Monetary Policy, Balance Sheet Effect and Economic Fluctuations in a Small Open Economy with Liability Dollarization and Imperfect International Substitutability of Assets

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1. Introduction

Macroeconomic stability is an important objective for many nations. One important policy instrument to achieve this objective is monetary policy. Due to many factors and complications that may influence the business cycle, conducting the appropriate monetary policy regime is not always obvious. The financial sector is an important factor for macroeconomic stability, both for developed and developing countries. An important factor which may influence the financial sector and the conduct of monetary policy in small open economies is the issue of liability dollarization.

For a number of developing countries which have deregulated their banking sector and have an open capital account, their banking system may be in the following situation. On one hand if their banks borrow external funds from international capital markets, it must be denominated in foreign currency, and they must repay the principal and interest to the creditors in foreign currency as well. On the other hand they use these funds to lend out to domestic firms and individuals in domestic currency, and are repaid by their debtors (thus they receive their income) in domestic currency. However the forward markets involving the domestic currencies of these countries are either non-existent or thin and illiquid. This makes it difficult and costly to hedge against exchange rate risk. In the literature this situation is termed as “liability dollarization”, “original sin” or currency mismatch (Eichengreen and Hausman, 1999, Calvo 2002 and Cepedes et.al. 2002, Nakamura, 2011).

There are discussion on the impact of liability dollarization on the choice of optimal monetary policy regime. However it seems that there is still a mixed result on this discussion. On one hand Cepedes et.al. (2004) by employing the Financial Accelerator model of Bernanke et.al. (1999), assuming sticky wages and using entrepreneurs as key economic agent in the model concluded that a flexible exchange rate and inflation targeting provide more macroeconomic stability compared to a fixed exchange rate regime in insulating the economy against external shocks. Following from the result of this study, Nakamura (2011) also found that a targeting rule to address the terms of trade fluctuations is not efficient. In contrast Choi and Cook (2004) which also employs the Bernanke et.al (1999) model but assumes sticky prices and using financial intermediary as the key economic agent in the financial accelerator model found that a fixed exchange rate rule that stabilizes banks’ balance sheets offer greater stability than does an interest rate rule that targets inflation.

A common feature of the above papers is that they do not explicitly include capital inflows in their analysis. In addition the aforementioned papers assumes perfect international substitutability of assets (perfect capital mobility), and does not consider alternative policies to stabilize the economy such as capital controls. Liu and Spiegel (2013) had found that by assuming imperfect capital mobility and capital controls, monetary policy aimed at smoothing the exchange rate (sterilization) is optimal. It should be noted that an important feature of business cycles in small open economies is that they are highly influenced by international capital flows and capital account pressures. This can be seen in the Asian
financial crises of 1997-1999 (Kaminsky and Reinhart, 1998 and Gwang, 1999). This also observed in the recent global financial crises (Liu and Spiegel, 2013).

It would be interesting to extend the preceding discussion relating the optimal monetary policy regime in a small open economy under liability dollarization. In particular we develop a financial accelerator model consistent with liability dollarization, and assume imperfect international substitutability of assets. In the model we will consider cases with capital controls and without capital controls. With this model in mind, we then pose the question of what is the optimal monetary policy regime in the cases considered. In particular we can ask, is monetary policy regime which aims at stabilizing the exchange rate optimal or is it flexible exchange rate inflation targeting which is optimal, in the cases considered? The optimum criteria to be considered in assessing alternative monetary policy rules would be those that would minimize the fluctuations of macroeconomic variables.

2. The model

This study develops an infinite horizon small open economy model with financial intermediaries (banks) and the economy is not a borrower. The banks provide credit to households by purchasing one period bonds denominated in domestic currency issued by households. These banks finance their lending in excess of their net worth by borrowing from foreign investors/creditors.

In this model we assume (1) the economy faces liability dollarization. In other words, given a world interest rate, and due to the issue of “original sin” domestic banks must borrow and redeem the principal and interest from the one period bonds in foreign currency, (2) there is an exogenous moral hazard problem in the forward foreign currency market involving the domestic currency so that we assume for simplicity that banks do not hedge against foreign exchange rate risk, (3) Household and non-financial firms cannot borrow in the international capital market, (4) Assume that there is imperfect international substitutability between domestic and foreign assets, (5) to ensure the existence of a unique positive stochastic discount factor, and because of the previous assumption, assume that (generalized) Uncovered Interest Rate Parity holds and the international financial market is complete, (6) banks lend out only to domestic borrowers and are repaid only in domestic currency. (7) Households purchase home produced intermediate goods and is rented to firms with returns equivalent to the relative price between home and foreign produced intermediate goods.

2.1. Households

A representative household choose consumption and labor and seeks to maximize preference. The households own production firms. The households obtain income from supplying labor and capital to firms and lumps sum profit from the non-competitive production firms. In addition the household accumulate debt at the end of the period t.

Using a modified version of Peters (2008), the optimization problem of the household can be written as to choose consumption and labor to maximize utility represented by the following utility function:
\[ U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\gamma a^t}{(\gamma - 1)\ln C^{\frac{\gamma}{\gamma - 1}}_t} + b^t \left( \frac{M_t}{P_t} \right)^{\frac{\gamma - 1}{\gamma}} + \eta \ln L_t \right], \beta \in (0,1) \]  

(1)

Subject to the following budget constraint:

\[ w_tL_t + \Delta_{dt} + \Delta_{ft} - \frac{M_t}{P_t} + \frac{M_{t-1}}{P_t} + \frac{B_t}{P_t} - \frac{R_t}{P_t} - \frac{B_{t-1}}{P_t} + R_{nt}K_t \geq C_t - T_t \]  

(2)

Denote \( B_t \) as the domestic one period bonds denominated in domestic currency issued by household and sold to banks at the end of period \( t \), \( W_t \) as the real wage and \( \Delta_{dt}, \Delta_{ft} \) as real profit from ownership of firms, \( C_t \) is a composite consumption index of imported and domestically produced consumption goods and \( B_{t-1} \) is the t-1 bonds redeemed by households from banks with paying interest \( (1+i_t) \).

The first order conditions for the household optimization problem are as follows:

\[ \lambda_t = \frac{aC_t^{\frac{1}{\gamma}}}{C_t^{\frac{1}{\gamma}} + b_t^r m_t^r} \]  

(3)

This is the inter-temporal substitution condition of the household. Denote \( 1+i_t \) as the gross rate of return of domestic risk free asset.

The intra-temporal substitution of the household between consumption and work is as follows:

\[ \lambda_t = \beta E_t \left( \frac{P_t \lambda_{t+1}}{P_{t+1}} \right) = \frac{aC_t^{\frac{1}{\gamma}}}{C_t^{\frac{1}{\gamma}} + b_t^r m_t^r} \]  

(4)

The stochastic discount factor is defined as:

\[ \lambda_t \frac{W_t}{P_t} = \frac{\eta}{L_t} \]  

(5)

While the inter-temporal substitution between consumption and purchasing home intermediate goods is as follows:
\[
\psi \left( \frac{k_{t+1}}{k_t} \right) + 1 = \beta E_t \left[ \frac{\lambda_{t+1}}{P_{t+1}} \left( \frac{R^{\text{id}}_{t+1}}{P_{t+1}} + 1 - \delta + \frac{\psi}{2} \left( \frac{k_{t+1}^2}{k_t^2} - 1 \right) \right) \right]
\]

\[
\frac{1}{R_t} = \beta E_t \left[ \frac{P \lambda_{t+1}}{P_{t+1} \lambda_t} \right]
\]

Denote $\delta$ and $\lambda_t$ as the depreciation rate of intermediate goods and the real exchange rate respectively.

2.2. Banking Sector

2.2.1. The setup of the banks' optimization problem

The main economic agent in this model is the bank sector. We define the setting for the banking sector as follows. The small open economy is a net debtor at the world real interest rate. Assuming asymmetric information between domestic households with international investors/lenders such that domestic households do not have access to issuing debt in foreign currency but can issue and sell one period bonds in domestic currency to domestic banks (domestic one period bonds denominated in domestic currency). However the banks has access to a complete asset market which can be traded internationally.

There is a unit measure of risk neutral bank owners which are subject to idiosyncratic and imperfectly observable risk. Assume that banks have access to foreign borrowing and a complete assets market which can be traded internationally. And assume that the bank can then intermediate these funds to domestic households by purchasing domestic one period bonds denominated in domestic currency issued by domestic households.

In excess to its net worth, the bank borrows from foreigners by issuing and selling domestic one period bonds denominated in foreign currency to foreign investors/lenders. The banks then use these funds to either lend in the domestic economy in domestic currency by purchasing domestic one period bonds denominated in domestic currency or purchase foreign one period bonds. Assume that foreign investors/lenders can only purchase domestic assets in the form of domestic one period bonds denominated in foreign currency issued by domestic banks.

Because the domestic banks can also purchase foreign one period bonds, thus the bank has two types of asset; domestic one period bonds denominated in domestic currency issued by domestic households and foreign one period bonds. On the liability side the bank has one type of liability; domestic one period bonds denominated in foreign currency issued by the bank and purchased by foreign investors/lenders. We assume that the domestic banks holds both types of assets in their portfolio.\(^1\)

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\(^1\) To justify this assumption is that banks want to diversify idiosyncratic risk between the small open economy and the world economy and because we assume it is very costly to hedge against foreign exchange rate risk in the futures market for the domestic market. So as a way to reduce this risk they may purchase some foreign currency denominated one period bonds. We can also assume that due to some exogenous moral hazard, the bank opt not to hedge against exchange rate risk in the costly futures market but instead alternatively they purchase foreign currency denominated short-term bonds.
Assume that there is a portfolio adjustment cost from altering the composition of the portfolio between foreign and domestic assets.

A banker begins period $t$ with net worth $nW_t$. At time $t$ the banker purchase domestic one period bonds denominated in domestic currency from households, $b^{i,d}_t$ which pays a risk free return $(1 + i^d_t)$ in domestic currency. In addition banks also choose to purchase one period foreign currency denominated bonds $b^{i,f}_t$ which gives non-diversifiable free return in foreign currency of $(1 + r_{f,t})$. Following Liu and Spiegel (2013) assume that $(1 + r_{f,t})$ is exogenous and follows a stationary process:

$$\ln(1 + r_{f,t}) = (1 - \rho_r)\ln(1 + r) + \rho_r \ln(1 + r_{f,t-1}) + \sigma_r e_{r_t}$$

with $\rho_r \in (0, 1)$. Denote $(1 + r)$ as the steady state foreign interest rate, $\sigma_r$ is the standard deviation of $(1 + r_{f,t})$ and is an i.i.d. $e_{r_t}$ is a standard normally distributed random variable.

To finance this purchase in excess of net worth the banks issues and sells domestic one period bonds denominated in foreign currency to foreign investors/lenders and convert the proceeds into domestic currency at the nominal spot exchange rate $e_t$. Denote $e_t$ as the domestic currency price of one unit of foreign currency. Assume that interest rate for the domestic one period bond denominated in foreign currency issued by the bank is $(1 + i^d_t)$ which must be paid in foreign currency. Assume also that the central bank of the domestic economy can impose capital control policy in the form of tax on capital inflow in the amount of $(1 - \tau)$. This imply that the gross rate of return that foreign investors receive from purchasing domestic one period bonds denominated in foreign currency issued by the domestic bank if the capital control tax was imposed $(\tau > 0)$ is $\left(1 + i^d_t\right) \left(1 - \tau\right)$, while if capital control tax is absent then the foreign investors receive the full return $\left(1 + i^d_t\right)$.

In addition assume that domestic assets are imperfect substitutes for foreign assets and there is a portfolio adjustment cost represented by the parameter $\Omega_d$. As a note, by the assumption of imperfect substitutability between foreign and domestic assets, the domestic bank would hold both asset despite the portfolio adjustment cost. Thus we can write the debt of the bank, in the form of domestic one period bonds denominated in foreign currency issued by bank “h” and sold to foreign investors, multiplied by adjustment cost in excess of net worth as:

$$d_t^f = \left(b^{i,d}_t + e_t b^{i,f}_t\right) \left[1 + \frac{\Omega_d}{2} \left(\Psi - \Psi^f\right)\right] - nW_t$$
\[ \Psi_t = \frac{b_{t,h}^{l,h}}{b_{t,h}^{l,h} + e_t b_{b_t}^{l,h}} \]

The term \( \Psi_t \) denotes the share of domestic currency bonds to total bonds while \( \Psi \) denotes the steady state share of domestic currency bonds to total bonds.

As in Edwards and Vegh (1997) assume that the banks require real resources to engage in intermediation in the domestic economy which is a fraction of the gross asset of the bank, but not for purchasing foreign one period bonds (the cost of purchasing foreign bonds is summarized by the portfolio adjustment cost specified in the equation above). Following Choi and Cook (2004) for the domestic currency lending, denote the nominal net receipts of banks after one period which also includes purchase of foreign assets as \( \omega_{t+1}^{i} (1 + i_t) b_{t}^{l,b} + e_{t+1} (1 + r_t) b_{t}^{l,f} \) measured in domestic currency.

Denote \( \omega \) as the technology of bank \( t \) at time \( t \), which is a random variable independent across banks and distributed uniformly over \([\omega_{t+1}^{i}, 1] \). Foreign investors/lenders can observe \( \omega_{t+1}^{i} \) only at some monitoring cost which is equals to a fraction of gross asset of the bank \( \mu [\omega_{t+1}^{i} (1 + i_t) b_{t}^{l,b} + e_{t+1} (1 + r_t) b_{t}^{l,f}] \).²

In this paper we assume that the optimization behavior of foreign lenders is exogenous to the economy.

Closely following Carlstrom and Fuerst (1997), the optimal financial contract can be described as follows. The bank chooses the quantity of domestic currency denominated lending (which is equivalent to the amount of domestic one period bonds denominated in domestic currency) \( b_{t}^{l} \) and foreign one period bonds \( b_{t}^{F} \) prior to the realization of the idiosyncratic risk \( \omega_{t}^{i} \), interest rate for domestic currency lending (from purchasing one period bonds denominated in domestic currency), \( 1 + i^d_t \), interest from purchasing foreign one period bonds \( 1 + r_t \), which is exogenous and \( 1 + i^d_t \), which is the interest rate that the bank must pay in foreign currency when issuing and selling domestic one period bonds in denominated in foreign currency to foreign investors. As in Bernanke et al. (1999) the interest rate \( 1 + i^d_t \) is state contingent to eliminate any aggregate risk to foreign investors.

The implied state contingent minimum efficiency level \( \omega \) at which default is avoided is:

² It should be noted that the nominal exchange rate attached to the expression of the nominal net receipt of the bank and the monitoring cost of observing \( \omega_{t}^{i} \) by lenders is the nominal spot exchange rate at period \( t+1 \), when \( \omega_{t+1}^{i} \) is known. So in these expression we should use \( e_{t+1} \) not \( e_t \). This is because the foreign asset pays a return in foreign currency, thus the relevant exchange rate should be at the time when pay off is realized which is in period \( t+1 \).
\[
\frac{1}{\omega_{e+1}^{t}} \left( 1 + i_{t} \right) b_{t}^{i,h} + e_{e+1}^{t} \left( 1 + r_{t} \right) b_{bt}^{i,f} \frac{1}{e_{e+1}} = \left( 1 + i_{t}^{*} \right) \left[ \frac{1 + \Omega_{b} + \frac{b_{bt}^{l,h}}{b_{t}^{l,h} + e_{t} b_{bt}^{l,f}} - \psi}{e_{t}} \right]^{2} - n \omega_{e}^{t}
\]

For a given realization of the spot nominal interest rate \(e_{e+1}^{t}\), and in the case of no default, the expected payoff for the bank is the proceeds that it receives from the domestic one period bonds denominated in domestic currency and foreign one period bonds minus the interest paid to foreign investors/lenders from their holding of domestic one period foreign currency denominated bonds multiplied by the probability of no default. In the case of default the bank receives nothing. Thus the expected payoff to the banker can be written as:

\[
f^{t}\left( \omega_{e+1}^{t} \right) \left[ (1 + i_{t}) b_{t}^{i,h} + e_{e+1}^{t} (1 + r_{t}) b_{bt}^{i,f} \right]
\]

Following Choi and Cook (2004), the fraction of expected payoff of the banker over the relevant range of \(\omega\) with positive probability can be written as:

\[
f^{t}(\omega) = \int_{\omega}^{1} \frac{\omega}{1 - \omega} d\omega - \int_{\omega}^{1} \frac{1}{1 - \omega} d\omega
\]

This is the share of net assets retained by the bank\(^3\).

The expected payoff of foreign lenders (investors) is the interest payment received net of capital control tax in the case of no default, or the value of the bank (net of liquidation cost) in the case of default, and can be represented as follows:

\[
g^{t}\left( \omega_{e+1}^{t} \right) \left[ (1 + i_{t}) b_{t}^{i,h} + e_{e+1}^{t} (1 + r_{t}) b_{bt}^{i,f} \right]
\]

The fraction of expected payoff over the relevant range of \(\omega\) with positive probability can be written as:

\[
g^{t}(\omega) = (1 - \tau_{t}) \int_{\omega}^{1} \frac{1}{1 - \omega} d\omega - \int_{\omega}^{1} \frac{(1 - \mu) \omega}{1 - \omega} d\omega
\]

This expression is the share of net asset which goes to foreign investors.\(^4\)

\(^3\) The derivation of expected payoff of bankers is presented in the appendix of this paper.
To construct the expression for the optimal financial contracting problem we assume the following. First assume that the bank has sufficient bargaining power to collect residual returns. Second assume constant dollar price of foreign goods. Finally assume that the foreign investors/lender receive an ex post average return equal to an exogenous risk free interest rate in foreign currency \((1 + i_{t})\). The optimal financial contracting problem can be written as follows:

\[
\text{Max } E_t \left[ f(\omega_{t+1}) \left\{ \frac{e_t}{e_{t+1}} \right\} \left[ (1+i_t)b_{i,t} + e_{t+1}(1+r_t) b_{b,t} \right] \right]
\]

Subject to

\[
g(\omega_{t+1}) \left[ \frac{1}{e_{t+1}} \right] \left[ (1+i_t)b_{i,t} + e_{t+1}(1+r_t) b_{b,t} \right] = \left( 1+r_t \right) \left( \frac{1}{e_{t+1}} \right) \left[ b_{i,t} + e_b^{l,b} \left( 1 + \frac{\Omega_b \left( b_{i,t} + e_b^{l,b} \right)}{b_{i,t} + e_b^{l,b}} - \psi \right) \right] - nW_t
\]

(7)

The first order conditions for the banker’s optimization problem are

\[
E_t \left[ \left[ \frac{1}{dp_{t+1}} \right] \left[ (1+i_t) \left( \frac{e_t}{e_{t+1}} \right) - (1+r_t) \right] \right] = \left( 1 - \frac{1}{e_{t+1}} \right) (1+r_t) \Omega_b \left( \frac{b_{i,t}}{b_{i,t} + e_b^{l,b} - \psi} \right)
\]

(8)

Denote \(dp_{t+1} = \frac{E_t(\lambda_{t+1})}{f'(\omega_{t+1}) + \lambda_{t+1}g'(\omega_{t+1})}\) as the gross default risk premium of banks which following Choi and Cook (2004) depends on the credit worthiness (and implying it depends also on the leverage ratio) of the bank\(^4\).

\[
E_t \left[ f'(\omega_{t+1}) + \lambda_{t+1}g'(\omega_{t+1}) \left( \frac{e_t}{e_{t+1}} \right) \left[ (1+i_t)b_{i,t} + e_{t+1}(1+r_t) b_{b,t} \right] \right] = 0
\]

The first of the above expressions for the necessary conditions can be viewed as a generalized UIP condition. This is because if the portfolio adjustment cost were zero under the assumption of perfect substitutability between foreign and domestic assets, then we get an open economy version of the supply curve of investment finance a la. Bernanke et. al (1999) where the existence of financial frictions in the form of liability dollarization produces a wedge in the standard UIP condition in the form of a (gross) default premium (which in Bernanke et. al (1999) it is referred to as external financing premium). And if the gross default risk premium (which is a function of the credit worthiness of the bank) is one

\(^4\) The derivation of expected payoff of foreign lenders/investors is presented in the appendix of this paper.

\(^5\) The derivation of the generalized UIP condition is presented in the appendix.
(which implies net default risk premium is zero) and if there is no capital controls so that \( \tau_i = 0 \) then this expression would reduce to a standard UIP condition.

As argued in Choi and Cook (2004) assume that the bank productivity \( \bar{\omega} \) is independent of any individual characteristic of the bank. With this assumption define the aggregate constraint of all banks in the economy as:

\[
ge\left(\bar{\omega}, \frac{1}{e_t} \right) \left[ (1 + i_{t-1})B_{t-1}^H + e_t (1 + r_{t-1}) \tilde{B}_{t-1}^F \right] = (1 + r_t) \left( \frac{1}{e_{t-1}} \right) \left( B_{t-1}^H + e_{t-1} \tilde{B}_{t-1}^F \right) \left[ 1 + \Omega_b \left( \frac{B_{t-1}^H}{B_{t-1}^H + e_{t-1} \tilde{B}_{t-1}^F} - \psi \right) \right] - nW_{t-1}
\]  

(9)

In addition the first order conditions in aggregate form can also be written as:

\[
E_t \left[ \left[ \frac{f(\bar{\omega}, \lambda_{t+1}) + \lambda_{t+1} g(\bar{\omega}, \lambda_{t+1})}{\lambda_{t+1}} \right] \left[ (1 + i_t) \left( \frac{e_t}{e_{t+1}} \right) - (1 + r_t) \right] \right] = \left( 1 - \frac{1}{e_t} \right) (1 + r_t) \Omega_b \left( \frac{B_t^H}{B_t^H + e_t \tilde{B}_t^F} - \psi \right)
\]  

(10)

For convenience redefine the proportion of the banks' portfolio held in domestic assets to be in real terms as follows:

\[
\Psi_t = \frac{B_t^H}{B_t^H + e_t \tilde{B}_t^F} = \frac{B_t^H}{P_t} \frac{P_t^H}{P_t^H + e_t P_t \tilde{P}_t^F} = \frac{B_t^H}{P_t} \frac{P_t^H}{P_t^H + e_t P_t \tilde{P}_t^F} = \frac{b_t^h}{b_t^h + s_t b_t^f}
\]  

(11)

Following Carlstrom and Fuerst (1997) and Bernanke et al. (1999), assume that bankers are risk neutral with constant subjective discount factor and that a fraction \( 1 - \gamma \) of bankers die in every period and in the final period they will consume all of their net worth. Using this assumption and redefine the debt of the bank in terms of the domestic currency using the redefinition in equation (12), we can aggregate the one period bonds issued and sold by domestic banks to foreign investors (which is also the supply of one period bonds in foreign currency issued by domestic banks) as:
\[ D_t = E_t \left( B_t^H + e_t B_{bi}^F \right) \left[ 1 + \frac{\Omega_b}{2} \left( \frac{B_t^H}{B_t^H + e_t B_{bi}^F} - \overline{\psi} \right)^2 \right] - NW, \]  
\[ = \left( b_t^k + s_t b_t^f \right) \left[ 1 + \frac{\Omega_b}{2} \left( \overline{\psi_t} - \overline{\psi} \right)^2 \right] - NW, \]  
(12)

This is basically the amount of borrowing in foreign exchange taken by the banks from the international capital market.

Following Corsetti, Dedola and Leduc (2011), the assumption of imperfect substitutability between domestic assets and foreign assets imply that the demand function for domestic one period bonds denominated in foreign currency issued by banks (in terms of domestic currency) is downward sloping. Using a modified version of Liu and Spiegel (2013), the demand for the one period bonds issued by banks can be written as follows:

\[ D_t = E_t \left[ \left( 1 + i_t \right) \frac{S_t}{s_t} \frac{\pi_{t+1}}{\pi_{t+1}} - (1 + r_t) - \frac{\beta}{\beta - \Gamma} \right] \]  
(13)

By assuming \( \Gamma(\cdot) > 0 \), this is interpreted as that an increase in the interest rate differential would increase the foreign demand for domestic one period bond denominated in foreign currency issued by the banks. In the view of the foreign investors, this implies that domestic one period bonds are imperfect substitutes for foreign bonds with similar maturity.

2.2.2. Discussion of the relation between net worth and default premium and defining the financial accelerator model

Now we establish the relation between nominal exchange rate and the default premium of the bank. This relationship is embedded in equations (9), (10) and (11). Note from equation (9) that all variables except \( \overline{\omega_t} \) and \( e_t \) are predetermined. Suppose the nominal exchange rate depreciates (which implies \( e_t \) rises). According to equation (9) and the expression for \( g \left( \overline{\omega_{t+1}} \right) \), this is matched by a rise in \( \overline{\omega_t} \).

From equation (12) and the expression for \( f \left( \overline{\omega_{t+1}} \right) \) a rise in \( \overline{\omega_t} \) requires a fall in \( f \left( \overline{\omega_{t+1}} \right) \) and thus the net worth of the bank falls. For a given level of total domestic debt this in turn implies that the ratio of debt-to-net worth (leverage ratio) rises. Following Choi and Cook (2004), around the steady state, the default risk premium \( dp_t \) is an increasing function of the leverage ratio, thus \( dp_t \) rises. In sum, a depreciation of the nominal exchange rate increases the default premium for the bank.

With the establishment of the relationship between net worth of the bank with the default premium through the change in the exchange rate, we can define the financial accelerator model for the banking
sector. Following Bernanke et al. (1999), the financial accelerator model for the banking sector can be defined by three following equations.

The first equation is the gross return of portfolio of the bank \( R^k \) which can be defined as follows.

\[
E_t \left( R^k_{t+1} \right) = E_t \left( \frac{(1+i_{t+1}) b_{t+1}^h + s_{t+1} (1+r_{t+1}) b_{t+1}^l}{b_t + s b_t^l} \right)
\]

(14)

The relationship between default premium and net worth can be defined as:

\[
E_t \left( \frac{R^k_{t+1}}{(1+r_{t+1})} \right) = \left( \frac{NW_{t+1}}{D_{t+1}} \right)^\gamma
\]

(15)

Denote \( e_{t+1} D_{t+1} \) as the amount of foreign currency borrowing in terms of the domestic currency.

The third equation is the equation for net worth:

\[
NW_t = \Gamma \left[ R^k_t \left( b_{t-1}^h + s_{t-1} b_{t-1}^l \right) - (1+r_{t-1}) D_{t-1} \right] + \frac{\beta}{(\beta - \Gamma)}
\]

\[
= \Gamma \left[ R^k_t \left( b_{t-1}^h + s_{t-1} b_{t-1}^l \right) - (1+r_{t-1}) \left( b_{t-1}^h + s_{t-1} b_{t-1}^l \right) \right] + \frac{\beta}{(\beta - \Gamma)}
\]

\[
= \Gamma \left[ R^k_t \left( b_{t-1}^h + s_{t-1} b_{t-1}^l \right) - (1+r_{t-1}) \left( b_{t-1}^h + s_{t-1} b_{t-1}^l \right) \right] + \frac{\beta}{(\beta - \Gamma)}
\]

(16)

This is the law of motion for the net worth of the banking sector.

2.3. The Firm

The model specification in this section follows from Peters (2008). The firms are owned by households. The production sector consists of three sectors. The final goods firms, the domestic intermediate goods firms and the imported intermediate goods firms.

2.3.1. Domestic and imported composite goods

The composite domestic and imported intermediate goods \( (Y_{di}, Y_{fi}) \) are by combining a continuum of differentiated domestic and imported intermediate goods \( (Y_{di}(i), Y_{fi}(j)) \). Denote the successive intermediate good as \( (P_{di}(j), P_{fi}(j), P_{di}, P_{fi}) \). Profit maximization results into the following demand functions for each goods “j” of domestic and imported differentiated goods, the Producer price index (PPI) and Importer Price Index (IPI).

\[
Y_{di}(j) = \left( \frac{P_{di}(j)}{P_{di}} \right)^{-\theta} Y_{di}
\]

(17)
\[ P_{dt} = \left( \int_0^1 \frac{P_{dt}(j)}{dj} \right)^{-\frac{1}{1-\theta}} \]  \hspace{1cm} (18)

\[ Y_{ft}(j) = \left( \frac{P_{ft}(j)}{P_{ft}} \right)^{-\theta} Y_{ft} \]  \hspace{1cm} (19)

\[ P_{ft} = \left( \int_0^1 \frac{P_{ft}(j)}{dj} \right)^{-\frac{1}{1-\theta}} \]  \hspace{1cm} (20)

2.3.2. The final goods firms

The final goods firm is perfectly competitive. It aggregates domestic and imported intermediate goods \( \left( Y_{dt}, Y_{ft} \right) \) to produce final goods \( Z_i \), with following aggregation technology:

\[ Z_i = \left[ \left( \frac{1}{1-\omega_f} \right)^\nu Y_{dt} + \omega_f^\nu Y_{ft} \right]^{\frac{1}{\nu-1}} \]  \hspace{1cm} (20)

The final goods can be used for consumption and investment so that the resource constraint can be written as:

\[ Z_i = C_i + I_i \]

Profit maximization with respect to equation (20) result in the following demand functions of domestic and imported differentiated goods:

\[ Y_{dt} = \left( 1-\omega_f \right) \left( \frac{P_{dt}}{P_t} \right)^{-\nu} Z_i \]  \hspace{1cm} (21)
\[ Y_{di} = (\omega_f) \left( \frac{P_{fi}}{P_t} \right)^{\omega} Z_i \]  

The consumer price index is defined as:
\[ P_i = \left[ \left( 1 - \omega_f \right) P_{dt}^{1-\nu} + \omega_f P_{fi}^{1-\nu} \right]^{\frac{1}{1-\nu}} \]  

2.3.3. The domestic intermediate goods producing firms and import intermediate goods firms

Each domestic intermediate goods producing firms is monopolistically competitive and adjust price according to Calvo (1983) and a fraction of \((1 - \phi)\) firms can reset their price, while remaining prices are unchanged. Each firm produce differentiated product by employing labor \(L_i\) and capital \(K_i\) from households and using the following technology:
\[ Y_i = K_i^\alpha \left[ A_i L_i \right]^{1-\alpha}, \quad \alpha \in (0,1) \]  

The exogenous technology shock \(A_i\) evolves according to the following law of motion:
\[ \ln A_i = \left( 1 - \rho_A \right) \ln A + \rho_A \ln A_{i-1} + \varepsilon_{A_i} \]  

The domestic intermediate goods can be used domestically or exported so that
\[ Y_i(j) = Y_{di}(j) + Y_{fi}(j) \]  

The foreign demand for domestic goods are defined as follows.
\[ Y_{x_i}(j) = \left( \frac{P_{dt}(j)}{e_i P_t^*} \right)^\theta Y_{xi} \]  

The aggregated foreign demand for domestic exports can be written as:
\[ Y_{xi} = \left( \frac{P_{dt}}{e_i P_t^*} \right)^{\tau - \theta} \]  

Foreign price \(P_t^*\) evolves according to the following law of motion:

13
\[
\ln \left( \frac{P_t}{P_{t-1}} \right) = \left(1 - \rho_{\pi} \right) \ln (\pi^*) + \rho_{\pi} \ln \left( \frac{P_{t-1}}{P_{t-2}} \right) + \epsilon_{\pi}
\]

(29)

Firms chooses price \( \overline{P}_{dt}(j) \) to maximize profit subject to equations (17) and (27). Profit maximization yield the following aggregated first order condition:

\[
\frac{W_t}{P_t} = (1 - \alpha) \frac{Y_t(j)}{L_t(j)} q_t \overline{P}_{dt}
\]

(30)

\[
\frac{R_t}{P_t} = \alpha \frac{Y_t(j)}{K_t(j)} q_t \overline{P}_{dt}
\]

(30)

\[
P_{dt}(j) = \frac{\theta}{1 - \theta} \frac{E_t \sum_{i=0}^{\infty} (\beta \phi)^i \lambda_{t+i} Y_{t+i} q_{t+i}}{E_t \sum_{i=0}^{\infty} (\beta \phi)^i \lambda_{t+i} Y_{t+i}} \overline{P}_{dt+1}
\]

(31)

Define \( \overline{P}_{dt} = \frac{P_{dt}}{P_t} \) and denote \( q_t \) is the Lagrangian multiplier applied to the production function constraint in the profit maximization.

Define the aggregate domestic price index as:

\[
P_{dt}^{1 - \theta} = \phi P_{dt}^{1 - \theta} + (1 - \phi) \overline{P}_{dt}
\]

(32)

2.3.4. Imported Intermediate Goods

Homogenous intermediate goods are imported by a continuum of \( j \in [0,1] \) importers which are of the Calvo (1983) type. A fraction of \( (1 - \phi) \) firms can reset their price, while remaining prices are unchanged. Firms chooses price \( \overline{P}_{fr}(j) \) to maximize profit subject to equation (19). Profit maximization results in the following first order condition:

\[
\overline{P}_{fr}(j) = \frac{\theta}{1 - \theta} \frac{P_{fr+1}}{E_t \sum_{i=0}^{\infty} (\beta \phi)^i \lambda_{t+i} Y_{fr+i}(j) P_{fr+1}}
\]

(33)
Define the real exchange rate as \( S_t = \frac{e_t P_t^*}{P_t} \). Define the aggregate import price index as:

\[
P_t^{1-\theta} = \phi P_{t-1}^{1-\theta} + (1 - \phi) P_t^{1-\theta}
\]

(34)

2.4. The Central Bank

Following Dib (2003) assume that the central bank of the small open economy actively manages short term interest \( R_t \), rate in response to deviation from output \( Y_t \), inflation \( \pi_t \), growth in money supply \( \mu_t \), and the real exchange rate \( S_t \). This a modified Taylor rule and can be written as:

\[
\ln \left( \frac{R_t}{R} \right) = (1 - \rho_s) \ln \left( \frac{Y_t}{Y} \right) + \rho_s \ln \left( \frac{\pi_t}{\pi} \right) + \rho_s \ln \left( \frac{S_t}{S} \right) + \ln \nu_t
\]

(35)

The variables without the time subscripts are in steady state. The monetary policy shock follows the following law of motion.

\[
\ln \nu_t = \rho_\nu \ln \nu_{t-1} + \epsilon_{\nu t}
\]

(36)

2.5. Equilibrium Conditions

Assuming symmetry among firms, the equilibrium condition for this model are:

3. Analysis

3.1. Calibration

The Calibration of the parameters for the households this model are as follows form Peters (2008). In this paper we consider estimates of parameters of two countries (Thailand and Mexico) used in that paper to proxy it as two different monetary policy rules. The parameters for Thailand serves as a proxy for monetary policy rule which place a positive weight on real exchange rate stabilization. On the other hand, Mexico serves as a proxy for monetary policy rule which does place stabilization of the real exchange rate as a monetary policy objective. The steady state values for portfolio of the banks follows from Liu and Spiegel (2013). Parameter value for the survival rate \( \Gamma \) follows Bernanke et al. (1999).

3.2. Impulse response function (IRF)

In this analysis will consider two cases of monetary policy rules of the specification presented in the previous section. The first case is in which monetary authority places a positive weight on real exchange rate stability. The second case is in which the monetary authority does not place any weight on stabilizing the real exchange rate. The weights for this analysis is taken from Peters (2008) and considering two countries. Thailand is taken for the case of positive weight on real exchange rate, while Mexico is taken for the case of zero weights on the real exchange rate.
3.2.1. Monetary policy shock

Consider a positive monetary policy shock which implies a monetary policy tightening in figure 1 and figure 2 in Appendix B. In both cases of the monetary policy rule considered, a monetary policy shock result in a fall in real money growth (\(\mu\)) and domestic inflation (\(\pi\)).

On the other hand the IRF for real exchange rate (s) and exports (yx) are different for the two cases. On impact, for the first case there was a minor depreciation in the real exchange rate which is followed by a small appreciation, then it depreciated again before appreciating and stabilizing. Export for this case export followed the real exchange rate and increased for 10 quarters before decreasing. For the second case, the real exchange rate appreciated before depreciated and stabilizing. Because of this pattern, export declined before increasing then stabilized. As can be expected, appreciation of the real exchange rate is larger on the second case (zero weight) then in the first.

However the IRF of monetary policy shock for the domestic real interest rate show a puzzle. For both cases there was a small decrease in the real interest rate before increasing then stabilizing. Due to this, in the first case (positive weight), investment and output rose while consumption fell. In the second case, consumption and investment in capital increased while output fell.

The IRF for the net worth of the bank sector show similar pattern in the two cases but differ in magnitude. In both case net worth fell on impact. The reason for this is maybe that although an appreciation of the real exchange rate decrease the domestic currency value of the banks foreign debt, but because the bank also holds foreign denominated assets, it values falls. The result from the IRF implies that the is that the positive impact on liabilities with respect to net worth is out weighted by the negative impact due to assets values. This result is consistent with the balance sheet affect in Nakamura (2011).

3.2.2. Foreign real interest rate shock

The pattern of response of the real exchange rate due to a positive shock in the foreign real interest rate is about similar. At impact the real exchange rate depreciates, then appreciates but in the first (positive weight) case appreciation occurs after fifteen quarters while in the second case (zero weight), appreciation occurs after three quarters. With respect to export, in the first case, export rise and stabilize, while in the second case, export rose up to approximately two quarters then fell and stabilized.

In both cases, growth of real money supply (\(\mu\)) and inflation (\(\pi\)) fell, however there is a puzzle in which the real domestic interest rate fell. In case one, output increase then stabilized while in case two, output increased up to approximately two periods then fell before stabilizing. Consumption fell in case one while it rose in case two, while investment rose in both cases before stabilizing.

Net worth of the bank sector rose in both cases. This result may be due to the rise in foreign real interest rates increases the value of foreign assets held by the banks and it outweighs the cost of increased liabilities of its foreign debt the in. This effect is again similar to Nakamura (2011) where bank assets dominate liabilities.

4. Conclusion

By specifying a general equilibrium model with liability dollarization with balance sheet effect and imperfect international substitutability of assets we can observe that there are some variations in the
fluctuations of the macroeconomic variable under consideration for the two monetary policy rule cases. This may indicate that the effect monetary policy rules in small open economies which stabilize the real exchange rate may have different effect on these variables as compared with monetary policy rules which does not focus on the stabilization of the real exchange rate as monetary policy objective.

In addition the result show that when the bank holds both foreign and domestic assets, the asset side of the balance sheet of the bank play dominant role in determining net worth of the bank. This implies that although the economy specified in this model falls under the problem of liability dollarization, if the bank holds both assets and if assets are internationally imperfect substitutes so that the asset side dominates, at least around the steady state, we can say that a depreciation of the real exchange rate does not necessarily result in a fall in the net worth of the bank.

In relation to policy, this result imply that stabilizing the asset side of the balance sheet of the banking sector may be more important than stabilizing the liability side. This is in line with some of the policy recommendations in the literature.
Appendix A.

A Deriving the generalized UIP condition

Start with the debt of the bank, in the form of domestic one period bonds denominated in foreign currency issued to foreign investors, multiplied by asset adjustment cost $\Omega_k$ in excess of net worth.

$$
\left( b_i^{t,b} + e_i b_i^{t,f} \right) \left[ 1 + \frac{\Omega_k}{2} \left( \frac{b_i^{t,f}}{b_i^{l,b} + e_i b_i^{l,f} - \psi} \right) \right] - n w_i
$$

$$
d_i = \frac{\left( b_i^{t,b} + e_i b_i^{t,f} \right) \left[ 1 + \frac{\Omega_k}{2} \left( \frac{b_i^{t,f}}{b_i^{l,b} + e_i b_i^{l,f} - \psi} \right) \right] - n w_i}{e_i}
$$

The term $\psi$ denotes the steady state share of domestic currency bonds in total bonds. Denote $d_i$ as the foreign currency denominated one period bond issued by the domestic banks and sold to foreign investors.

Following Choi and Cook (2004) for the domestic currency lending, denote the nominal net receipts of banks after one period which also includes purchase of foreign assets

as $\omega_{i+1} \left( 1 + i_i \right) b_i^{t,b} + e_{i+1} \left( 1 + r_i \right) b_i^{t,f}$ measured in domestic currency. Denote $\omega_{i}$ as the technology of bank $i$ at time $t$, which is a random variable independent across banks and distributed uniformly over $[\omega, 1]$. Foreign lenders can observe $\omega_{i+1}$ only at some monitoring cost which is equal to a fraction of gross asset of the bank $\mu \omega_{i+1} \left( 1 + i_i \right) b_i^{t,b} + e_{i+1} \left( 1 + r_i \right) b_i^{t,f}$. 6

The implied state contingent minimum efficiency level $\omega$ at which default is avoided is:

$$
\omega_{i+1} \left( 1 + i_i \right) b_i^{t,b} + e_{i+1} \left( 1 + r_i \right) b_i^{t,f} = \left( 1 + i_i \right) e_{i+1} \left[ 1 + \frac{\Omega_k}{2} \left( \frac{b_i^{t,f}}{b_i^{l,b} + e_i b_i^{l,f} - \psi} \right) \right] - n w_i
$$

Assuming that the amount of asset purchased and foreign debt issued domestic banks is predetermined, then the minimum efficiency level of $\omega$ at which default is avoided can be written as:

6 It should be noted that the nominal exchange rate attached to the expression of the nominal net receipt of the bank and the monitoring cost of observing $\omega_{i+1}$ by lenders is the nominal spot exchange rate at period t=1, when $\omega_{i+1}$ is known. So in these expressions we should use $e_{i+1}$ not $e_i$. This is because the foreign asset pays a return in foreign currency, thus the relevant exchange rate should be at the time when payoff is realized which is in period t=1.
\[
\frac{\omega_i}{(1+i_{i-1})}b_{t-1}^{i,h} + e_i(1+r_{i-1})b_{i-1}^{i,f} = \left(1+i_{i-1}^{\varepsilon}\right) \left(1+\frac{\Omega_i}{2} \left[\frac{b_{i-1}^{i,h} + e_i b_{i-1}^{i,f}}{b_{i-1}^{i,h} + e_i b_{i-1}^{i,f} - \psi} \right]^2 \right) - nw_{i-1}
\]

For a given realization of the spot nominal interest rate \(e_{i+1}\), and in the case of no default, the expected payoff for the bank is the proceeds that it receives from the domestic currency and foreign currency bonds minus the interest paid to foreign lenders multiplied by the probability of no default. In the case of default the bank receives nothing. Thus the expected payoff to the banker can be written as:

\[
Pr\left(\omega_i \geq \omega_i \right) \left[ E\left(\omega_i, \omega_{i-1} \geq \omega_i \right) (1+i_{i-1})b_{i-1}^{i,h} + e_i(1+r_{i-1})b_{i-1}^{i,f} \right] - Pr\left(\omega_i \leq \omega_i \right) \left[ \omega_i (1+i_{i-1})b_{i-1}^{i,h} + e_i(1+r_{i-1})b_{i-1}^{i,f} \right]
\]

\[
= Pr\left(\omega_i \geq \omega_i \right) \left[ E\left(\omega_i, \omega_{i-1} \geq \omega_i \right) (1+i_{i-1})b_{i-1}^{i,h} + e_i(1+r_{i-1})b_{i-1}^{i,f} \right] - \omega_i \left[ (1+i_{i-1})b_{i-1}^{i,h} + e_i(1+r_{i-1})b_{i-1}^{i,f} \right]
\]

\[
= f\left(\overline{\omega}_i\right) \left[ (1+i_{i-1})b_{i-1}^{i,h} + e_i(1+r_{i-1})b_{i-1}^{i,f} \right] - \omega_i \left[ (1+i_{i-1})b_{i-1}^{i,h} + e_i(1+r_{i-1})b_{i-1}^{i,f} \right]
\]

Following Choi and Cook (2004), the fraction of expected payoff of the banker over the relevant range of \(\overline{\omega}\) with positive probability can be written as:

\[
f\left(\overline{\omega}\right) = \sum_{\omega} \frac{1 - \omega}{1 - \omega} d\omega - \frac{1}{\sum_{\omega} \frac{1 - \omega}{1 - \omega} d\omega}
\]

This is the share of net assets retained by the bank.

The expected payoff of foreign lenders (investors) is the interest payment received net of capital control tax in the case of no default, or the value of the bank (net of liquidation cost) in the case of default, and can be represented as follows:

\[
Pr\left(\omega_i \geq \omega_i \right) \left[ (1+i_{i-1}) \left( b_{i-1}^{i,h} + e_i b_{i-1}^{i,f} \right) \left[ 1 + \frac{\Omega_b}{2} \left( b_{i-1}^{i,h} + e_i b_{i-1}^{i,f} - \psi \right)^2 \right] - nw_{i-1} \right]
\]

\[
+ (1-\mu) E\left(\omega_i \left[ (1+i_{i-1})b_{i-1}^{i,h} + e_i(1+r_{i-1})b_{i-1}^{i,f} \right]
\]

19
\[
\begin{align*}
&= \Pr\left(\omega_i \geq \omega_i^*\right)\left(1 - \tau_{t-1}\right)\left[\left(1 + i_{t-1}\right)b_{t-1} + e\left(1 + r_{t-1}\right)b_{t-1}^{I_f}\right]\Pr\left(\omega_i^* \leq \omega_i^*\right)\left[\left(1 + i_{t-1}\right)b_{t-1} + e\left(1 + r_{t-1}\right)b_{t-1}^{I_f}\right] \\
&= \left\{\Pr\left(\omega_i \geq \omega_i^*\right)(1 - \tau_{t-1})\omega_i^* + (1 - \mu)E\left(\omega_i^* \mid \omega_i \leq \omega_i^*\right)\Pr\left(\omega_i^* \leq \omega_i^*\right)\left[\left(1 + i_{t-1}\right)b_{t-1} + e\left(1 + r_{t-1}\right)b_{t-1}^{I_f}\right]\right\} \\
&= g\left(\omega_i^*\right)\left[(1 + i_{t-1})b_{t-1} + e\left(1 + r_{t-1}\right)b_{t-1}^{I_f}\right] \\
\end{align*}
\]

The fraction of expected payoff over the relevant range of $\omega$ with positive probability can be written as:

\[
g\left(\omega_i^*\right) = \left(1 - \tau_{t-1}\right)\omega_i^* - \frac{\mu}{1 - \tau_{t-1}}\omega_i^* - \frac{\mu}{1 - \tau_{t-1}}\omega_i^* d\omega
\]

This expression is the share of net asset which goes to foreign investors.

To construct the expression for the optimal financial contracting problem we further assume the following. First assume that the bank has sufficient bargaining power to collect residual returns. Second, assume constant dollar price of foreign goods. Third, assume that the monitoring cost of foreign lenders is non-stochastic. Finally, assume that the foreign lender receive an ex post average return equal to an exogenous risk free interest rate in foreign currency $\left(1 + r_{t-1}\right)$. The optimal financial contracting problem can be written as follows:

\[
\begin{align*}
\max_{b_{t-1}^{I_f}, e_{t-1}} & \mathbb{E}_t\left[f\left(\omega_i^*\right)\left(\frac{e_{t-1}}{e_i}\right)\left[(1 + i_{t-1})b_{t-1} + e\left(1 + r_{t-1}\right)b_{t-1}^{I_f}\right]\right] \\
\text{Subject to} & \\
& g\left(\omega_i^*\right)\left[(1 + i_{t-1})b_{t-1} + e\left(1 + r_{t-1}\right)b_{t-1}^{I_f}\right] = (1 + r_{t-1})\left(\frac{1}{e_{t-1}}\right)\left[b_{t-1}^{I_f} + e\left(1 + r_{t-1}\right)b_{t-1}^{I_f}\right] + \frac{\Omega_{t-1}}{2} \left(\frac{b_{t-1}^{I_f}}{b_{t-1}^{I_r} + e\left(1 + r_{t-1}\right)b_{t-1}^{I_f}} - \psi\right)^2 - \pi_{t-1} \\
\end{align*}
\]

Writing the above problem in Lagragian form:

\[
\begin{align*}
\max_{b_{t-1}^{I_f}, e_{t-1}} & \mathbb{E}_t\left[f\left(\omega_i^*\right)\left(\frac{e_{t-1}}{e_i}\right)\left[(1 + i_{t-1})b_{t-1} + e\left(1 + r_{t-1}\right)b_{t-1}^{I_f}\right]\right] \\
+ & \left(\lambda_{t-1}\right)\left(g\left(\omega_i^*\right)\left[(1 + i_{t-1})b_{t-1} + e\left(1 + r_{t-1}\right)b_{t-1}^{I_f}\right] - (1 + r_{t-1})\left(\frac{1}{e_{t-1}}\right)\left[b_{t-1}^{I_f} + e\left(1 + r_{t-1}\right)b_{t-1}^{I_f}\right] + \frac{\Omega_{t-1}}{2} \left(\frac{b_{t-1}^{I_f}}{b_{t-1}^{I_r} + e\left(1 + r_{t-1}\right)b_{t-1}^{I_f}} - \psi\right)^2 - \pi_{t-1}\right) \\
\end{align*}
\]
Denote $\lambda_{t-1}$ as Lagrange multiplier. The first order condition for domestic currency lending $b_{t-1}^{l,b}$ is:

$$
E_t \left[ f \left( \overline{\omega}_i \right) \left( \frac{e_{t-1}^{(1)}}{e_{t}} \right) (1 + i_{t-1}) + \lambda_t \left( g \left( \overline{\omega}_i \right) \left( \frac{e_{t-1}^{(1)}}{e_{t}} \right) - \left( 1 + r_{t-1} \right) \left[ 1 + \frac{\Omega_b}{2} \frac{b_{t-1}^{l,f}}{b_{t-1}^{l,h} + e_{t-1} b_{t-1}^{l,h}} \right]^2 \right) \right] = 0
$$

$$
-(1 + r_{t-1}) \left( b_{t-1}^{l,h} + e_{t-1} b_{t-1}^{l,f} \right) \Omega_b \left( \frac{b_{t-1}^{l,b}}{b_{t-1}^{l,b} + b_{t-1}^{l,f}} - \psi \right) \left( \frac{b_{t-1}^{l,b} + e_{t-1} b_{t-1}^{l,f} - b_{t-1}^{l,b}}{\left( b_{t-1}^{l,h} + e_{t-1} b_{t-1}^{l,f} \right)^2} \right) = 0
$$

$$
\Rightarrow E_t \left[ f \left( \overline{\omega}_i \right) \left( \frac{e_{t-1}^{(1)}}{e_{t}} \right) (1 + i_{t-1}) + \lambda_t \left( g \left( \overline{\omega}_i \right) \left( \frac{e_{t-1}^{(1)}}{e_{t}} \right) - \left( 1 + r_{t-1} \right) \left[ 1 + \frac{\Omega_b}{2} \frac{b_{t-1}^{l,f}}{b_{t-1}^{l,h} + e_{t-1} b_{t-1}^{l,h}} \right]^2 \right) \right] = 0
$$

$$
-(1 + r_{t-1}) \Omega_b \left( \frac{b_{t-1}^{l,b}}{b_{t-1}^{l,b} + b_{t-1}^{l,f}} - \psi \right) \left( \frac{e_{t-1} b_{t-1}^{l,f}}{b_{t-1}^{l,h} + e_{t-1} b_{t-1}^{l,f}} \right) = 0 \quad (7)
$$

The first order condition for purchasing one period foreign bonds $b_{t-1}^{l,f}$ is:

$$
E_t \left[ f \left( \overline{\omega}_i \right) \left( \frac{e_{t-1}^{(1)}}{e_{t}} \right) e_{t} (1 + r_{t-1}) + \lambda_t \left( g \left( \overline{\omega}_i \right) \left( \frac{e_{t-1}^{(1)}}{e_{t}} \right) e_{t} (1 + r_{t-1}) - e_{t-1} (1 + r_{t-1}) \right) \left[ 1 + \frac{\Omega_b}{2} \left( \frac{b_{t-1}^{l,f}}{b_{t-1}^{l,h} + e_{t-1} b_{t-1}^{l,h}} \right) \right]^2 \right] = 0
$$

$$
-(1 + r_{t-1}) \left( b_{t-1}^{l,h} + e_{t-1} b_{t-1}^{l,f} \right) \Omega_b \left( \frac{b_{t-1}^{l,b}}{b_{t-1}^{l,b} + b_{t-1}^{l,f}} - \psi \right) \left( \frac{-b_{t-1}^{l,h}}{\left( b_{t-1}^{l,h} + e_{t-1} b_{t-1}^{l,f} \right)^2} \right) = 0
$$

$$
\Rightarrow E_t \left[ f \left( \overline{\omega}_i \right) \left( \frac{e_{t-1}^{(1)}}{e_{t}} \right) e_{t-1} \left( 1 + r_{t-1} \right) + \lambda_t \left( g \left( \overline{\omega}_i \right) \left( \frac{e_{t-1}^{(1)}}{e_{t}} \right) e_{t-1} \left( 1 + r_{t-1} \right) - \left( 1 + r_{t-1} \right) \left[ 1 + \frac{\Omega_b}{2} \left( \frac{b_{t-1}^{l,f}}{b_{t-1}^{l,h} + e_{t-1} b_{t-1}^{l,h}} \right) \right]^2 \right) \right] = 0
$$
\[-\left(\frac{1}{e_{t-1}}\right)(1+r_{t-1})\Omega_b\left(\frac{b_{l-1}^{l,h}}{b_{l-1}^{l,h} + b_{l-1}^{l,f}} - \psi\right)\left(\frac{-b_{l-1}^{l,h}}{b_{l-1}^{l,h} + e_{t-1}b_{l-1}^{l,f}}\right)\right\] = 0 \tag{8}

Subtracting the two equations as follows (7) – (8):

\[E_t\left[f\left(\omega_t\right)\left(1 + i_{t-1}\right)\left(\psi + (1 + r_{t-1})\right) + \lambda_t g\left(\omega_t\right)\left(1 + i_{t-1}\right)(1 + r_{t-1})\right] + \lambda_t \left[1 - \frac{1}{e_{t-1}}\right] \left(1 + r_{t-1}\right)\Omega_b \left(\frac{-b_{l-1}^{l,f}}{b_{l-1}^{l,h} + e_{t-1}b_{l-1}^{l,h}} - \psi\right)\]

\[\Rightarrow E_t\left[f\left(\omega_t\right) + \lambda_t g\left(\omega_t\right)\left(1 + i_{t-1}\right)(1 + r_{t-1})\right] - \lambda_t \left[1 - \frac{1}{e_{t-1}}\right] \left(1 + r_{t-1}\right)\Omega_b \left(\frac{-b_{l-1}^{l,f}}{b_{l-1}^{l,h} + e_{t-1}b_{l-1}^{l,h}} - \psi\right)\]

\[\Rightarrow E_t\left[1 - \frac{1}{\lambda_t}\right] \left(1 + i_{t-1}\right)(1 + r_{t-1})\Omega_b \left(\frac{-b_{l-1}^{l,f}}{b_{l-1}^{l,h} + e_{t-1}b_{l-1}^{l,h}} - \psi\right)\]

\[\Rightarrow E_t\left[\frac{1}{dp_{r-1}}\right] \left(1 + i_{t-1}\right)(1 + r_{t-1})\Omega_b \left(\frac{-b_{l-1}^{l,f}}{b_{l-1}^{l,h} + e_{t-1}b_{l-1}^{l,h}} - \psi\right)\]

Denote \(dp_r = \frac{E_r(\lambda_{r-1})}{f(\omega_{r+1}) + \lambda_{r+1} g(\omega_{r+1})}\) as the gross default risk premium of banks which depends on credit worthiness.
Appendix B. Impulse Response Diagrams

Figure 1: Monetary Policy Shock (with positive weight on real exchange rate in policy rule)

Figure 2: Monetary Policy Shock (zero weight on real exchange rate in policy rule)
Figure 3: Shock to foreign real interest rate (case 1)

Figure 4: Shock to foreign real interest rate (case 2)
References


