Examining The Macroeconomic Implications of Shocks to Foreign Official Holdings of U.S Treasuries: A New Keynesian Approach

John Nana Francois*

This version: October 17, 2014†

Abstract

Motivated by Tobin's (1969) imperfect substitution of bonds, this paper introduces an endogenous financial cost term into a standard New Keynesian model that allows for imperfect substitution between government bonds of different maturities. We extend current research on the relationship between foreign official holdings of U.S Treasury and long term interest rates by investigating the following. First, we examine the portfolio rebalancing channel effect of shocks to foreign official holdings on the long term interest rates in a general equilibrium model. Second, we examine the impact of these shocks on major macroeconomic variables in the U.S through a feedback from the endogenous term structure of interest rates generated by the model. The simulated model suggests that, shocks to foreign official holdings has a positive impact on inflation and consumption but have a negative impact on labor hours worked and output. We also find that shocks to foreign official holdings affect the long term rate negatively through the portfolio rebalancing channel (stock effect). This negative effect is consistent with results in the literature.

JEL Classification: E00, E43, F30

Keywords: Foreign official Holdings, long-term interest rate, portfolio rebalancing

*Department of Economics, University of Kansas, email:jfrancois@ku.edu.

†I want to thank the macro seminar group at the University of Kansas Economics Department for useful comments. I am grateful to Matteo Falagiarda for making his codes available for replication. Any errors henceforth are my responsibility

1 Introduction

There is a large literature that attempts to explain the effect of international capital inflows into the U.S Treasury on the long term interest rate. Particularly, empirical research have shown that capital inflows affect the long-term interest rates negatively. Moreover, the literature have shown that the decoupling of the long term rates from short term rates in the mid-2000s, a phenomenon known as the Greenspan Conundrum, was partly due to foreign official inflows, (See for instance Warnock and Warnock (2009), Bernanke, Reinhart, and Sack (2004), Beltran et al (2013), Kohn (2013)). However, these models do not examine the complete impact shocks to foreign official holdings have on the macro-economy.

Motivated by Tobin's (1969) imperfect substitution of bonds, we introduce an endogenous financial cost term into a standard New Keynesian model. The endogenous financial cost term allows us to simultaneously examine the portfolio rebalancing channel effect of shocks to foreign official holdings on long term interest rate and the impact these shocks have on the macro-economy. To appreciate why it is important to investigate the impact of these shocks on the macro-economy, figure 1 compares monthly long term bond holdings by foreign official institution and the Federal reserve bank of the U.S between 1996-2011. Figure 1 shows that foreign official holdings has consistently been higher than the Federal reserve holdings over this period. The striking observation from Figure 1 is that, even at the time of the quantitative easing (QE2- from November 2010 to June 2011), foreign official holdings ($ 3 trillion) was twice the Fed holdings ($1.5 trillion) of long term bonds. Notice that the Federal reserve is using the quantitative easing (Q.E) at the Zero Lower Bound in hopes of lowering long term rate to stimulate investment which in the long-run meets its goal of stabilizing prices and achieving full employment. Given the Federal reserves' goal of holding government debt, it is important to investigate the impact of shocks to these foreign official holdings on the macro-economy. [1]

Consistent with the point above, we show the portfolio rebalancing channel (sometimes referred to as the stock effect) effect of foreign official holdings on the long term rates in this framework. The portfolio rebalancing channel argument abstracts from the fact that large asset purchases by other entities apart from households reduces relative supply of that asset available to households; this bids up the price of that asset leading to the fall in the yield of that asset.[2] For instance, Falagiarda (2014) evaluates the quantitative easing (QE2-from November 2010 to June 2011) in the U.S and the first phase of Asset Purchase Facility (APF-from March 2009 to January 2010) in the U.K in a DSGE model by looking at the portfolio rebalancing channel of the quantitative easing. Their model exhibits similar features as the model we use in this paper.[3] However, while the quantitative easing had a clear policy implication of stimulating the economy at the zero lower bound, not much is known about how shocks to foreign official holding affects the macro-economy.

This paper thus examines two important issues, i. the portfolio rebalancing channel

[1]We must however make it clear that we are not in anyway saying foreign official holdings is similar to the quantitative easing . We are only making this comparison because of their stock effects given their magnitude. Moreover, we analyse the shocks to foreign official holdings prior to the financial crisis.

[2]Krishnamurthy and Vissing -Jorgensen (2011) give an extensive study of the portfolio rebalancing channel. See also Greenwood and Vayanos (2013) for supply and demand effects on interest rates

[3]For more on portfolio adjustment friction see for instance Andrés,Lopez-Salido and Nelson (2004) and Marzo, Söderstroöm and Zagaglia (2007).

effect of foreign official holdings shocks on the long term interest rate in a general equilibrium model and ii. the implications of these shocks on the macro-economy. The framework we employ is a standard New Keynesian model that allows for imperfect substitution of government bonds of different maturities through an endogenous financial cost term in the form of a portfolio adjustment cost. The introduction of the cost term is in light of Tobin (1969) and more recently Andrés et al (2004). We argue that households perceive entering the long term bond market as riskier, that is, longer term bonds are illiquid relative to the same investment in shorter term bonds. Thus as households hold long-term bonds, they hold additional short-term bonds to compensate themselves of the loss of liquidity. In that sense, households self-impose a reserve requirement on their long-term investment. This formulation allows us to examine the portfolio rebalancing channel through which foreign official holding shocks affects the long term rates.

Recent surveys of central banks suggest that most reserve managers do change their reserve portfolios in response to changes in Treasury prices and other macroeconomic variables. Foreign official institutions such as foreign central banks sort of optimize their foreign reserve portfolio, hence foreign official holding are in fact endogenous (See for instance Borio, Galati and Heath (2008); Pringle and Carver (2002)). However modeling the objective function of these foreign central banks is problematic since it is not clear what their objective function is. Moreover, total foreign official holdings is a pool of holdings from different central banks across the world. This makes it even more difficult to specify an objective function for these central banks.[4] To keep the model simple and tractable while avoiding some of the issues mentioned earlier, we model the share of foreign official holdings to outstanding bond supply as exogenous. Specifically, the share of foreign official holdings to outstanding bond supply follow an AR(1) process but shocks from the macro-economy can affect foreign official holding itself. We refer to this formulation of foreign official institutions in our model as pseudo-exogenous.[5] This allows us to examine the impact of shocks to foreign official holdings on the macro-economy and foreign official institutions reaction to macroeconomic shocks.

Using the modified New Keynesian model described above, we find the following results. First, we find that when households value liquidity, shocks to foreign official holdings affect the long term rates negatively through the portfolio rebalancing channel. The negative effect of shocks to foreign official holdings on the long term rates is consistent with results in the literature. [6] Second, we find that shocks to foreign official holdings has a positive impact on inflation and consumption but a negative impact on labor hours worked and output. The Federal reserves' reaction to the increase in inflation possibly explains the Greenspan conundrum in the mid-2000s. We show this result from the models generated impulse response to shocks to foreign official holding.

1.1 Related Literature

The relationship between foreign official holdings and the long term interest has been thoroughly established in the literature with the general consensus that, foreign official holdings affect the long term rates negatively (See for instance Bernanke, Reinhart and Sack (2004);

[4]The objective function of the Bank of Japan for instance will be very different from the Bank of England.

[5]Details of this formulation is shown in section 3.5

[6]See Warnock and Warnock (2009),Bernanke, DeMarco, and Kamin (2011), Bernanke, DeMarco, Kamin and Tyron (2011) Beltran et al (2012), Kohn (2013)

Sierra (2010)). Various studies have explained this relationship by examining different measurements of foreign purchases in different methodologies over different periods of time.

Bernanke (2005), argues that unconventional movements of the long term rates is as a result of a global savings glut, now referred to as GSG hypothesis. The GSG hypothesis explains that increased capital inflows from countries in which desired savings greatly exceeded desired investment including Asia emerging markets and commodity exporters were an important reason that US longer term interest rates during this period were lower than expected. Warnock and Warnock (2009), control for various macroeconomic factors and then estimate that had there been no foreign flows into U.S bonds in 2004, the 10 year treasury yield would be 150 basis points higher in 2005. Similarly, Beltran et al (2012) estimate that if foreign official inflows into U.S Treasuries were to decrease in a given month by 100 billion, 5-year Treasury rates would rise by about 40-60 basis points.

These papers have however focused largely on the effect foreign official holding on the long term interest rate and have been silent on how shocks to foreign official holdings impact the macro-economy. This paper addresses this issue in New Keynesian model with adjustment costs. In a consumption based model, Kohn (2013) quantify the effects of foreign official purchases on the long term interest rate and the term premium. The model they employ assumes that foreign official purchases finances consumption growth and through equilibrium outcomes, they in turn affect the long term rate and the term premium. This paper instead examines the portfolio rebalancing effect of how shocks to foreign official holding affect the long-term interest rates in a general equilibrium model. We find that shocks to foreign official holdings indirectly finances consumption. Moreover, their model does not investigate the macroeconomic implications of shocks to foreign official holdings. We fill this gap by examining response of the macro-economy to shocks to foreign official holdings in richer model.

The rest of the paper is organized as follows Section 2 gives a short stylized facts about foreign official holding and selected macro-variables. Section 3 considers the model and its features. Section 4 presents the results from calibrated model and Section 5 concludes.

2 Stylized Facts

This section provides some descriptive statistic for the two yields studied in this paper (1 and 10-year) and their spread (10-year minus 1-year) from 1996:01-2007:12. It further establishes the correlation between the yields,spread and inflation with foreign official holdings. Table 1 reports the summary statistics in the form of the mean and standard deviation of the two yields (1 year and 10 year) and their correlation with total foreign official bond holding (FOH). The mean of the two yields suggests that the average yield curve is upward sloping. Further, the 1-year yield is more volatile than the 10-year. The two yields are negatively correlated with foreign official bond holdings with a higher correlation the higher the maturity. The spread is negatively correlated with FOH. Inflation on the other hand, is positively correlated with foreign official holdings. To appreciate the magnitude of foreign official holdings, figure 1 compares the long term bond holdings of foreign official institution and the Fed. From the figure, foreign official holdings has consistently been higher than the Fed holdings. The striking observation from Figure 1 is that, even at the time of the quantitative easing (QE2- from November 2010 to June 2011), foreign official holdings was two folds of the Fed holdings of long term bonds. Figure 2 depicts the mean of foreign official bond holdings and outstanding bonds (Short and Long Term) in the U.S from 1996:01 to

2007:12.[7]

The average foreign official holding of long term bonds is about 31% (approximately \$766.6 billion) of outstanding long term bonds, which is about \$2.4 trillion. The average foreign official holding of short term bonds is about 50% (approximately \$311.3 billion) of outstanding short term bonds, which stands at \$607.9 billion. Figure 2 on the other hand depicts the portfolio shares of long term and short term foreign holdings by foreign official institutions. Long term bond holding by foreign official institutions is about 71% of total foreign holdings while short term holdings is about 29%. These shares somewhat reveals a possible preference of bonds with longer maturities over bonds with shorter maturity by foreign official institutions.

Lastly, Figure 3. graphs the level yields of the 1-year and 10-year bond with the spread between the 10-year and 1-year bond. Between July 2004 and July 2006, the 1-year interest rate increased from 1.24% to 5.22% (approximately 320% increase) but the 10-year interest rate only increased from 3.89% to 5.09% (approximately 34 %). The huge increase in short term rates followed by the sluggish increase in long term rates was referred to as the Greenspan Conundrum.

3 The Model

There is a representative agent who populates the economy and supplies labor inputs for firms. A monopolistically competitive firm then hires labor to produce differentiated goods. The government sector conducts both fiscal and monetary policy. Finally, there is also an agent whose share of government bond supply evolves exogenously.

3.1 Households

There is a representative agent who lives infinitely. The agent gains utility by choosing consumption bundle C_t, real money holdings M_t/P_t and labor hours N_t according to the utility function

$$u\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{(C_t - \vartheta C_{t-1})^{1-\gamma}}{1-\gamma} + \frac{1}{1-\eta}\left(\frac{M_t}{P_t}\right)^{1-\eta} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{1}$$

The parameter $\gamma > 0$ is the coefficient of risk aversion, $\eta > 0$ is the elasticity of money demand, ϑ is the habit formation parameter and $\varphi \geq 0$ is the inverse of the Frish elasticity of labor supply. The representative agent thus maximizes her life-time utility

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, \frac{M_t}{P_t}, N_t) \tag{2}$$

with $\beta \in (0,1)$ as the discount factor. Since there is a continuum of consumption goods available for purchase, C_t corresponds to a Dixit-Stiiglitz aggregate of consumption;

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

[7]The computation of the means of outstanding bonds excludes System Open Market Account, this is the name of the Fed's account

where $j \in (0,1)$ represent the continuum of differentiated final goods and $\varepsilon > 1$ governs the elasticity of substitution between different final goods
The period budget constraint is given as:

$$\frac{B_t}{P_t R_t} + \frac{B_{L,t}^H}{P_t R_{L,t}}(1+\rho_t) + \frac{M_t}{P_t} = \frac{B_{t-1}}{P_t} + \frac{B_{L,t-1}^H}{P_t R_t} + \frac{M_{t-1}}{P_t} + \frac{W_t}{P_t} N_t - C_t \tag{4}$$

The agent allocates her wealth between money holding and two zero-coupon bonds which differ in maturity, these bonds are purchased at their nominal prices. The bonds are short term bonds and long term bonds denoted B_t^H and $B_{L,t}^H$ respectively. B_t^H pays R_t and B_t^H pays $R_{L,t}$. The budget constraint of households reveals an active secondary market as proposed by Ljunquist and Sargent (2004). The right hand side of the household budget constraint shows that long term bonds $B_{L,t-1}^H$ is priced with money market rates. The agent carries over long term bonds purchased at time $t-1$ and sells it on the secondary market at the rate $1/R_t$. [8]

Following Falagiarda and Marzo (2012), Harrison (2012) and Falargiada (2014), we assume that intratemporal trading between bonds of different maturities is costly to agents thus they pay a cost whenever they shift the portfolio allocation between short and long term bonds, I model the endogenous cost function as :

$$\rho_t = \frac{\phi_L}{2}\left(\kappa_L \frac{B_t}{B_{L,t}} - 1\right)^2 Y_t \tag{5}$$

where $\phi_L > 0$ and κ_L is the inverse of steady state house holding of short-term to long-term bonds. This implies that ρ_t is zero at steady state. The financial friction term allows for imperfect substitutability between long and short term bonds. There are several motivation for including the transaction cost friction. However following Andrés et al (2004), I argue that households perceive entering the long term bond market as riskier, that is, they are illiquid relative to the same investment in shorter term bonds. Thus as they purchase long-term bonds, they hold additional short-term bonds to compensate themselves of the loss of liquidity. In effect households self-impose a reserve requirement on their long-term investment.[9]

3.2 Optimality Conditions

The first order conditions for the optimizing consumer's problem is given as:

$$C_t : (C_t - \vartheta C_{t-1})^{-\gamma} - \beta\vartheta E_t (C_{t+1} - \vartheta C_t)^{-\gamma} = \lambda_t \tag{6}$$

$$N_t : N_t^{\varphi} = \lambda_t \left(\frac{W_t}{P_t}\right) \tag{7}$$

[8]At time $t-1$, an agent who buys long term bonds and plans to sell them in period t faces price uncertainty as R_t is not known at time $t-1$. The price R_t follows from a simple arbitrage arguments, in period t, these bonds represent identical sure claims to consumption goods at the time of the end of the maturity as newly issued one-period bonds in period t. See also Falagiarda (2014)

[9]Other justifications for including the portfolio friction is the theory of preferred habitat by Vayanos and Vila (2009). Secondly, as in Falagiarda (2014), on can rationalize these costs as proxies for the shares of resources devoted to covering information costs or costs of managing bond portfolio.

$$M_t : \left(\frac{M_t}{P_t}\right)^{-\eta} + \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t \quad (8)$$

$$B_t^H : E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \frac{\lambda_t}{R_t} + \frac{\lambda_t \kappa_L \phi_L}{R_{L,t}} \left(\kappa_L \frac{b_t^H}{b_{L,t}^H} - 1\right) Y_t \quad (9)$$

$$B_{L,t} : E_t \frac{\lambda_{t+1}}{\pi_{t+1} R_{t+1}} = \frac{\lambda_t}{R_{L,t}} + \frac{\lambda_t \phi_L}{2R_{L,t}} \left(\kappa_L \frac{b_t^H}{b_{L,t}^H} - 1\right)^2 Y_t - \frac{\lambda_t \phi_L \kappa_L b_t^H}{R_{L,t} b_{L,t}^H} \left(\kappa_L \frac{b_t^H}{b_{L,t}^H} - 1\right) Y_t \quad (10)$$

Equation (6) represents the marginal utility of wealth and it depends on the marginal utility of consumption today and the expected marginal utility of consumption tomorrow due to the presence of habit formation. Equation (7) relates real wage to the marginal rate of substitution between labor hours and consumption. Equation (6) and (8) can be combined to obtain an expression for money demand. Finally, equation (9) and (10) are the Euler equations for short and long term bond holdings respectively. As it is standard in the literature, we will show below that those two equations implicitly reveal a term structure relationship linking long and short term rates. [10]

3.3 Rebalancing Channel (Stock Effect)

From equations (9) and (10) it is clear that the quantity of long term bonds held by foreign official institutions affects the pricing of the long term interest rate. However to gain more insight of the channel through which foreign official holdings affect the long term rate, I combine the log-linearized first order conditions of short and long term bond holdings. Thus equations (9) and (10) yields [11]:

$$\tilde{R}_{L,t} = \tilde{R}_t + \eta_1 E_t \tilde{R}_{t+1} + \eta_2 E_t(\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1}) - \eta_3(\tilde{b}_t^H - \tilde{b}_{L,t}^H) \quad (11)$$

where $\eta_{13} = \phi_L(1 + \kappa_L) > 0$ and $\eta_i (i = 1, 2, 3)$ are convolutions of steady state values and parameters. From equation (11), long term bonds depends positively on long term bond supply. On the other hand, due to imperfect asset substitutability short term bond supply relates negatively to the long-term rate. The portfolio rebalancing channel (stock effect) is captured in the last term of equation (11). Increases in long term bond holdings by foreign official institutions reduces the supply of long-term bonds available for households. Long term rates then fall given the positive relationship between long term bond supply and the long term rate.

3.4 Firms

I assume a continuum of firms indexed by $i \in [0, 1]$. Firm i produces a differentiated good given the same technology, hence the production function of a typical firm is given by

$$Y_t(i) = a_t N_t(i)^{1-\alpha}$$

[10] Similar results can be found in Falargiada (2014), Andrës et al (2004) and Marzo et al (2008)

[11] Derivation results of equation is 11 is similar to Falagiarda (2014)

where $Y_t(i)$ is output, $N_t(i)$ is the number of work hours hired from labor and a_t is a common technology available to all firms and evolves exogenously. I define aggregate labor and output as $N_t = \int_0^1 N_t(i)di$ and $Y_t = (\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di)^{\frac{\varepsilon}{\varepsilon-1}}$ respectively.

As in Calvo(1983), the representative firm sells its output in monopolistically competitive market and sets nominal prices on a staggered basis. A firm may set its price with probability $1-\theta$ in any given period, independent of the time elapsed since the last adjustment. Hence, each period a measure of $1-\theta$ firms reset their prices, while a fraction θ simply adjusts prices at a pace of steady-state inflation, π (i.e., non adjusting firms follow the rule, $P_t(i) = P_{t-1}(i)\pi$). Thus, θ^k will be the probability that the price set at time t will still hold at time $t+k$.

A representative firm's maximization problem is thus given by:

$$\max_{P^*} E_t \sum_{k=0}^{\infty} \theta^k \{\lambda_{t,t+k} Y_{t+k}(i)(P_t^* - P_{t+k} MC_{t+k})\}$$

subject to the demand schedule $Y_{t+k}(i) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$, where P_t^* denotes the price the firm chooses by resetting prices at time t and $Y_{t+k} = C_{t+k} + G_{t+k}$. The latter equation is due to the fact that in addition to household consumption, government purchases differentiated products produced by firms. The optimality condition for the firm maximization problem is given by:

$$\sum_{k=0}^{\infty} \theta^k \{\lambda_{t,t+k} Y_{t+k}(i)(P_t^* - \frac{\varepsilon}{\varepsilon-1} P_{t+k} MC_{t+k})\} = 0 \qquad (12)$$

The equation that describes the dynamics of aggregate price level is given by

$$P_t = [\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \qquad (13)$$

Similar to the results in Andrés et al (2004) the presence of habit formation in household preference implies that the marginal cost term in the supply-side equations depends on past, present and future output as well as government expenditure. Hence the log-linearized supply-side equation around a zero steady state inflation rate are given by

$$\tilde{\pi}_t = \beta E_t(\tilde{\pi}_{t+1}) + \kappa \tilde{mc}_t \qquad (14)$$

$$\tilde{mc}_t = \left(\frac{\psi+\alpha}{1-\alpha}\right)\tilde{y}_t - \left(\frac{1+\psi}{1-\alpha}\right)\tilde{a}_t - \tilde{\lambda}_t \qquad (15)$$

$$\tilde{\lambda}_t = \delta_1(\tilde{y}_t - \tilde{g}_t) + \delta_2(\tilde{y}_{t-1} - \tilde{g}_{t-1}) + \delta_3(\tilde{y}_{t+1} - \tilde{g}_{t+1}) \qquad (16)$$

where $\delta_1 = \frac{-\gamma(1+\beta\vartheta^2)}{(1-\vartheta)(1-\beta\vartheta)}$, $\delta_2 = \frac{\gamma\vartheta}{(1-\vartheta)(1-\beta\vartheta)}$, $\delta_3 = \frac{\beta\vartheta\gamma}{(1-\vartheta)(1-\beta\vartheta)}$, and $\kappa = \frac{(1-\theta\beta)(1-\theta)(1-\alpha)}{\theta(1+\alpha(\varepsilon-1))}$ (See, for example Andres, Lopez-Salido, and Vallez (2004) and Gali, Gertler, and Lopez-Salido (2001))

3.5 Foreign official Holdings

Recent surveys of central banks suggest that most reserve managers do change their reserve portfolios in response to changes in Treasury prices and other macroeconomic variables. That is foreign central banks sort of optimize their foreign reserve portfolio, hence foreign official holding are in fact endogenous (See for instance Borio, Galati and Heath (2008); Pringle and Carver (2002)). However modeling the objective function of these foreign central banks is problematic since it is not clear what their objective function is. I therefore model foreign official holdings as pseudo-exogenous.[12] Particularly, long term bond holding by foreign central banks is denoted by $B^F_{L,t}$ and is a time varying share x_t of outstanding long term debt. Similarly, B^F_t is the short term bond holding by foreign central banks and it is a time varying share z_t of outstanding short term debt. These time varying shares x_t and z_t follow an AR(1) process.

$$B^F_{L,t} = x_t B_{L,t} \tag{17}$$

and

$$B^F_t = z_t B_t \tag{18}$$

$$\log\left(\frac{x_t}{X}\right) = \rho_x \log\left(\frac{x_{t-1}}{X}\right) + \varepsilon^x_t \tag{19}$$

$$\log\left(\frac{z_t}{Z}\right) = \rho_z \log\left(\frac{z_{t-1}}{Z}\right) + \varepsilon^z_t \tag{20}$$

where X and Z are the steady state values of x_t and z_t respectively

3.6 Demand for Bonds

Households and foreign central banks demand outstanding bonds of different maturity so that.

$$B_{L,t} = B^H_{L,t} + B^F_{L,t} \tag{21}$$

$$B_t = B^H_t + B^F_t \tag{22}$$

where B_t and $B_{L,t}$ are outstanding government money market bonds and long term bonds respectively.

3.7 The Government

Government expenditure is financed by seigniorage revenues, issuance of long-term and short-term bonds. Thus the government budget constraint is given as

$$\frac{B_t}{P_t R_t} + \frac{B_{L,t}}{P_t R_{L,t}} + \frac{M_t - M_{t-1}}{P_t} = \frac{B_{t-1}}{P_t} + \frac{B_{L,t}}{P_t R_t} + G_t \tag{23}$$

where

$$G_t \equiv \left[\int_0^1 G_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

[12]With the exception of Beltran et al (2012) and Sierra (2010) almost all previous studies have treated foreign official inflows as exogenous

Furthermore, I model the issuance of new long term bonds to follow an AR(1) process so that shocks to foreign official demand for long term bonds only affects the composition of outstanding government debt (See for instance Marzo et al.(2008) and Falagiarda (2014)).

$$\log\left(\frac{B_{L,t}}{B}\right) = \rho_{bL}\log\left(\frac{B_{L,t-1}}{B_L}\right) + \varepsilon_t^{bL} \tag{24}$$

To guarantee dynamic stability and a unique equilibrium in the model, I assume that government expenditure is set according to the following fiscal rule: [13]

$$\log\left(\frac{G_t}{G}\right) = \phi_G\log\left(\frac{B_{t-1}}{B}\right) + \varepsilon_t^G \tag{25}$$

where $\phi_G \in (0,1)$
Lastly, the central bank sets the short term rate R_t according to Taylor (1993);

$$\log\left(\frac{R_t}{R}\right) = \rho_R\log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\rho_\pi\log\left(\frac{\pi_t}{\pi}\right) + (1-\rho_R)\rho_Y\log\left(\frac{Y_t}{Y}\right) + \varepsilon_t^R \tag{26}$$

hence R_t responds to lagged R_t, inflation and output through ρ_R, ρ_π and ρ_Y respectively with an interest rate smoothing component. The monetary policy shock ε_t^R is an i.i.d with zero mean and standard deviation σ_R.

3.8 Resource Constraint

The total output of the economy is not only allocated to consumption and government expenditure but also to a portfolio adjustment cost term which is in the unit of output. Thus the model is completed by a resource constraint given as:

$$Y_t = C_t + G_t + \frac{B_{L,t}}{P_t R_{L,t}}\rho_t \tag{27}$$

4 Calibration

The model is calibrated to match the behavior of monthly U.S data prior to the financial crisis in 2008. A subset of the parameters are chosen based on previous studies and are standard in the literature. Specifically, following for instance Christiano, Eichenbaum and Evans (2005), Smets and Wouters(2007) or Marzo et al (2008), we set the habit formation parameter to 0.7. The coefficient of risk aversion parameter is calibrated to 2 as the elasticity of substitution values used in the real business cycle literature is 0.5; the elasticity of real money balances is set equal to 7 as in Marzo et al (2007) and Falagiarda (2014).The inverse of Frisch elasticity of labor supply is set to 1 (Marzo et al 2008); the coefficient of marginal cost is set to 0.014 as in Andrés et al (2004); the value of share of capital in production is calibrated to 0.36 (Christiano, Eichenbaum and Evans (2005)). Finally the portfolio adjustment friction is set to 0.01 (see, for instance, Falagiarda (2014)). The autoregressive coefficient and standard deviation of technology process is calibrated to have a value 0.95 and 0.01 respectively, Christiano and Eichenbaum (1992). We set the autoregressive coefficient

[13]See Andrés et al (2004) for a similar formulation of transfer payment

of long term outstanding bonds ρ_{bL} as 0.90 as in Falagiarda (2014) There is a free parameter which is the coefficient for the AR process for government expenditure and it assumes a value between 0 and 1. In the simulation we set the value of ϕ_G as 0.9. The parameters for monetary policy is calibrated following Falagiarda (2014).

The variables x_t and z_t can be pinned down by equations (17) and (18) respectively, however we use flow data as proxy. Specifically, we use monthly data on foreign official inflows of long term bonds and short term bonds to represent x_t and z_t respectively in the estimation of ρ_x and ρ_z.[14] we choose to use foreign official inflows since it gives a more accurate account of the speed of purchases of bonds by foreign official banks. Moreover, this variable excludes value changes which makes it a more accurate measure for estimation relative to foreign official holdings. Due the volatile nature of the inflow series (Figure 4), we estimate the parameters ρ_x, ρ_z using an Autoregressive Conditional Heteroskedasticity of order one (ARCH (1)). The parameters ρ_x, ρ_z are therefore set as 0.62 and 0.2 respectively and they are both significant at the 1% level. The ARCH(1) coefficient for the error terms are both significant at the 0.05 level (Tables 1 & 2 in the appendix report the estimation results).

4.1 Model response to foreign official holding shock

To illustrate the transmission of the shock to foreign official holding in our framework, we examine the economy's response to a positive shock to foreign official holding. Figure 6 depicts the models impulse response to a foreign official holdings shock. A positive shock ($\sigma_x = 0.2581$) to long term bond holdings by foreign official institutions increases the amount of long term bond holdings of foreign institutions. This reduces the amount of long term bonds available to households by 20 percent and hence a reduction in household short term bond holding. Through the portfolio rebalancing channel shown in equation (11), the reduction of household long term bond holdings then reduces the long-term yield by 25 basis point. The shocks negative impact on long term interest rate is consistent with the results in the literature.

With the fall in long term interest rate, the shock increases consumption through a feedback from the term structure which in turn increases inflation by 30 basis point. A possible explanation for the increase in consumption could be the low long-term interest rates. Since household can consume more today, they increase their leisure, hence labor hours worked falls and hence fall in output. The increase in inflation reduces money holdings by household. Government expenditure falls through a feedback from the fiscal rule. Finally, the Fed reacts to the increase in inflation by increasing the short term rates. The simultaneous increase of the short term rates and the fall in the long term rates decreases the spread. This last result is a possible explanation for the Greenspan conundrum in mid-2000s.

4.2 Model response to long term bond supply shock

Figure 7 depicts the models response to a positive shock to long term bond supply. The shock increases both household and foreign official long term bond holdings. Since households value liquidity and hence self-impose a reserve requirement on their long-term investment, they increase their short term bond holdings. The increase in short term bond holdings by

[14] Monthly data on long term inflows were made available by Beltran et al (2012). We computed the monthly short term inflows using Bertaut-Tyron data

households finances government expenditure thus an increase in government expenditure.

Consumption on the other hand falls hence fall in inflation. The fall in inflation increases money demand, while labour hours increase due to the fall in consumption. One explanation for consumption falling could be that, household are investing more in bonds than consumption. Another possible explanation is that, household cut consumption and save today to pay for future taxes due to the increase government spending. The initial increase in government spending increases output but the corresponding fall in consumption leads output to fall eventually. Lastly, Fed reacts to the low inflation by reducing by the short term rates. This in turn decreases the long term rate initially but the increase in household long term bond holdings bids up the long term rate. The fall in the short term rate and the eventual rise of the long term rate increases the spread.

5 Concluding Remarks

In this paper we have explained the portfolio rebalancing effect of shocks to foreign official holdings on the long term interest rate in a general equilibrium model. We also examined how shocks to foreign official holdings impacts the macro-economy through a feedback from the term structure generated in the model. In light of Tobin (1969), we introduced imperfect substitution between different types of bonds into a standard New Keynesian model. Together with other model features, this introduces quantity of bonds held by households directly into the long-term rate pricing function. This allowed us to simultaneously examine the portfolio rebalancing effect of foreign official holdings shocks on the long term rates and their impact on the macro-economy.

The principal contribution of our research relative to prior work on foreign official holdings of U.S Treasuries is, while prior work focused mainly on the effect of foreign official holdings on the long term interest rate, our work analyses the impact of shocks to foreign official holdings on the macro-economy. We also contribute to the literature by specifically examining the portfolio rebalancing channel effect of shocks to foreign official holdings in a general equilibrium model.

The results from the simulated model suggests that shocks to foreign official holdings has a positive impact inflation and consumption but has a negative impact on labor hours worked and output. We also find that shocks to foreign official holdings affect the long term rate negatively through the portfolio rebalancing channel (stock effect). This negative effect is consistent with results in the literature

6 Appendix

Log-Linearized Model

$$(\beta\theta\gamma(Cc_{t+1} - \theta Cc_t) - \gamma(Cc_t - \theta Cc_{t-1}))(C - \theta C)^{-\gamma-1} = \lambda\tilde{\lambda}_t \tag{28}$$

$$\tilde{m}_t = \frac{1}{\eta}\left(\frac{\pi}{\pi-\beta}\tilde{\lambda}_t - \frac{\beta}{\pi-\beta}E_t(\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1})\right) \tag{29}$$

$$\hat{w}_t = \varphi\hat{n}_t - \hat{\lambda}_t \tag{30}$$

$$E_t\frac{\beta}{\pi}(\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1}) = \frac{\tilde{\lambda}_t}{R} - \frac{\tilde{R}_t}{R} - \frac{\kappa_L\phi_L}{R_L}(b_t^h - b_{L,t}^h) \tag{31}$$

$$E_t\frac{\beta}{\pi R}(\lambda_{t+1} - \pi_{t+1} - R_{t+1}) = \frac{\lambda_t}{R_L} - \frac{R_{L,t}}{R_L} - \frac{\phi_L}{R_L}(b_t^h - b_{L,t}^h) \tag{32}$$

$$\frac{b^h}{R}\tilde{b}_t^h - \frac{b^h}{R}\tilde{R}_t + \frac{b_L^h}{R_L}\tilde{b}_{L,t}^h - \frac{b_L^h}{R_L}\tilde{R}_{L,t} + m\tilde{m}_t = \frac{b^h}{\pi}\tilde{b}_{t-1}^h - \frac{b^h}{\pi}\tilde{\pi}_t + \tag{33}$$

$$\frac{b_L^h}{\pi R}\tilde{b}_{L,t-1}^h - \frac{b_L^h}{\pi R}\tilde{R}_t - \frac{b_L^h}{\pi R}\tilde{\pi}_t + \frac{m}{\pi}\tilde{m}_{t-1} - \frac{m}{\pi}\tilde{\pi}_t + Y\tilde{y}_t - C\tilde{c}_t \tag{34}$$

$$\tilde{y}_t = \tilde{a}_t + (1-\alpha)\tilde{n}_t \tag{35}$$

$$b_L^h\tilde{b}_{L,t}^h = b_L\tilde{b}_{L,t} - xb_L\tilde{b}_{L,t} - x\tilde{x}_t b_L \tag{36}$$

$$b^h\tilde{b}_t^h = b\tilde{b}_t - zb\tilde{b}_t - z\tilde{z}_t b \tag{37}$$

$$\frac{b}{R}(\tilde{b}_t - \tilde{R}_t) + \frac{b_L}{R_L}(\tilde{b}L, t - \tilde{R}_{L,t}) + m\tilde{m}_t = \frac{b}{\pi}(\tilde{b}_{t-1} - \tilde{\pi}_t) + \frac{b_L}{\pi}(\tilde{b}_{L,t-1} - \tilde{\pi} - \tilde{R}_t) + \frac{m}{\pi}(\tilde{m}_t - \tilde{\pi}_t) + G\tilde{g}_t \tag{38}$$

$$\tilde{R} = \rho_R\tilde{R}_{t-1} + (1-\rho_R)\rho_\pi\tilde{\pi}_t + (1-\rho_R)\rho_Y\tilde{y}_t + \varepsilon_t^r \tag{39}$$

$$\tilde{a}_t = \rho_a\tilde{a}_{t-1} + \varepsilon_t^a \tag{40}$$

$$\tilde{z}_t = \rho_z\tilde{z}_{t-1} + \varepsilon_t^z \tag{41}$$

$$\tilde{x}_t = \rho_x\tilde{x}_{t-1} + \varepsilon_t^x \tag{42}$$

$$\tilde{b}_{L,t} = \rho_{bL}\tilde{b}_{L,t-1} + \varepsilon_t^l \tag{43}$$

$$\tilde{g}_t = \phi_G\tilde{b}_{t-1} + \varepsilon_t^g \tag{44}$$

Proof of Market Clearing Condition

For each good i we have $Y_t(i) = C_t(i) + G_t(i) + \rho_t Y_t(i)$. Also

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t; G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} G_t; Y_t = (\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di)^{\frac{\varepsilon}{\varepsilon-1}}$$

From the last equation we obtain

$$(1-\rho_t)Y_t = (\int_0^1 ((1-\rho_t)Y_t(i))^{\frac{\varepsilon-1}{\varepsilon}} di)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\rho_t = \frac{\phi_L B^H_{L,t}}{2P_t R_{L,t}} \left(\kappa_L \frac{B_t}{B_{L,t}} - 1\right)^2$. Since $(1-\rho_t)Y_t(i) = C_t(i) + G_t(i)$, substituting into the last equation yields

$$(1-\rho_t)Y_t = (\int_0^1 (C_t(i) + G_t(i))^{\frac{\varepsilon-1}{\varepsilon}} di)^{\frac{\varepsilon}{\varepsilon-1}}$$

now substituting $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t; G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$ into the latter equation gives

$$(1-\rho_t)Y_t = (C_t + G_t)\left[\int_0^1 \left[\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}\right]^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

As $P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$ it implies $\left[\int_0^1 \left[\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}\right]^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ consequently we have the market clearing condition as

$$(1-\rho_t)Y_t = C_t + G_t \Rightarrow Y_t = C_t + G_t + \rho_t Y_t$$

ARCH(1) Estimation of ρ_x and ρ_z

The parameters ρ_x and ρ_z are estimated as follows:

$$foi_{L,t} = \beta_x + \rho_x foi_{L,t-1} + \varepsilon^x_t \tag{45}$$

and

$$foi_{S,t} = \beta_z + \rho_z foi_{S,t-1} + \varepsilon^z_t \tag{46}$$

where $foi_{L,t}$ and $foi_{S,t}$ are foreign official inflows of long term bonds and foreign official inflows of short term bonds respectively.

Due to the volatility nature of the time series of $foi_{L,t}$ and $foi_{S,t}$ (See Figure 4), the error term ε^x_t and ε^z_t are modelled such that they follow ARCH(1) process:

$$(\varepsilon^x_t)^2 = \alpha^x_0 + \alpha^x_1(\varepsilon^x_{t-1})^2 + \upsilon^x_t \tag{47}$$

and

$$(\varepsilon^z_t)^2 = \alpha^z_0 + \alpha^z_1(\varepsilon^z_{t-1})^2 + \upsilon^z_t \tag{48}$$

Below is the estimation results.

Table 1: Estimation results : arch

Variable	Coef.	(Std. Err)
Equation 44 : $\text{foi}_{L,t}$		
$\text{foi}_{L,t-1}$	0.624	(0.073)
Intercept	0.163	(0.039)
Equation 46 : ARCH(1)		
L.arch	0.315	(0.128)
Intercept	0.137	(0.022)

Table 2: Estimation results : arch

Variable	Coef.	(Std. Err.)
Equation 45 : $\text{foi}_{S,t}$		
$\text{foi}_{S,t-1}$	0.200	(0.048)
Intercept	0.172	(0.102)
Equation 47 : ARCH(1)		
L.arch	0.839	(0.367)
Intercept	1.234	(0.252)

References

[1] Andrés, J, López-Salido, J.D., and Nelson, E. 2004. “Tobin's Imperfect Asset Substitution In Optimizing General Equilibrium.”, Journal of Money, Credit and Banking, 36(4):665-690.

[2] Beltran O. Daniel, Kretchmer Maxwell, Marquez Jaime and Thomas P. Charles 2012 “ Foreign Holdings of U.S Treasury Yields” International Finance Discussion Yield “Conundrum” from a Macro-Finance Perspective” Federal Reserve Bank of San Franciso, Working Paper Series.

[3] Bernanke, B.S 2005 “The Global Savings Glut and the U.S Current Account Deficit, “Homer Jones Lecture, April 14, 2005. `http://www.federalreserve.gov/boarddocs/speeches/2005/20020414/default.htm`

[4] Bernanke S. Ben, Bertaut Carol, DeMarco P. Laurie and Kamin Steven (2011). “ International Capital Flows and the Return to Safe Assets in the United States, 2003-2007” International Finance Discussion Papers.

[5] Borio, C., Galati, G. and Heath, A. (2008) “FX Reserve Management: Trends and Challenges.” BIS Papers 40, Bank for International Settlements.

[6] Christiano, L.J. and Eichenbaum, M. (1992) “Current Real-Business-Cycle Theories and Aggregate labor-Market Flactuations.” American Economic Review, 82(3):430-450.

[7] Christiano, L.J, Eichenbaum, M. and Evans, C.L. (2005). “Nominal Rigidities and The Dynamic Effects of a Shock to monetary policy.” Journal of Political Economy, 119(1):1-45

[8] Calvo, Guillermo 1983 “Staggered Prices in a Utility-Maximization Framework.” *Journal of Monetary Economics*, 12:3,383-98.

[9] Falagiarda, M. 2013 ”Evaluating Quantitative Easing: A DSGE Approach.” Unpublished Manuscript.

[10] Falagiarda, M. and Marzo, M. 2012 “A DSGE Model With Endogenous Term Structure.” Quaderni DSE Working Paper 830, Department of Economics, Universitá di Bologna.

[11] Harrison, R. 2011 “Asset Purchase Policies and Portfolio Balance Effects: A DSGE Analysis”. In Chadha J. and Holly, S., editors, Interest Rates, Prices and Liquidity: Lessons From The Financial Crisis, chapter 5

[12] Harrison, R. 2012 “Asset Purchases Policy at The Effective Lower Bound For Interest Rates”. Bank of England Working Papers 444,Bank of England.

[13] Kohn, David 2013 “Addicted to Debt: Foreign Purchases of U.S. Treasuries and the Term premium”

[14] Krishnamurthy, A. and Vissing-Jorgensen, A 2012 “The Aggregate Demand for Treasury Debt”, Journal of Political Economy, volume 120, No. 2

[15] Ljungqvist, L. and Sargent, T (2004), Recursive macroeconomic theory. MIT Press.

[16] Marzo, M., Söderstroöm,U, and Zagaglia, P. (2008). “The Term Structure of Interest Rates and the Monetary Transmission Mechanism.”, Unpublished Manuscript

[17] Pringle, R. and Carver, N (2002) “How Countries Manage Reserve Assets.” Chapter 1. London: Central Banking Publications.

[18] Rudesbush, G.D., E.T Swanson, and T. Wu 2006 “The Bond Yield “Conundrum” from a Macro-Finance Perspective,” Monetary and Economic Studies, 24(S1),83-1009

[19] Sierra, J. (2010) “International Capital Flows and Bond Risk Premia.” Working Papers 10-14, Bank of Canada.

[20] Smets, F. and Wouters, R (2007). “Shocks and Friction in US Business Cycles: A Bayesian DSGE Approach.” American Economic Review, 97(3):586-606.

[21] Taylor (1993) .....

[22] Tobin, J. 1969 “A General Equilibrium Approach to Monetary Theory”. Journal of Money, Credit and Banking, 1(1):15-29.

[23] Tobin, J. 1982 “Money and Finance in the Macroeconomic process.” Journal of Money, Credit and Banking, 14(2):171-204.

[24] Vayanos, D and Vila,J.L. 2009 “A Preferred-Habitat model of Term Structure of Interest Rates.” Working Paper 15487, National Bureau of Economic Research.

[25] Warnock E. Francis and Warnock C. Veronica (2005) “International Capital Flows and U.S Interest Rates” Unpublished paper, International Finance Discussion Papers 1014, Board of Governors of the Federal Reserve System (U.S)

Table 3: **Descriptive Statistics, 1996:01-2007:12**

Statistics	1-year	10-year	Spread	Inflation
Mean	4.0973	5.1166	1.0193	0.3996
Standard deviation	1.6214	0.8707	1.0675	0.4596
Correlation (FOH)	-0.1433	-0.5522	-0.2327	0.2013

Note: The 1-year and 10 year

Table 4: **Parameter calibration**

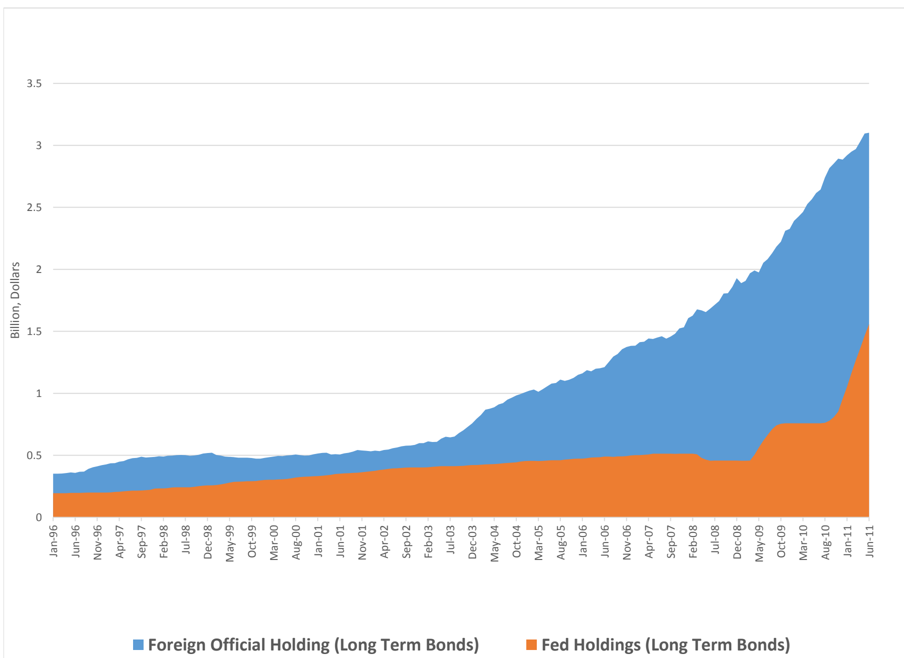
Description	Notation	Value
Discount factor	β	0.994
Habit formation	ϑ	0.7
Coefficient of risk aversion	γ	2
Elasticity of money demand	η	7
Inverse of elasticity of labor supply	φ	1
Coefficient of marginal cost	κ	0.014
Share of capital in production	α	0.36
Portfolio adjustment friction	ϕ_L	0.01
Monetary policy		
Monetary policy response to output	ρ_Y	0
Monetary policy response to inflation	ρ_π	1.97
Monetary policy inertia	ρ_R	0.97
Autogressive Coefficients		
Technology shock	ρ_a	0.95
Government spending	ϕ_G	0.90
LT bonds shock	ρ_{bL}	0.90

Table 5: **Calibration of Key Parameter and Steady State Values**

Description	Notation	Value
Total LT bonds Outstanding per GDP	$B_{L,t}$	0.220
Total ST bonds Outstanding per GDP	B_t	0.056
LT bonds held by households per GDP	$B_{L,t}^H$	0.151
LT bonds held by households per GDP	B_t^H	0.028
LT bonds held by FOH per GDP	$B_{L,t}^F$	0.068
ST bonds held by FOH per GDP	$B_{L,t}^F$	0.028
Share of FOH LT bonds(x_t) shock Coef.	ρ_x	0.62†
Share of FOH ST bonds (z_t) shock Coef.	ρ_z	0.20†
Magnitude of LT Foreign Official Inflow	σ_x	0.2581

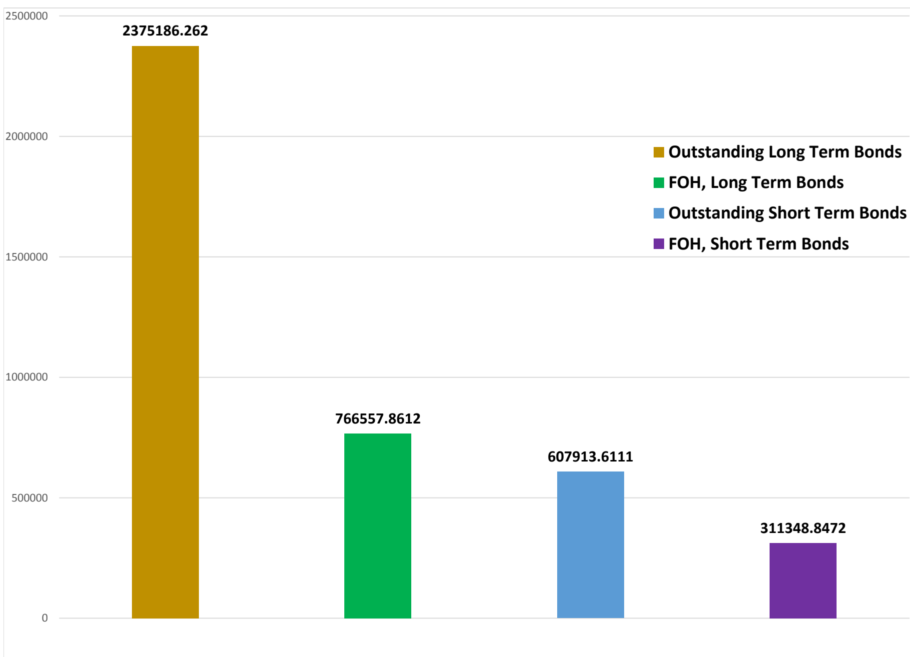
Notes:† denotes significance at 0.01 level.

Figure 1: Long Term Bond Holdings, FED vs Foreign Official Institutions



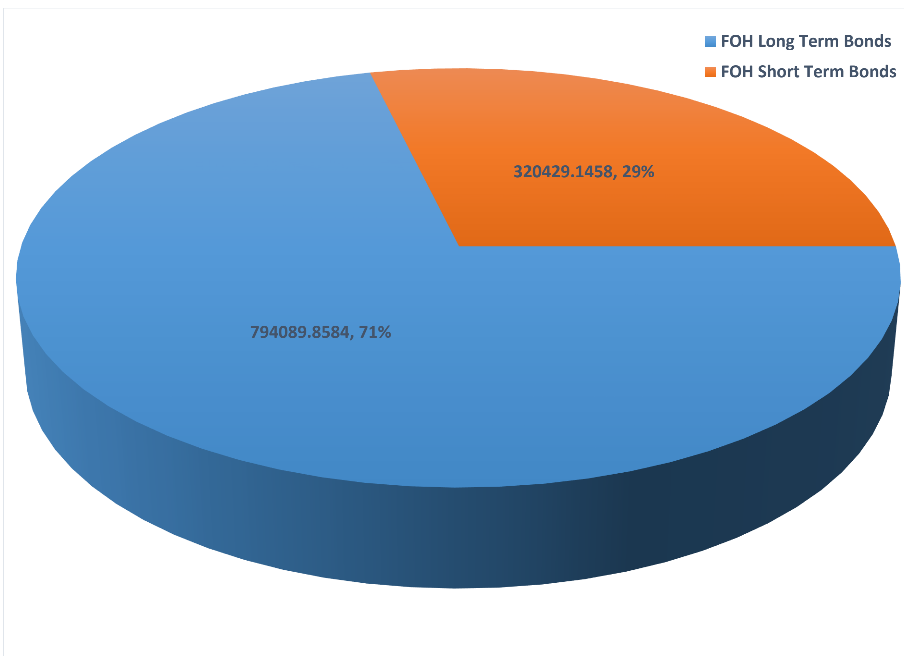
Source: Bertaut-Tyron Dataset.

Figure 2: Mean of FOH and Outstanding Bonds in the US, 1996:01-2007:12



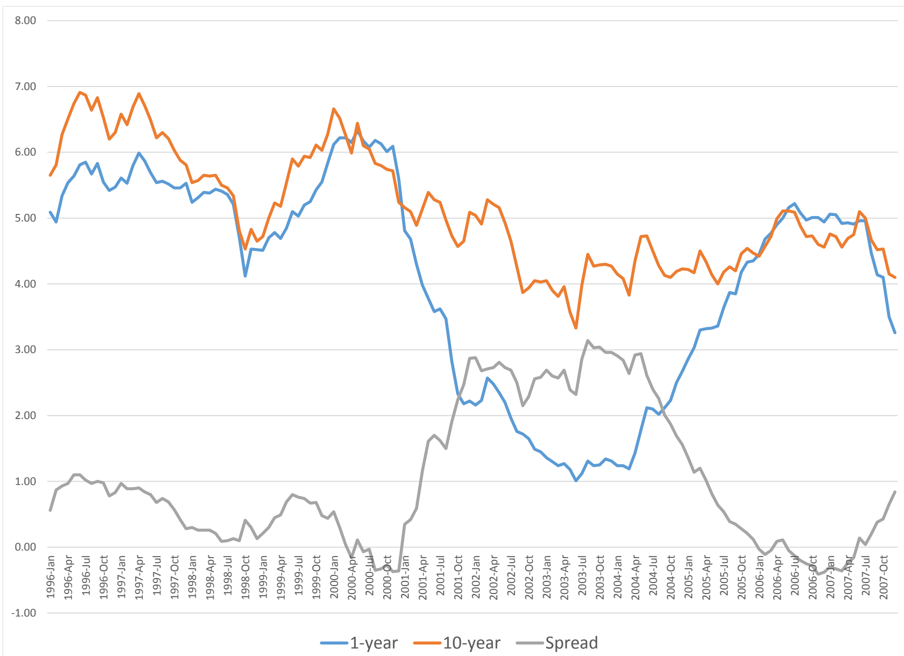
Source: Bertaut-Tyron Dataset. Values are in Millions of Dollars and FOH stands for Foreign Official Holdings. The computation of the means of outstanding bonds excludes System Open Market Account, this is the name of the Fed's account.

Figure 3: Shares of Short and Long Term Foreign Official Bond Holdings to Total Foreign Official Holding, 1996:01-2007:12



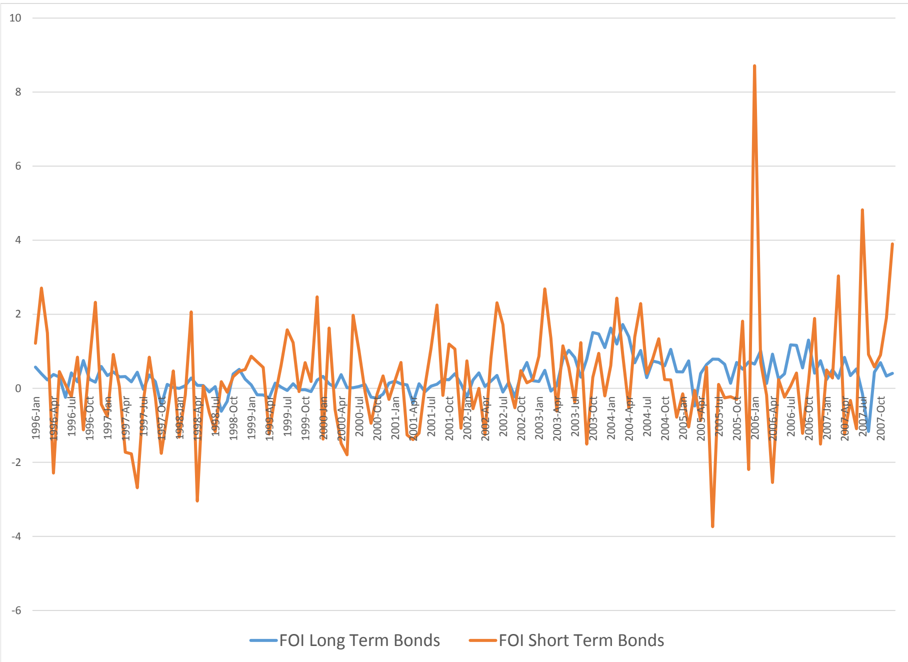
Source:Bertaut-Tyron Dataset

Figure 4: Monthly Yields and Spread of 1-year and 10-year Bond, 1996-2007



Source: FRED
Note: The Spread is computed as the 10-year interest rate minus the 1-year interest rate

Figure 5: Foreign Official Inflow into U.S, 1996-2007



Source: Bertaut-Tyron Dataset.

Figure 6: Impulse Response to Long Term Foreign Official Bond Holding Shock

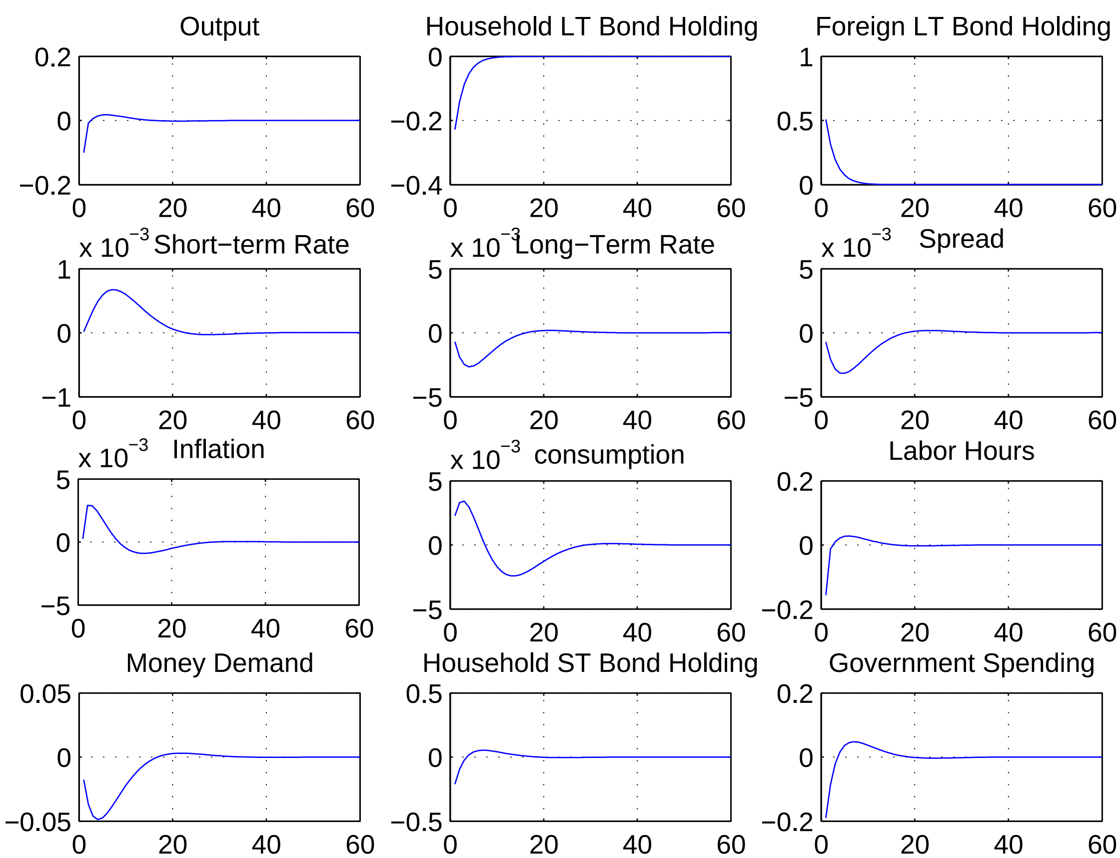


Figure 7: Impulse Response to Long Term Bond Supply Shock