + S_tA_{t+1} + P_t(C_t + Ip_t) = R_{t-1}A^d_t + S_t\varphi_{t-1}R_{f_{t-1}}A_t \\
+ R^K_{t-1}PK_{t-1} + DIV_t + W_tL_t - T_t
\end{bmatrix}
\]

where

\[ T_t = \tau^C_t P_tC_t + \tau^K_t R^K_{t} PK_{t-1} + \tau^L_t W_tL_t - \tau^K_t P_tPK_{t-1} \]

where \( A^d_t \) and \( A_t \) are the net stocks of risk-free domestic and foreign currency bonds that mature in period \( t \). \( R_{t-1} \) is the interest rate on domestic currency bonds and \( R_{f_{t-1}} \) is the interest rates on foreign currency bonds adjusted by a risk premium, \( \varphi_{t-1} \). This risk premium is an endogenous function which is a decreasing function of stationary holding of foreign assets of the entire domestic economy, and an increasing function of risk premium shock, \( \chi_{t-1} \). The endogenous risk premium function has the following form

\[ \varphi_{t-1} = exp(-\left(\frac{A_{t-1}}{A_{ss}}\right) + \chi_{t-1}) \]

where

\[ ln\chi_{t-1} = \rho_{\chi}ln\chi_{t-2} + \epsilon_{t-1}^{\chi}. \]

\( \chi_{t-1} \) can be interpreted as bias in the household’s date \( t - 1 \) forecast of the date \( t \) exchange rate, \( S_t \), based on yesterday information about interest rate (Kollmann, 2002). Besides, the stock of bonds from domestic government is
assumed to be non-negative. In fact, the household is not allowed to borrow from the government. The foreign bonds are bought from the world market. Here, the model allows this stock of bonds to be negative or positive. If the stock is negative, it means that the household is a net borrower and a net lender, otherwise.

Moreover, $R^K_t$, $W_t$, $T_t$, $DIV_t$, and $S_t$ represent, respectively, the rental rate of capital, the rental rate of labor, the total taxes received by the government from the household, the dividend received by household and the nominal exchange rate. Finally, $\tau^C_t$, $\tau^K_t$, and $\tau^L_t$ are the tax rates on consumption (generally known as the value added tax in most of European countries), capital, the and labor income respectively.

Therefore, the household maximizes (1) subject to (4) and (5) from which the first order conditions are derived which respect to domestic currency bonds, international currency bonds, private capital stock at time $t$, and labor hours:

\[
1 = \beta E_t \left[ \left( \frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left( \frac{1 + \tau^C_t}{1 + \tau^{C, t}_{t+1}} \right) \frac{1}{\pi^{t+1}} R^K_t \right],
\]

(8)

\[
1 = \beta E_t \left[ \left( \frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left( \frac{1 + \tau^C_t}{1 + \tau^{C, t}_{t+1}} \right) \frac{1}{\pi^{t+1}} e_{t+1} \varphi_t R^K_t \right],
\]

(9)

\[
1 = \beta E_t \left[ \left( \frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left( \frac{1 + \tau^C_t}{1 + \tau^{C, t}_{t+1}} \right) (R^K_t (1 - \tau^K_{t+1}) + \delta \tau^K_{t+1} + (1 - \delta)) \right],
\]

(10)

\[
\psi = \left[ \left( \frac{1}{C_t + \gamma GC_t} \right) \left( \frac{1 - \tau^L_t}{1 + \tau^L_t} \right) W_t \frac{1}{\phi_t} \right],
\]

(11)
\(e_{t+1}\) is the depreciation factor of the nominal exchange rate between period \(t + 1\) and \(t\) which is defined as:

\[
e_{t+1} = \frac{S_{t+1}}{S_t}.
\]

### 2.2 Uncovered Interest Parity

Combining equations (6) and (7) yields

\[
R_t = E_t e_{t+1} \varphi_t R_{f_t}.
\]

Then, the log-linearization around the steady state of (13) gives the uncovered interest parity (UIP) condition with an endogenous risk premium, \(\varphi_t\):

\[
E_t \hat{S}_{t+1} - \hat{S}_t = \hat{R}_t - \hat{R}_{f_t} - \hat{\varphi}_t.
\]

I define \(\hat{S}_t = \frac{S_t - S_{ss}}{S_{ss}}\), \(\hat{R}_t = \frac{R_t - R_{ss}}{R_{ss}}\), \(\hat{R}_{f_t} = \frac{R_{f_t} - R_{f_{ss}}}{R_{f_{ss}}}\), and \(\hat{\varphi}_t = \frac{\varphi_t - \varphi_{ss}}{\varphi_{ss}}\) where the subscript \(ss\) means steady state value.

I assume here that \(\varphi_t\) is one at steady state in order to solve the model. The departure of the UIP condition was first empirically observed by Fama (1984) is measured here by \(\varphi_t\). This endogeneous risk premium function is introduced following Schmitt-Grohé and Uribe (2003) in order to make the small open economy model stationary.

### 2.3 The firms

- **Final good**

The economy produce the final good according the aggregate technology

\[
Z_t = \left[ (1 - \alpha_m)^{\frac{1}{\varphi}} (Y_t^d)^{\frac{(\varphi-1)}{\varphi}} + (\alpha_m)^{\frac{1}{\varphi}} (Y_t^m)^{\frac{(\varphi-1)}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}}
\]

where \(Z_t\) is the final good output at time \(t\); \(Y_t^d\) is the quantity of domestic intermediate goods used to produced the final good output; \(Y_t^m\) is the quantity
of imported intermediate goods used to produce the final good output; $\vartheta > 0$ is the elasticity of substitution between the imported and domestic intermediate goods in the production of the final good output; and $\alpha^m$ is the share of imported intermediate good in the production of the composite final good. Also, it determines the steady state degree of openness (here is expressed in terms of imports to GDP ratio) of the country to the rest of the world.

Besides, I assume the firm operates in a perfectly competitive market. It takes the price of output, $P_t$, as given. Therefore, if $P^d_t$ and $P^m_t$ are prices of domestic and imported intermediate goods sold in the domestic market, the problem of the firm consists of cost minimizing of the production of the final composite output by combining the inputs (intermediate goods) subject to (15) above which yields the following demands for intermediate goods required to produce the final composite good:

$$Y^m_t = \alpha^m \left( \frac{P^m_t}{P_t} \right)^{-\vartheta} Z_t,$$  \hspace{1cm} (16)

and

$$Y^d_t = (1 - \alpha^m) \left( \frac{P^d_t}{P_t} \right)^{-\vartheta} Z_t,$$  \hspace{1cm} (17)

where

$$P_t = \left[ (1 - \alpha^m)(P^d_t)^{1-\vartheta} + \alpha^m(P^m_t)^{1-\vartheta} \right]^{1/(1-\vartheta)}.$$  \hspace{1cm} (18)

Since the firm operates in a perfectly competitive market, the price, $P_t$, is also its marginal cost. Finally, $P_t$ is the consumer price index (CPI).

- **The Domestic Intermediate goods**

The domestic intermediate good $s$ is produced using the technology

$$Y_t(s) = \theta_t K_{t-1}(s)^{\alpha} L_t(s)^{1-\alpha}$$  \hspace{1cm} (19)
where $Y_t$ is the final good output at period $t$, and $\theta_t$ is the exogenous
domestic productivity shock which is identical to all $s$ firms.

$$K_t(s) = GK_t + PK_t(s), \quad (20)$$

$K_t$ is the effective (total) capital stock and $GK_t$ is the exogenous
capital stock available at time $t$ and is identical to all firms.

Assume in the first stage that the wage rate is $W_t$ and the rental capital
rate is $R^K_t$. Then, the problem for the firms consists of choosing $L_t$ and $K_t$, taking $W_t$ and $R^K_t$ as given, which maximize their profit function:

$$\Pi^d_{t+j}(s) = P^{sd}_{t+j}(s)Y_{t+j}(s) - R^K_t K_{t+j}(s) - W_t L_{t+j} \quad (21)$$

subject to (19) whose first order conditions are:

$$W_t = (1 - \alpha) \frac{MC_t Y_t(s)}{L_t(s)} \quad (22)$$

where $MC_t$ is the marginal cost and $P^{sd}_{t}$ is the price of the intermediate
goods in domestic currency,

$$R^K_t = \alpha \frac{MC_t Y_t(s)}{K_t(s)}. \quad (23)$$

The total domestic production is used domestically, $Y^d_t$ or exported to
the world market, $Y^x_t$, so that

$$Y_t = Y^d_t + Y^x_t, \quad (24)$$
where $Y_t^x$ is the total export for the small open country whose demand function in the world market is

$$Y_t^x = \alpha^x \left( \frac{P_t^x}{P^*_t} \right)^{-\eta} Z_t^*.$$  

(25)

$P_t^x$ represents the price of the export intermediate good in the world market, $P_t^*$ the exogenous world price level, $\alpha^x$ the share of the small open economy’s export in the world market, and $Z_t^*$ the world output level. $Z_t^*$ is exogenous in the small open economy.

Since the intermediate goods’ firms operate in a monopolistic competition market, they gain profits. Then, in the second stage the firms set the price à la Calvo (1983) that maximizes the expected discounted real profits. Therefore, in each period, a fraction of $(1 - \lambda)$ firms are able to reset their prices, while others keep unchanged their prices. Usually, the period from which the price is unchanged is $\frac{1}{(1-\lambda)}$.

Thus, the firms solve the following problem

$$\max_{P_t^d(s)} E_t \sum_{j=0}^{\infty} (\beta \lambda)^j \left( \frac{C_{t+j+\gamma} \delta_{t+j} + GC_{t+j}}{P_{t+j}^d} \right) \Pi_{t+j}^d(s)$$

subject to (19).

The following demand function is derived as the solution of the above problem:

$$Y_{t+j}^d(s) = \left( \frac{P_t^d(s)}{P_{t+j}^d} \right)^{-\nu} Y_{t+j}^d$$  

(26)
whose first order condition with respect to $P^d_t$ is

$$P^d_t = \frac{\nu}{\nu - 1} \frac{E_t \sum_{j=0}^{\infty} (\beta \lambda)^j \left( \frac{C_t + \gamma GC_t}{C_{t+j} + \gamma GC_{t+j}} \right) Y_{t+j}(s) MC_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta \lambda)^j \left( \frac{C_t + \gamma GC_t}{C_{t+j} + \gamma GC_{t+j}} \right) Y_{t+j}(s) / P^d_{t+j}}. \quad (27)$$

The import price index is

$$(P^d_t)^{1-\nu} = \lambda (P^d_{t-1})^{1-\nu} + (1 - \lambda)(P^d_t)^{1-\nu}. \quad (28)$$

Analogously, the firms can choose a new export price at time $t$ by solving the following problem

$$\max_{P^x_t(s)} E_t \frac{\sum_{j=0}^{\infty} (\beta \lambda)^j \left( \frac{C_t + \gamma GC_t}{C_{t+j} + \gamma GC_{t+j}} \right) \Pi^x_{t+j}(s)}{P^x_{t+j}(s)}$$

above subject to (19) and the demand function

$$Y^x_{t+j}(s) = \left( \frac{P^x_t(s)}{P^x_{t+j}} \right)^{-\nu} Y^x_{t+j}. \quad (29)$$

The solution gives $P^x_t$ as

$$P^x_t = \frac{\nu}{\nu - 1} \frac{E_t \sum_{j=0}^{\infty} (\beta \lambda)^j \left( \frac{C_t + \gamma GC_t}{C_{t+j} + \gamma GC_{t+j}} \right) Y_{t+j}(s) MC^x_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta \lambda)^j \left( \frac{C_t + \gamma GC_t}{C_{t+j} + \gamma GC_{t+j}} \right) Y_{t+j}(s) / P^x_{t+j}}. \quad (30)$$

where

$$MC^x_t = \frac{P^d_t}{P^x_t S_t}, \quad (31)$$

and $S_t$ is the nominal exchange rate expressed here as the domestic currency price of foreign currency.

The import price index is

$$(P^x_t)^{1-\nu} = \lambda (P^x_{t-1})^{1-\nu} + (1 - \lambda)(P^x_t)^{1-\nu}. \quad (32)$$
The Imported Intermediate goods

As for the domestic intermediate goods, the imported intermediate producers set their price à la Calvo (1983). The importer sets its price, $P^*_{m}(s)$, that maximizes the expected discounted profits below, taking as given the nominal exchange rate, $S_t$, and the world price level, $P^*_w$,

$$
\max_{P^*_{m}(s)} \mathbb{E}_t \left( \sum_{j=0}^{\infty} (\beta \lambda)^j \left( \frac{C_{t+j} + \gamma G_{C_{t+j}}}{C_{t+j} + \gamma G_{C_{t+j}}} \right) \Pi^m_{t+j}(s) \right)
$$

subject to

$$
Y^m_{t+j}(s) = \left( \frac{P^*_{m}(s)}{P^m_{t+j}} \right)^{-\nu} Y^m_{t+j}.
$$

where

$$
\Pi^m_t = (P^*_{m}(s) - MC^m_t) \left( \frac{P^*_{m}(s)}{P^m_t} \right)^{-\nu} Y^m_t,
$$

where the marginal cost for importer is

$$
MC^m_t = \frac{S_t P^*_t}{P^m_t}.
$$

The solution for $P^*_{m}$ implies the following first order condition

$$
P^*_{m} = \frac{\nu}{\nu - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta \lambda)^j \left( \frac{C_{t+j} + \gamma G_{C_{t+j}}}{C_{t+j} + \gamma G_{C_{t+j}}} \right) Y^m_{t+j}(s) MC^m_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta \lambda)^j \left( \frac{C_{t+j} + \gamma G_{C_{t+j}}}{C_{t+j} + \gamma G_{C_{t+j}}} \right) Y^m_{t+j}(s) / P^m_{t+j}}.
$$

The import price index is

$$
(P^m_t)^{1-\nu} = \lambda (P^m_{t-1})^{1-\nu} + (1 - \lambda)(P^*_{m})^{1-\nu}.
$$
2.4 The Government

The government alleviates tax on all sources of income for households. Also, the government issues one period debt, $D_t$, that matures at $t$, and balances its budget at each period. I consider the government final good purchases, $G_t$, as exogenous.

\[ P_t G_t + D_{t-1} R_{t-1} = D_t + T_t \]  
(38)

where $T_t = \tau^C_t P_t C_t + \tau^K_t R^K_t P K_{t-1} + \tau^L_t W_t L_t - \tau^K_t P_t P K_{t-1}$.

I define the real government debt normalized by steady state real GDP as $B_t \equiv \left( \frac{D_t}{P_t Z_{ss}} \right)$.

The Government spending is decomposed into

\[ G_t = G C_t + I g_t \]  
(39)

where $G C_t$ and $I g_t$ are the government consumption and investment at period $t$. I assume here that, the public capital evolves according to

\[ G K_{t+1} = I g_t + (1 - \delta) G K_t. \]  
(40)

Therefore, the tax policy rules are set as

\[ \frac{\tau^K_t}{\tau^K_{ss}} = \left( \frac{B_t}{B_{ss}} \right)^\gamma^K \left( \frac{G C_t}{G C_{ss}} \right)^\gamma^{G C} \left( \frac{I G_t}{I G_{ss}} \right)^\gamma^I \left( \frac{\theta_t}{\theta_{ss}} \right)^\gamma^\theta \left( \frac{\pi_t}{\pi_{ss}} \right)^\gamma^\pi, \]  
(41)

\[ \frac{\tau^K_t}{\tau^K_{ss}} = \left( \frac{B_t}{B_{ss}} \right)^\gamma^K \left( \frac{G C_t}{G C_{ss}} \right)^\gamma^{G C} \left( \frac{I G_t}{I G_{ss}} \right)^\gamma^I \left( \frac{\theta_t}{\theta_{ss}} \right)^\gamma^\theta \left( \frac{\pi_t}{\pi_{ss}} \right)^\gamma^\pi, \]  
(42)

\[ \frac{\tau^K_t}{\tau^K_{ss}} = \left( \frac{B_t}{B_{ss}} \right)^\gamma^K \left( \frac{G C_t}{G C_{ss}} \right)^\gamma^{G C} \left( \frac{I G_t}{I G_{ss}} \right)^\gamma^I \left( \frac{\theta_t}{\theta_{ss}} \right)^\gamma^\theta \left( \frac{\pi_t}{\pi_{ss}} \right)^\gamma^\pi. \]  
(43)
It is important to indicate that the variables with subscripts "ss" denote the steady state values. Besides, I distinguish, as Kollmann(2008) three types of feedback policy rules:

- the "simpler rules" which stipulate that the any tax rate above reacts only to the real government debt;
- the "reacher rules" which link any tax rate to real government debt, government consumption, government investment, productivity shock, and inflation; and
- the "baseline rules" which link any tax rate to real government debt, government consumption, government investment, and productivity shock.

I assume that the government commits to setting the policy parameters $\phi_B$, $\phi_G$, $\phi_I$, $\phi_\theta$, and $\phi_\Pi$ at values that maximize the unconditional expected value of household utility subject to the restriction that the unconditional mean of real debt has to close to its steady state value (Kollmann 2008) as

$$|EB_t - B_{ss}| < 0.01.$$  \hspace{1cm} (44)

The equation (44) is set to rule out the long-run values of debt and taxes that differ greatly from the values observed in reality which has been showed by Aiyagari et al.(2002) from the optimal Ramsey fiscal policy.

2.5 The Central Bank

Following Taylor (1993, 1999), the central bank set the short-run nominal interest rate, $R_t$, in response to inflation and output gaps.

$$\frac{R_t}{R_{ss}} = \left(\frac{\pi_t}{\pi_{ss}}\right)^{\phi_\pi} \left(\frac{Z_t}{Z_{ss}}\right)^{\phi_Z} \left(\frac{e_t}{e_{ss}}\right)^{\phi_e}$$  \hspace{1cm} (45)
Therefore, two policy rules will be discussed in this paper:

- the "simpler rules" is defined as the nominal short-run interest rate reacts to inflation and output gap (e.g., Taylor, 1993, 1999; Kollmann, 2002, 2008) and

- the "reacher rules" link the nominal short-run to inflation, output and depreciation factor of the nominal exchange rate gap.

As the government, the central bank also commits to setting the policy parameters $\phi_\pi, \phi_Z$, and $\phi_e$ at the values that maximize the unconditional expected value of household utility.

### 2.6 Relative Prices and International Risk Sharing

Since the model is solved in real terms, I have defined some relative prices in order to simplify many things.

\[ \psi_{t}^{\text{lop}} = MC_t^m = \frac{S_t P_t^*}{P_t^m} = \frac{\pi_t^*}{\pi_t} \psi_{t-1}^{\text{lop}} \]  
(46)

describes the deviation from the law of one price (LOP) because I impose that $\psi_{t}^{\text{lop}}$ is different from one. However, in steady state $\psi_{ss}^{\text{lop}}$ must be equal to one.

\[ \psi_t^x = \frac{P_t^x}{P_t} = \frac{\pi_t^x}{\pi_t} \psi_{t-1}^x \]  
(47)

\[ \psi_t^d = \frac{P_t^d}{P_t} = \frac{\pi_t^d}{\pi_t} \psi_{t-1}^d \]  
(49)
\[
\psi_t^m = \frac{P_t^m}{P_t} = \frac{\pi_t^m}{\pi_t} \psi_{t-1}.
\] (50)

Let now define the real exchange rate and the international risk sharing respectively as

\[
RER_t = \frac{S_t P_t^*}{P_t} = S_t \psi_t^*.
\] (51)

\[
RER_t = \left( \frac{C_t^* + \gamma GC_t^*}{C_{t+1}^* + \gamma GC_{t+1}^*} \right) \left( \frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right)
\] (52)

### 2.7 Market clearing conditions

The intermediate goods markets clear as firms meet all demand at posted prices. The market for final good, labor, and capital rental clear when:

\[
Z_t = C_t + I p_t + G_t,
\] (53)

\[
L_t = \int_0^1 L_t(s) ds,
\] (54)

and

\[
K_t = \int_0^1 K_t(s) ds.
\] (55)

The bond market clearing requires as Kollmann (2002), I assume that foreigners do not hold bonds denominated in the currency of small open economy. Therefore, the bonds markets clear:

\[
A_t^d = 0,
\] (56)

and
\[
\psi_t^* \psi_t Y_t^e - \psi_t^* Y_t^{m} = A_t - \varphi_t R_{t-1} A_{t-1}
\] (57)

The equation (56) determines the evolution of the net foreign assets following Schmitt-Grohé and Uribe (2003). It represents the balance of payments that includes the risk premium on foreign assets holding by households. The left hand side of equation (56) represents the balance of trade, while the right hand side represents the balance of capital. Finally, I define the net asset position as \( NFA_t = \frac{A_t}{\tau^* Z^\star} \).

### 2.8 Exogenous variables

I consider nine shocks in this model: the domestic productivity, the world inflation, the world interest rate, the world output, the UIP, the government consumption, the government investment, the preference, and the labor supply which are expressed as following:

\[
\log Z_t^* = \rho_Z \log Z_{t-1}^* + \epsilon_Z.
\] (58)

\[
\log \pi_t^* = \rho_\pi \log \pi_{t-1}^* + \epsilon_{\pi^*}
\] (59)

\[
\log I_g_t = \rho_{Ig} \log I_{g_{t-1}} + \epsilon_{Ig}
\] (60)

\[
\log GC_t = \rho_{GC} \log GC_{t-1} + \epsilon_{GC}
\] (61)

\[
\log \chi_t = \rho_{\chi} \log \chi_{t-1} + \epsilon_{\chi}
\] (62)

\[
\log R_{f_t} = \rho_{Rf} \log R_{f_{t-1}} + \epsilon_{Rf}
\] (63)
\[ \log \theta_t = \rho \log \theta_{t-1} + \epsilon_\theta \]  

### 2.9 Solution method and Welfare measures

I solve my model using Sims’ (2000) second-order accurate method. The welfare is evaluated through a second-order Taylor expansion of the utility function around the steady state which gives

\[ E(U(C_t, GC_t, L_t)) \approx U(C_t, GC_t, L_t) + E(\hat{C}_t + \gamma \hat{GC}_t) - L_{ss} E(\hat{L}_t) \]

\[ - Var(\hat{C}_t + \gamma \hat{GC}_t) \]

where \( Var(\hat{C}_t + \gamma \hat{GC}_t) \) is the variance of \((\hat{C}_t + \hat{GC}_t)\).

By expressing the welfare as the permanent relative change in private consumption and government consumption (compared to the steady state), \( \xi \), which gives

\[ E(U(C_t, GC_t, L_t)) : U((1 + \xi)(C_t, GC_t), L_t) \approx U(C_t, GC_t, L_t) + E(\hat{C}_t + \gamma \hat{GC}_t) - L_{ss} E(\hat{L}_t) \]

\[ - Var(\hat{C}_t + \gamma \hat{GC}_t). \]

Hence, the welfare \( \xi \) can be decomposed into two components \( \xi^m \) and \( \xi^v \) as following:

\[ U((1 + \xi^m)(C_t, GC_t), L_t) = U(C_t, GC_t, L_t) + E(\hat{C}_t + \gamma \hat{GC}_t) - L_{ss} E(\hat{L}_t) \]

\[ U((1 + \xi^v)(C_t, GC_t), L_t) = U(C_t, GC_t, L_t) - Var(\hat{C}_t + \gamma \hat{GC}_t). \]

\( \xi^m \) and \( \xi^v \) represent the mean of private consumption plus non-investment government spending, and hours worked, and variance of private consumption plus non-investment government spending. By applying the equation (2) into the previous equations yields

\[ \ln(1 + \xi) = E(\hat{C}_t + \gamma \hat{GC}_t) - L_{ss} E(\hat{L}_t) - Var(\hat{C}_t + \gamma \hat{GC}_t) \]
\[
\ln(1 + \xi^m) = E(\hat{C}_t + \gamma \hat{G}C_t) - L_{ss}E(\hat{L}_t)
\] (70)

\[
\ln(1 + \xi^v) = -\text{Var}(\hat{C}_t + \gamma \hat{G}C_t)
\]
\[
= \text{Var}(\hat{C}_t) + \gamma^2 \text{var}(\hat{G}C_t) + 2 \text{cov}(\hat{C}_t, \hat{G}C_t)
\] (71)

Therefore,

\[
(1 + \xi) = (1 + \xi^m)(1 + \xi^v).
\] (72)

2.10 Parameters (non-policy)

Most of non-policy parameters is calibrated to quarterly data for France, Germany, the U.K and Netherlands from 1977 to 2007. The period is chosen because of the availability of quarterly data. Other parameters follow the works of Bouakez and Rebei (2007, Kollmann (2001), and Baxter and King (1993), and . The real domestic and foreign interest rate are set to be equal at steady state which are derived from the first order conditions with respect to domestic and foreign bonds, \(R_{ss} = Rf_{ss} = \pi_{ss}^r\). The inflation (CPI) is set to 1.005 which means that is 2% annually. Therefore, \(\beta = \pi_{ss}^r R_{ss}^{-1}\).

The indexation of prices, \(\nu\) is set to 6 which corresponds to Kollmann (2001)'s the price-marginal cost steady state markup factor for intermediate goods \(\frac{\nu}{(\nu - 1)} = 1.2\). The price elasticities of substitution between import and domestic intermediate goods, in one hand, and between the export and foreign intermediate goods are set \(\vartheta = \eta = 0.6\), in other hands. Moreover, the elasticity of output with respect to capital, \(\alpha\), is set to 0.24 which is consistent also with Kollmann (2002) and Bouakez and Rebei (2007). The depreciation rate of the capital stock, \(\delta\), is set to 0.025 which corresponds to many previous studies (e.g., Bouakez and Rebei, 2007; Kollmann, 2002, 2008, and 2010....). Following Kollmann (2002), \(\lambda = 0.75\), the average interval that producers of intermediate goods change their prices à la Calvo.
Furthermore, the risk premium parameter, $\alpha^A$, the steady state import/$Z_{ss}$ ratio, $\alpha^m$, the steady state export/$Z_{ss}^*$ ratio, and the degree of complementarity (substitution) between private and government consumptions, $\gamma$, are set to 0.8, 0.3, 0.0075, and 0.39 respectively. The last parameter is consistent with Baxter and King (1993).

Finally, the parameters’ values related to all shocks ($\rho$ and variances $\epsilon^-$) are included in the appendix C.

3 Results

The main results of the simulation are reported in table 1 according to the baseline model (sticky prices with optimized policy) and all other alternative models (flexible prices, producer currency pricing, and fixed exchange rate regime). The tables 2 and 3 report the variation decomposition of main variables and the predicted standard deviations and mean values of important variables. The impulse response functions are shown in the appendix A.

As for Kollmann (2002), the statistics/responses for the domestic interest rate, $NFA_t$, and different tax rates refer to differences of these variable from steady state values. However, the statistics of the remaining variables are referred to relative deviations from steady state values.

3.1 Results for the baseline model (sticky prices)

The results are reported on the table 1 column (1). The optimized policy rules’ coefficients are the following:

- Monetary Policy rule: $R_t = R_{ss} + 3.857\tilde{\pi}_t^d - 0.5612\hat{Z}_t - 0.8531\hat{e}_t$

- Income tax rate rule: $\tau_t^C = \tau_{ss}^C + 1.5\hat{B}_t - 0.25\hat{G}_t - 0.02\hat{G}_t^C - 0.1\hat{\theta}_t + 0.1\tilde{\pi}_t - 0.2\hat{e}_t$

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• Capital tax rate rule: \( \tau^K_t = \tau^K_{ss} + 1.25\hat{B}_t - 0.64\hat{G}C_t - 0.5\hat{I}G_t - 0.1\hat{\theta}_t + 0.1\hat{n}_t - 0.2\hat{\epsilon}_t \)

• Income tax rate rule: \( \tau^L_t = \tau^L_{ss} + 1.75\hat{B}_t - 0.5\hat{G}C_t - 0.4\hat{I}G_t - 0.45\hat{\theta}_t + 0.2\hat{n}_t - 0.1\hat{\epsilon}_t \).

The simple rules which link the nominal gross interest rate on bonds to both output and inflation gap for monetary policy and each tax rate to public debt give the welfare level closed to above rules. In fact, the welfare level for the richer rule (as given above) is \( \zeta = 0.760664\% \) against for the simple rule of \( \zeta = 0.734483\% \). Besides, when I consider the monetary policy rule without the factor of depreciation rate of exchange rate (\( \hat{\epsilon}_t \)), the welfare drop from \( \zeta = 0.760664\% \) to \( \zeta = 0.757956\% \) which corresponds to a gap of 0.002708\%. Therefore, when the central bank adjust the nominal rate to the factor of depreciation rate of exchange rate, the welfare gain is 0.002708\%. This result is consistent with Kollmann (2002) who finds a gain of 0.002\%. The level of the welfare is decomposed into mean (\( \zeta^m = 0.084656\% \)) and variance (\( \zeta^v = 0.760664\% \)).

The negative sign of the coefficient related to the output gap explained by two important economic intuitions. The first intuition is the direct impact of risk premium shock on nominal exchange rate. In fact, an increase in risk premium shock depreciates the nominal exchange rate. Therefore, from the Uncovered Interest Rate equation (14), since the foreign nominal interest rate is exogenous, only the domestic nominal interest should decrease in order to off-set the depreciation of nominal exchange rate. The second intuition is the consequence of the first one. The depreciation of nominal exchange rate of the domestic currency gives incentive to the domestic intermediate firms to export more which increase the domestic output. This result is confirmed by Kollmann 2002 who finds the same sign for output gap. However, in terms of magnitude of the coefficient of output gap, our results are strongly different. Kollmann 2002 finds \(-0.01\) and my paper finds \(-0.58\). This difference is due to the combined impact of increase on both risk premium and government spending shocks on nominal exchange rate and output in my paper. Both
shocks depreciate nominal exchange rate and increase the output which exacerbates the coefficient of output gap through the equation (14). Kollmann 2002 does not have a government sector in the model.

The standard deviation for the domestic PPI inflation is 0.7818% for the richer rule with $\hat{e}$ against 0.6375% for the simple rule. However, the standard deviation for the CPI is lower than the domestic PPI 0.6345%. Furthermore, the standard deviation for output, consumption, and private investment are 1.7317%, 1.5245%, and 11.8239% respectively. The predicted standard deviation for the private investment is very higher than other variables. Standard deviation for private consumption and output is relatively closed to Kollmann (2002) which were about 2%. The standard deviation of net foreign asset and the real exchange rate are 3.35504% and 2.0903% respectively.

The mean of the hours worked, output, inflation (CPI), inflation (PPI), private capital stock and total capital stock are respectively 0.738%, 0.9124%, 0.0102%, 0.0123%, 2.0722% and 1.4645% below their steady state. However, the mean of the private consumption is about 0.1531% above its steady state. Furthermore, the mean of the stock of foreign asset is above its steady state by an amount which is about 0.2957% of the steady state of the real GDP. Moreover, the mean of the real exchange rate shows an appreciation of 0.0966% with respect to its steady state and the mean of nominal exchange exhibits a depreciation of 0.1073% with respect to its steady state. Finally, the mean of import of intermediate goods is about 8.7088% below its steady state.

The government purchases are positive correlated with private consumption, real output, private capital stock, and hours worked. However, there negatively correlated with both real and nominal exchange rate. More specifically, the coefficient of correlation between the government purchases and the macroeconomic variable listed above are respectively 0.0078, 0.046, 0.0121, 0.0477, -0.1404 and -0.2038 for private consumption, real output, private capital stock, hours worked, nominal and real exchange rate.
When I consider the decomposed government purchases, the government consumption is highly correlated to major macroeconomic variables relative to government investment. In fact, the coefficient of correlation between government consumption and other macroeconomic variables are respectively 0.0073, 0.0466, 0.0112, 0.0479, -0.1406 and -0.2056. However, the coefficient of correlation between government investment and other macroeconomic variables are respectively 0.0035, 0.0010, 0.0063, 0.0029, -0.0108 and -0.0079.

The variance decomposition of the major macroeconomic variables indicates that the productivity shock is the main source of volatility of these variables. In fact, the productivity shock explains 78.78% of the variance of real output, 77.69% of private consumption, 85.23% of hours worked, 80.23% of private investment, 83.30% of the CPI inflation, 86.96% of producer domestic inflation, 50.31% of real exchange rate, and 65.10% of nominal exchange rate. Besides, the world inflation shock constitutes the second main source of volatility after the productivity shock.

The dynamic responses to the different shocks show that the real GDP and private consumption raises in response to one percent increase in world output shock, world inflation shock, and domestic productivity shock. However, the world interest rate, and the risk premium lower the real GDP after two first quarters. The private consumption lowers in risk premium shock and world interest rate. The real exchange rate depreciates in response to an increase in world output, risk premium shock, and world interest rate. However, the real exchange rate appreciates after an increase in world inflation shock(after the first two quarters) and productivity shock. Furthermore, the nominal exchange rate appreciates after an increase in world output shock(after the first two quarters), and world inflation (after 7 quarters). Nevertheless, the nominal exchange rate depreciates after a positive shock to the risk premium shock, world interest rate, and domestic productivity. Finally, a positive world interest rate shock (for the first 10 quarters), domestic productivity shock (after the first 7 quarters), risk premium shock (for the
last thirteen quarters), and world inflation increase the nominal gross domestic interest rate. However, the nominal gross domestic interest rate falls after a positive world output (after the first ten quarters), world interest rate for the first fifteen quarters, and the domestic productivity shock (the first eight quarters). These results are confirmed by Kollmann, 2002.

3.2 Results for Flexible price

Under the richer rule, the welfare level is \( \zeta = 2.404015\% \) whose gain is driven mostly by its mean component, \( \zeta^m = 2.182918\% \). Besides, the contribution of the variance component is not negligible, \( \zeta^v = 0.221098\% \). This result seems confirm the literature (Obdiftfeld and Rogoff (1995, 1998, 2000, and 2001), Betts and Devereux 1996, Devereux and Engel 2003 and 2006, Kollmann 2002, and Engel 2013). In fact, the literature predicts high welfare under flexible price than under sticky price.

The standard deviation for the domestic PPI inflation is 0.2196\% which is slightly lower than the CPI inflation which is 0.3850\%. Moreover, the standard deviation for output, consumption, and private investment are 0.2247\%, 0.3790\%, and 7.9329\% respectively. The predicted standard deviation for the private investment is very higher than other variables which means that the private investment is more volatile than other variables. Finally, the standard deviation of net foreign asset and the real exchange rate are 0.7389\% and 0.7161\% respectively. In comparison to the sticky price model, this model is less volatile which confirm the literature (Chari, Kehoe, and McGrattan, 2000).

The mean of hours worked and output are respectively 0.5494\% and 0.1334\% below their steady state. However, the mean of the private consumption, private investment, inflation (CPI), inflation (PPI), private capital stock and total capital stock are respectively 1.0825\%, 1.6750\%, 0.0494\%, 0.1502\%, 1.6750\%, and 1.1838\% above their steady states. Moreover, the mean of the stock of foreign asset is above its steady state by an amount which is about
0.5523% of the steady state of the real GDP. Furthermore, the mean of the real exchange rate shows a depreciation of 3.8717% with respect to its steady state and the mean of nominal exchange exhibits a depreciation of 2.8836% with respect to its steady state. Finally, the mean of import of intermediate goods is about 3.7675% above its steady state.

3.3 Results for Producer Currency Pricing

The previous models assume that the intermediate goods are sold in consumers’ currencies which implies the deviation from the law of one price and full exchange rate pas-through. In the subsection, I assume that the LOP holds. Therefore, under the optimized rule, the welfare is $\zeta = 2.458651\%$ whose gain is driven mostly by its mean component, $\zeta^m = 2.232015629\%$. Besides, the contribution of the variance component is not negligible, $\zeta^v = 0.226635783\%$. As I mentioned in the previous subsection, the literature has predicted that the producer currency pricing generates the welfare level higher than the consumer currency pricing, especially under the flexible exchange rate regime which is under analysis (e.g., Devereux and Engel 1998, Engel 2013).

The standard deviation for the domestic PPI inflation, 0.1615%, is slightly lower than the CPI inflation which is 0.2039%. Moreover, the standard deviation for output, consumption, and private investment are 0.2229%, 0.3765%, and 7.6400% respectively. The predicted standard deviation for the private investment is very higher than other variables which means that the private investment is more volatile than other variables. Finally, the standard deviation of net foreign asset and the real exchange rate are 0.5436% and 0.7261% respectively.

In comparison to the price to market pricing, the standard deviation of the domestic PPI inflation is lower than the domestic CPI inflation with lowest for PCP (0.1615%) against 0.2196% for flexible price. Therefore, the welfare maximizing monetary and fiscal policy rules entails the stabilization of the
domestic PPI inflation which also confirms by previous works (e.g., Devereux and Engels, 1998; Betts and Devereux, 2000; Kollmann, 2002; Senay and Sutherland, 2010).

3.4 Results for fixed Exchange Rate

Under the richer rules, the welfare level is $\zeta = 0.034306\%$ which is mostly dominated by its variance component, $\zeta^v = 0.033085\%$. The contribution of the mean component is negligible, $\zeta^m = 0.001221\%$. Therefore, by pegging the exchange rate, the welfare is reduced significantly in comparison to the flexible exchange rate regime ($\zeta = 0.760664\%$). These results are confirmed by the previous studies (Engel 2013; Devereux and Yetman 2012; Kollmann 2002).

Besides, the volatility of the main macroeconomic variable such as nominal and real exchange rate, private consumption, output, net foreign assets, and private investment is high under this regime than the flexible one. In fact, the standard deviation of nominal and real exchange rate, private consumption, output, net foreign assets, and private investment are respectively 6.52%, 4.26%, 1.81%, 3.03%, 8.23%, and 25.02%.

Moreover, the means of private consumption, hours worked, private capital stock, import of intermediate goods, and export of intermediate output are below their steady state, respectively, by 2.76%, 2.64%, 1.54%, 0.11%, and 12.48%. However, the means of output and domestic intermediate goods are above their steady state values by 1.57% and 0.63% respectively. Furthermore, the mean of the real exchange rate shows a depreciation of 1.65% with respect to its steady state value. The net foreign assets is above its steady state by an amount which is about 0.46% of the steady state of the real GDP.
3.5 Impact of Decomposed Government purchases on Real Exchange Rate and other macroeconomic variables

Following Kollmann 2002 and 2010, and Basu and Kollmann 2013, the model generates the government purchases-exchange rate puzzles for all the models. Specifically, the impact of government purchases on both the real exchange rate is reported in the different impulse response functions. Based on the sticky price model (baseline model), a positive government consumption contributes more to the depreciation of both nominal and real exchange rates than a positive government investment. In fact, the nominal exchange rate depreciates by 5% immediately, then attains a peak of 13% after ten quarters, and falls at 4% by the end of the twenty fifth quarter due to a positive government consumption shock. Nevertheless, the depreciation of nominal exchange rate after a positive government investment is not only about 0.4% immediately, but also does not exceed 1.5% for the entire period of simulation. Moreover, the depreciation of real exchange rate attains 9% due to a positive government consumption, while a positive government investment depreciate the real exchange rate by no more than 1% for the all quarters.

For the flexible prices’ model, the results seems little different from the sticky price one. A positive government consumption shock induces a depreciation of both real and nominal exchange rates by 11% in high. In contrary, the results are ambiguous for a positive government investment shock. The depreciation is confirmed fully for the real exchange rate by 0.2% in high. But it does not generated the depreciation of nominal exchange after seven quarters. In fact, any positive government investment will appreciate the latter in long run (after the first seven quarters).

The transmission mechanism through which the government purchases generate the depreciation of both nominal and real exchange rates does not differ much of the previous works by Kollmann (1995, 2010) and Basu and Kollmann (2013). In fact, in the literature, so far, there are two main links
the international risk sharing and marginal productivity of public capital stock. This paper adds two additional links through which the decomposed government purchases depreciate the real exchange rate: marginal utility of private consumption and marginal productivity of private capital stock.

An increase in the exogenous government consumption has positive impact on private consumption. This result is a puzzle because it contradicts the prediction of both classical and Keynesian. It well known as the crowdind-out of private consumption assumption. By this mechanism, the decrease in private consumption increase the relative consumption on the definition of the international risk sharing which appreciate the real exchange rate. However, when there is crowdind-in of the private consumption, the relative consumption will decrease, then the real exchange rate will depreciate. Another way to see it is through the marginal utility of consumption.

Moreover, the previous work on the topic by Kollmann (1995, 2010) and Kollmann and Basu (2013) predict the link between the real exchange rate and government purchases through the marginal productivity of labor which increases the output. This paper finds another channel, the marginal productivity of the private capital stock since the latter works did not include the private capital. By this channel, the government investment has a double impact on output: it boosts not only the marginal productivity of total capital through the productivity public capital, but also the marginal productivity of labor which both have a strong impact on output.

The last prediction of the previous works is the completeness of the market of asset market. Since in the paper I consider the case of incomplete asset market in the sense that the domestic bonds are not allowed to be purchased by the foreign households. Therefore, the latter limits the international risk sharing which plays an important role on generating the depreciation of the real exchange rate after a raise on government consumption through the crowding-in of private consumption (see Kollmann, 2010).
Furthermore, a raise on both government consumption and invest has a positive impact of output with a delay of two quarters, private consumption in long run, hours worked, gross return on bonds, the next foreign asset position, the export of domestic intermediate goods, domestic intermediate goods, and imports. However, it creates inflation (both CPI and PPI).

4 Conclusion

This paper aims to evaluate the impact tax and monetary policy rules with decomposed government purchases on welfare, real exchange rate and business cycle in a small open economy using a new-Keynesian dynamic stochastic general equilibrium framework. The model predicts that the government consumption has more impact than investment on both private consumption and investment, but less impact on the real GDP. Moreover, the government purchases-real exchange rate puzzle is generated by the model. In this sense, the government consumption contributes more on generating the puzzle than the investment. Moreover, the productive and complement government purchases have positive impact on welfare for any policy rules. The optimized policy rules have a pronounced anti-inflation stance and entail significant nominal and real exchange rate volatility for monetary policy. For tax policy rules, the public debt stance is the optimized rules.
References


A. The complete Non-Linear Model

\[ 1 = \beta E_t \left[ \left( \frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left( \frac{1 + \tau^C_t}{1 + \tau^C_{t+1}} \right) \frac{1}{\pi^C_{t+1}} R_t \right] \quad (73) \]

\[ 1 = \beta E_t \left[ \left( \frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left( \frac{1 + \tau^C_t}{1 + \tau^C_{t+1}} \right) \frac{1}{\pi^C_{t+1}} \phi_{t+1} R_f \right] \quad (74) \]

\[ 1 = \beta E_t \left[ \left( \frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left( \frac{1 + \tau^C_t}{1 + \tau^C_{t+1}} \right) \left( P^K_t (1 - \tau^K_{t+1}) + \delta \tau^K_{t+1} + (1 - \delta) \right) \right] \quad (75) \]

\[ \psi = \left[ \left( \frac{1}{C_t + \gamma GC_t} \right) \left( \frac{1 - \tau^L_t}{1 + \tau^C_t} \right) W_t \frac{1}{\phi_t} \right] \quad (76) \]

\[ 1 = (1 - \alpha^m)(\psi^d_t)^{1-\varphi} + \alpha^m(\psi^m_t)^{1-\varphi} \quad (77) \]

\[ J^d_t = \frac{\nu}{\nu - 1} N^d_t \quad (78) \]

\[ P^*_t = \left[ \frac{1 - \lambda \pi^{d-1}_{t+1}}{1 - \lambda} \right]^\frac{1}{1-\varphi} \quad (79) \]

\[ J^d_t = y^d_t P^*_t + \lambda \beta E_t \left[ \left( \frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left( \frac{P^*_t}{P^d_{t+1}} \right) \pi^{d-1}_{t+1} J^d_t \right] \quad (80) \]

\[ N^d_t = y^d_t MC_t + \lambda \beta E_t \left[ \left( \frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \pi^{d-1}_{t+1} N^d_t \right] \quad (81) \]
\[ J_t^x = \frac{\nu}{\nu - 1} N_t^x \] (82)

\[ P_t^{*x} = \left[ \frac{1 - \lambda^{\nu} t_{t+1}^{\nu-1}}{1 - \lambda} \right]^{\frac{1}{1-\nu}} \] (83)

\[ J_t^x = y_t^* P_t^{*x} + \lambda \beta E_t \left[ \left( \frac{C_t + \gamma G C_t}{C_{t+1} + \gamma G C_{t+1}} \right) \left( \frac{P_t^{*x}}{P_t^{*x}} \right) n_{t+1}^{\nu-1} J_t^x \right] \] (84)

\[ N_t^x = y_t^* M C_t^x + \lambda \beta E_t \left[ \left( \frac{C_t + \gamma G C_t}{C_{t+1} + \gamma G C_{t+1}} \right) n_{t+1}^{\nu-1} N_t^x \right] \] (85)

\[ J_t^m = \frac{\nu}{\nu - 1} N_t^m \] (86)

\[ P_t^{*m} = \left[ \frac{1 - \lambda^{m} t_{t+1}^{m-1}}{1 - \lambda} \right]^{\frac{1}{1-m}} \] (87)

\[ J_t^m = y_t^m P_t^{*m} + \lambda \beta E_t \left[ \left( \frac{C_t + \gamma G C_t}{C_{t+1} + \gamma G C_{t+1}} \right) \left( \frac{P_t^{*m}}{P_t^{*m}} \right) n_{t+1}^{m-1} J_t^m \right] \] (88)

\[ N_t^m = y_t^m \left( \frac{R E R_t}{\psi_t^m} \right) + \lambda \beta E_t \left[ \left( \frac{C_t + \gamma G C_t}{C_{t+1} + \gamma G C_{t+1}} \right) n_{t+1}^{m-1} N_t^m \right] \] (89)

\[ PK_t = (1 - \delta) PK_{t-1} + I p_t \] (90)

\[ GK_t = (1 - \delta) GK_{t-1} + I g_t \] (91)

\[ K_t = PK_t + GK_t \] (92)
\[ Y_t = \theta_t K_t^\alpha L_t^{1-\alpha} \]  

(93)

\[ Y_t = Y_t^d + Y_t^x \]  

(94)

\[ Y_t^d = (1 - \alpha^m)(\psi_t^d)^{-\theta} Z_t \]  

(95)

\[ Y_t^m = \alpha^m(\psi_t^m)^{-\theta} Z_t \]  

(96)

\[ Y_t^x = \alpha_x(\psi_t^x)^{-\eta} Z_t^* \]  

(97)

\[ \psi_t^{lop} = MC_t^m = \frac{RER_t}{\psi_t^m} \]  

(98)

\[ \psi_t^* = \frac{\pi_t^*}{\pi_t} \psi_{t-1}^* \]  

(99)

\[ \psi_t^x = \frac{\pi_t^x}{\pi_t^x} \psi_{t-1}^x \]  

(100)

\[ \psi_t^d = \frac{\pi_t^d}{\pi_t} \psi_{t-1}^d \]  

(101)

\[ \psi_t^m = \frac{\pi_t^m}{\pi_t} \psi_{t-1}^m \]  

(102)

\[ MC_t^x = \frac{\pi_t^d}{\pi_t^x} \frac{1}{e_t} MC_{t-1}^x \]  

(103)
\[ Z_t = C_t + Ip_t + G_t \quad (104) \]

\[ G_t = GC_t + Ig_t \quad (105) \]

\[ W_t = (1 - \alpha)MC_tY_t \left( \frac{1}{L_t} \right) \quad (106) \]

\[ R^K_t = \alpha MC_tY_t \left( \frac{1}{K_t} \right) \quad (107) \]

\[ \psi^*_t Y^*_t - \psi^*_t Y^*_m = A_t - \varphi_t Rf_{t-1}A_{t-1} \quad (108) \]

\[ G_t + D_{t-1} R_{t-1} = D_t + \tau^C_t C_t + \tau^K_t R^K_{t-1} K_{t-1} + \tau^L_t W_t L_t + \delta \tau^K_t K_{t-1} \quad (109) \]

\[ \varphi_t = \exp\left( - \left( \frac{A_t}{A_{ss}} \right) + \chi_t \right) \quad (110) \]

\[ e_t = \frac{S_t}{S_{t-1}} \quad (111) \]

\[ B_t = \frac{D_t}{Z_{ss}} \quad (112) \]

\[ RER_t = S_t \frac{P^*_t}{P_t} = S_t \psi^*_t \quad (113) \]

\[ \frac{R_t}{R_{ss}} = \left( \frac{\pi^d_t}{\pi^d_{ss}} \right)^{\phi_m} \left( \frac{Z_t}{Z_{ss}} \right)^{\phi_Z} \left( \frac{e_t}{e_{ss}} \right)^{\phi_e} \quad (114) \]
\[
\frac{\tau^C}{\tau^s} = \left( \begin{array}{c} B_t \\ B_s \end{array} \right) \gamma^B_C \left( \begin{array}{c} GC_t \\ GC_s \end{array} \right) \gamma^{GC}_C \left( \begin{array}{c} IG_t \\ IG_s \end{array} \right) \gamma^{IG}_C \left( \begin{array}{c} \theta_t \\ \theta_s \end{array} \right) \gamma^\theta_C \left( \begin{array}{c} \pi_t \\ \pi_s \end{array} \right) \gamma^\pi_C (115)
\]

\[
\frac{\tau^K}{\tau^s} = \left( \begin{array}{c} B_t \\ B_s \end{array} \right) \gamma^K_B \left( \begin{array}{c} GC_t \\ GC_s \end{array} \right) \gamma^{GC}_K \left( \begin{array}{c} IG_t \\ IG_s \end{array} \right) \gamma^{IG}_K \left( \begin{array}{c} \theta_t \\ \theta_s \end{array} \right) \gamma^\theta_K \left( \begin{array}{c} \pi_t \\ \pi_s \end{array} \right) \gamma^\pi_K (116)
\]

\[
\frac{\tau^L}{\tau^s} = \left( \begin{array}{c} B_t \\ B_s \end{array} \right) \gamma^B_L \left( \begin{array}{c} GC_t \\ GC_s \end{array} \right) \gamma^{GC}_L \left( \begin{array}{c} IG_t \\ IG_s \end{array} \right) \gamma^{IG}_L \left( \begin{array}{c} \theta_t \\ \theta_s \end{array} \right) \gamma^\theta_L \left( \begin{array}{c} \pi_t \\ \pi_s \end{array} \right) \gamma^\pi_L (117)
\]

\[
\log Z^* = \rho_Z \log Z^*_{t-1} + \epsilon_Z. \quad (118)
\]

\[
\log \pi^* = \rho_\pi \log \pi^*_{t-1} + \epsilon_\pi. \quad (119)
\]

\[
\log Ig_t = \rho_{Ig} \log Ig_{t-1} + \epsilon_{Ig}. \quad (120)
\]

\[
\log GC_t = \rho_{GC} \log GC_{t-1} + \epsilon_{GC}. \quad (121)
\]

\[
\log \chi_t = \rho_\chi \log \chi_{t-1} + \epsilon_\chi. \quad (122)
\]

\[
\log Rf_t = \rho_{Rf} \log Rf_{t-1} + \epsilon_{Rf}. \quad (123)
\]

\[
\log \theta_t = \rho_\theta \log \theta_{t-1} + \epsilon_\theta. \quad (124)
\]

\[
\log \zeta_t = \rho_\zeta \log \zeta_{t-1} + \epsilon_\zeta. \quad (125)
\]

\[
\log \phi_t = \rho_\phi \log \phi_{t-1} + \epsilon_\phi. \quad (126)
\]
B. Log Linear Model

\[
\begin{bmatrix}
\begin{pmatrix}
\frac{C_{ss}}{C_{ss} + \gamma GC_{ss}}
\end{pmatrix} (\hat{C}_{t+1} - \hat{C}_t) + \gamma \begin{pmatrix}
\frac{C_{ss}}{C_{ss} + \gamma GC_{ss}}
\end{pmatrix} (\hat{GC}_{t+1} - \hat{GC}_t)
\end{pmatrix}
+ \begin{pmatrix}
\tau_{C_{ss}}
\end{pmatrix} (\hat{\tau}_{t+1} - \hat{\tau}_t) + \hat{\pi}_{t+1} - \hat{R}_t = 0
\end{bmatrix}
\] (127)

\[
\begin{bmatrix}
\begin{pmatrix}
\frac{C_{ss}}{C_{ss} + \gamma GC_{ss}}
\end{pmatrix} (\hat{C}_{t+1} - \hat{C}_t) + \gamma \begin{pmatrix}
\frac{C_{ss}}{C_{ss} + \gamma GC_{ss}}
\end{pmatrix} (\hat{GC}_{t+1} - \hat{GC}_t)
\end{pmatrix}
+ \begin{pmatrix}
\tau_{C_{ss}}
\end{pmatrix} (\hat{\tau}_{t+1} - \hat{\tau}_t)
+ \hat{\pi}_{t+1} - \hat{R}_t - \hat{\epsilon}_{t+1} - \hat{\phi}_t = 0
\end{bmatrix}
\] (128)

\[
\begin{bmatrix}
\begin{pmatrix}
\frac{C_{ss}}{C_{ss} + \gamma GC_{ss}}
\end{pmatrix} (\hat{C}_{t+1} - \hat{C}_t) + \gamma \begin{pmatrix}
\frac{C_{ss}}{C_{ss} + \gamma GC_{ss}}
\end{pmatrix} (\hat{GC}_{t+1} - \hat{GC}_t)
\end{pmatrix}
+ \begin{pmatrix}
\frac{1}{(R_{K_{ss}}(1 - \tau_{K_{ss}}) + \delta \tau_{K_{ss}} + (1 - \delta))}
\end{pmatrix} (\hat{R}_{K_{ss}}(1 - \tau_{K_{ss}})(\hat{\tau}_{K_{t+1}} - \hat{\tau}_{K_{t+1}}) + \delta \tau_{K_{ss}} \hat{\tau}_{K_{t+1}})
\] (129)

\[
\begin{bmatrix}
\begin{pmatrix}
\frac{C_{ss}}{C_{ss} + \gamma GC_{ss}}
\end{pmatrix} (\hat{C}_t) + \gamma \begin{pmatrix}
\frac{C_{ss}}{C_{ss} + \gamma GC_{ss}}
\end{pmatrix} (\hat{GC}_t)
\end{pmatrix}
+ \begin{pmatrix}
\frac{1}{(1 + \tau_{L_{ss}})}
\end{pmatrix} (\hat{\tau}_{t+1} - \hat{\tau}_t)
+ \hat{\phi}_t = 0
\] (130)

\[
\hat{P}K_t = (1 - \delta)\hat{P}K_{t-1} + \delta \hat{I}_p_t
\] (131)

\[
\hat{G}K_t = (1 - \delta)\hat{G}K_{t-1} + \delta \hat{I}_g_t
\] (132)

\[
\hat{K}_t = \frac{PK_{ss}}{K_{ss}} \hat{P}K_t + \frac{GK_{ss}}{K_{ss}} \hat{G}K_t
\] (133)

\[
\hat{Y}_t = \hat{\theta}_t + \alpha \hat{K}_{t-1} + (1 - \alpha)\hat{L}_t
\] (134)
\[ \dot{Y}_t = \frac{Y^d_{ss} \dot{Y}^d_t}{Y_{ss}} + \frac{Y^x_{ss} \dot{Y}^x_t}{Y_{ss}} \]  

(135)

\[ \dot{Y}^d_t = -\vartheta \hat{\psi}^d_t + \dot{Z}_t \]  

(136)

\[ \dot{Y}^m_t = -\vartheta \hat{\psi}^m_t + \dot{Z}_t \]  

(137)

\[ \dot{Y}^x_t = -\eta \hat{\psi}^x_t + \dot{Z}_t^\star \]  

(138)

\[ \hat{\psi}^L_{OP} = \dot{MC}_t^m = R\dot{ER}_t - \hat{\psi}^m_t \]  

(139)

\[ \hat{\psi}^\star_t = \hat{\pi}^\star_t - \hat{\pi}_t + \hat{\psi}^\star_{t-1} \]  

(140)

\[ \hat{\psi}^x_t = \hat{\pi}^x_t - \hat{\pi}_t + \hat{\psi}^x_{t-1} \]  

(141)

\[ \hat{\psi}^d_t = \hat{\pi}^d_t - \hat{\pi}_t + \hat{\psi}^d_{t-1} \]  

(142)

\[ \hat{\psi}^m_t = \hat{\pi}^m_t - \hat{\pi}_t + \hat{\psi}^m_{t-1} \]  

(143)

\[ \dot{MC}_t^x = \hat{\pi}^x_t - \hat{\pi}_t + \dot{\epsilon}_t + \dot{MC}_{t-1}^x \]  

(144)

\[ \dot{Z}_t = \frac{C_{ss} \dot{C}_t}{Z_{ss}} + \frac{Ip_{ss} \dot{Ip}_t}{Z_{ss}} + \frac{G_{ss} \dot{G}_t}{Z_{ss}} \]  

(145)

\[ \dot{G}_t = \frac{GC_{ss} \dot{G}_t}{G_{ss}} + \frac{Ig_{ss} \dot{I}_g_t}{G_{ss}} \]  

(146)
\[ \dot{W}_t = \dot{MC}_t + \dot{Y}_t - \dot{L}_t \]  
(147)

\[ \dot{R}_t^K = \dot{MC}_t + \dot{Y}_t - \dot{K}_t \]  
(148)

\[ \begin{bmatrix}
\dot{A}_t \\
\frac{(1-Rf_{ss}\varphi_{ss})}{Rf_{ss}\varphi_{ss}} \dot{A}_{t-1} \\
- \left( \frac{\psi^{f*}_{ss}Y^x_{ss}}{(\psi^{f*}_{ss}Y^x_{ss} - \psi^{m*}_{ss}Y^m_{ss})} \right) (\dot{Y}^x_t + \dot{\psi}^*_t) \\
+ \left( \frac{\psi^{f*}_{ss}Y^m_{ss}}{(\psi^{f*}_{ss}Y^x_{ss} - \psi^{m*}_{ss}Y^m_{ss})} \right) (\dot{Y}^m_t + \dot{\psi}^*_t)
\end{bmatrix} = 0 \]  
(149)

\[ \begin{bmatrix}
\dot{G}C_t + \dot{I}g_t + \frac{1}{\beta} (\dot{D}_t + \dot{R}_{t-1} - \dot{\pi}_t) \\
= \dot{D}_t + \frac{\tau C_{ss}}{G_{ss}} (\dot{\tau}^C_t + \dot{C}_t) \\
+ \frac{\tau E R K_{ss}}{G_{ss}} (\dot{\tau}_t^K + \dot{R}_{t-1}^K + \dot{K}_{t-1}) \\
+ \frac{\tau E W_{ss} L_{ss}}{G_{ss} \delta r_{ss}} (\dot{\tau}_t^L + \dot{W}_t + \dot{L}_t) \\
- \frac{\tau E W_{ss} L_{ss}}{G_{ss} \delta r_{ss}} (\dot{\tau}_t^K + \dot{K}_{t-1})
\end{bmatrix} \]  
(150)

\[ \dot{R}ER_t = \dot{S}_t + \dot{\psi}^*_t \]  
(151)

\[ \dot{e}_t = \dot{S}_t - \dot{S}_{t-1} \]  
(152)

\[ \dot{B}_t = \dot{D}_t - \dot{\pi}_t \]  
(153)

\[ \dot{\pi}^d_t = \beta E_t \dot{\pi}^d_t + \frac{(1-\beta\lambda)(1-\lambda)}{\lambda} \dot{MC}_t \]  
(154)

\[ \dot{\pi}^m_t = \beta E_t \dot{\pi}^m_t + \frac{(1-\beta\lambda)(1-\lambda)}{\lambda} (\dot{R}ER_t - \dot{p}s^m_t) \]  
(155)
\[ \hat{\pi}_t^x = \beta E_t \hat{\pi}_t^x + \frac{(1 - \beta \lambda)(1 - \lambda)}{\lambda} \hat{MC}_t^x \] (156)

\[ \hat{\phi}_t = (-\alpha^A \hat{A}_t + \hat{c} \hat{h}_t) \] (157)

\[ \hat{\pi}_t = (1 - \alpha^m)(\psi_{ss}^{d})^{(\theta - 1)} \hat{z}_t^d + \alpha^m(\psi_{ss}^{m})^{(\varphi - 1)} \hat{\pi}_t^m \] (158)

\[ \hat{R}_t = \phi_\pi \hat{\pi}_t^d + \phi_Z \hat{Z}_t + \phi_e \hat{e}_t \] (159)

\[ \hat{\tau}_C^t = \gamma_B \hat{B}_t + \gamma_{GC} \hat{GC}_t + \gamma_I \hat{I}_t + \gamma_\theta \hat{\theta}_t + \gamma_C \hat{\pi}_t + \gamma_e \hat{e}_t \] (160)

\[ \hat{\tau}_K^t = \gamma_B \hat{B}_t + \gamma_{GC} \hat{GC}_t + \gamma_I \hat{I}_t + \gamma_\theta \hat{\theta}_t + \gamma_K \hat{\pi}_t + \gamma_e \hat{e}_t \] (161)

\[ \hat{\tau}_L^t = \gamma_B \hat{B}_t + \gamma_{GC} \hat{GC}_t + \gamma_I \hat{I}_t + \gamma_\theta \hat{\theta}_t + \gamma_L \hat{\pi}_t + \gamma_e \hat{e}_t \] (162)

\[ \hat{\pi}_t^* = \rho \hat{\pi}_t^*_{t-1} + \epsilon_t^* \] (163)

\[ \hat{\pi}_t^* = \rho \hat{\pi}_t^*_{t-1} + \epsilon_t^* \] (164)

\[ G\hat{C}_t = \rho_{GC} G\hat{C}_{t-1} + \epsilon_t^{GC} \] (165)

\[ \hat{I}_g_t = \rho_{Ig} \hat{I}_g_{t-1} + \epsilon_t^{Ig} \] (166)

\[ \hat{R}_f_t = \rho_{Rf} \hat{R}_f_{t-1} + \epsilon_t^{Rf} \] (167)
\[ \hat{\theta}_t = \rho_\theta \hat{\theta}_{t-1} + \epsilon_\theta^t \] (168)

\[ \hat{\chi}_t = \rho_\chi \hat{\chi}_{t-1} + \epsilon_\chi^t \] (169)
C. Sticky Price Model's Impulse Responses

Figure 1: Impulse Response for 1% increase in World Output
Figure 2: Impulse Response for 1% increase in World Output
Figure 3: Impulse Response for 1% increase in World Inflation
Figure 4: Impulse Response for 1% increase in World Inflation

- $yd$ (output)
- $ym$ (import prices)
- $yx$ (export prices)
- $rfa$ (real flows)
- $s$ (interest rate)
- $rer$ (real exchange rate)
- $r$ (real balance}

The impulse responses show the effects of a 1% increase in world inflation on various economic indicators over a 25 period horizon.
Figure 5: Impulse Response for 1% increase in Government Consumption
Figure 6: Impulse Response for 1% increase in Government Consumption

- $y_d$
- $y_m$
- $y_x$
- $n_{ta}$
- $s$
- $r_e$
- $r$
Figure 7: Impulse Response for 1% increase in Government Investment

The figure above shows the impulse response for various variables following a 1% increase in government investment. The variables include:

- $z$: The response variable for the scalar variable.
- $y$: The response variable for the output.
- $c$: The response variable for consumption.
- $ip$: The response variable for investment.
- $l$: The response variable for labor.
- $pid$: The response variable for private domestic investment.
- $pim$: The response variable for private domestic investment.
- $pix$: The response variable for private foreign investment.

Each graph depicts the change in these variables over time, with the x-axis representing time (in months) and the y-axis representing the change in the variable value. The red line indicates the baseline value, and the gray line shows the deviation from this baseline.
Figure 8: Impulse Response for 1% increase in Government Investment
Figure 9: Impulse Response for 1% increase in Risk Premium
Figure 10: Impulse Response for 1% increase in Risk Premium
Figure 11: Impulse Response for 1% increase in World Interest Rate
Figure 12: Impulse Response for 1% increase in World Interest Rate

- $y_d$
- $y_m$
- $y_x$
- $n_{fa}$
- $s$
- $r_{er}$
- $r$
Figure 13: Impulse Response for 1% increase in Domestic Productivity
Figure 14: Impulse Response for 1% increase in Domestic Productivity
D. Flexible Price Model’s Impulse Responses

Figure 15: Impulse Response for 1% increase in World Output
Figure 16: Impulse Response for 1% increase in World Output
Figure 17: Impulse Response for 1% increase in World Inflation
Figure 18: Impulse Response for 1% increase in World Inflation
Figure 19: Impulse Response for 1% increase in Government Consumption
Figure 20: Impulse Response for 1% increase in Government Consumption
Figure 21: Impulse Response for 1% increase in Government Investment
Figure 22: Impulse Response for 1% increase in Government Investment

- $yd$
- $ym$
- $yx$
- $nfa$
- $s$
- $rer$
- $r$
Figure 23: Impulse Response for 1% increase in Risk Premium

![Impulse Response Graphs]

- z
- y
- c
- ip
- l
- pi
- pid
- pim
- pix
Figure 24: Impulse Response for 1% increase in Risk Premium
Figure 25: Impulse Response for 1% increase in World Interest Rate
Figure 26: Impulse Response for 1% increase in World Interest Rate
Figure 27: Impulse Response for 1% increase in Domestic Productivity
Figure 28: Impulse Response for 1% increase in Domestic Productivity

Graphs showing the impulse response for various variables.
Table 1: Baseline model. Optimized policy rule

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<tr>
<td><strong>Welfare (% equivalent variation in consumption)</strong></td>
<td></td>
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<tr>
<td>ζ</td>
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<td>0.22</td>
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