Work Environment and Moral Hazard

Anthony M. Marino*

Marshall School of Business

University of Southern California

Los Angeles, CA 90089-0804

E-mail: amarino@usc.edu

April 2, 2015

Abstract

We consider a firm's provision of safety and health measures (working conditions) in a hidden action agency problem in which effort and working conditions interact in multiplicatively separable (neutral) manner in the cash flow process. Under this common formulation, the firm under supplies working conditions and effort at its second best, regardless of the share of accident damages borne by the firm. At this optimum, increases in the damage share paid by the firm decrease the compensation to the agent so as to render working conditions and effort unchanged. Shifting the damage share then does not impact the firm's or the agent's welfare. We show that direct regulation of working conditions can improve total surplus, but that the regulation of the damage share is ineffectual. Under first order approximations, we also examine the effects of changes in the hazard level of the job and the efficiency of working conditions. Finally, we show that our results can be changed if the neutral interaction between effort and working conditions is violated. JEL Codes: L2, J32, J33, M5, M12.

*I thank the Marshall School of Business for generous research support. Ricardo Alonso, Odilon Camara, Yanhui Wu and an anonymous referee provided valuable advice and comments.

1. Introduction

Work environment is influenced by many factors, including company culture and other psychological elements making up the intangible aspects of how one feels being a part of a particular organization. In addition, work environment is affected by the in-kind physical goods and services supplied to employees apart from monetary compensation.

There are at least three general categories of in-kind benefits being used in today's organizations. First there are technological or directly productive perks that are complementary inputs in conjunction with an agent's work effort which go into in producing cash flow for the firm. These inputs can be in excess of basic capital inputs necessary to carry out assignments. Examples of technological perks might include high quality computer and communication equipment, certain business machines, a high speed internet connection, and transportation services.

A second class of perks take the form of personal services which are indirectly productive in the sense that the provision of these services allows the agent to focus on his or her directly productive activities. Examples include child care services, meal service, the services of a masseuse, hair cutting service, on site laundry and alteration services, and the use of fitness equipment and facilities. The first two categories of benefits typically improve the marginal productivity of the agent's effort and/or reduce effort cost, and, at the same time, they may provide an increase in the agent's utility as a function of the amount of effort exerted in the firm.

A third category of work environment variables are those which affect the safety, health and general welfare of workers. Examples include safety rails, proper instructions on how to use equipment, creation of safe and comfortable computer workstations, respiratory protection, blood borne pathogen needle stick prevention, good climate control and ventilation, control of chemical exposure, general workplace hazard reduction, elements of employee wellness programs, and information programs meant to improve health, safety and welfare. These costly work environmental inputs can produce many benefits for organizations and their workers. In the United States, this part of work environment is subject to workers' compensation law as well as direct regulatory constraints (e.g., OSHA requirements.).

In this paper, we study the third category of work environment and the incentives for a firm to provide a good work setting as it relates to health, safety, accident avoidance, and the prevention of sickness. We want to examine this topic in the context of an underlying moral hazard agency problem, so that we are able to consider the interaction of optimal work environment with the optimal contract. Our model is designed to be simple and to clearly emphasize the key relationships among the share of expected employee accident costs borne by the firm, employee compensation, and optimal work environment.[1] While the provision of a better work environment is costly to the firm, there are two key potential benefits associated with providing a more healthy and a safer environment for workers. First, productivity of an agent can be affected, and, second, the probability that damages will accrue to both the firm and to the employee is reduced. Given that work environment as pertains to health and safety is heavily regulated, our model will characterize the share of possible damages internalized by the firm as a parameter and one minus that share will be internalized by the employee. Changes in regulation and/or the legal system would induce changes in that parameter.

Our model characterizes effort and working conditions as complements or substitutes in the creation of cash flow by the agent, with the leading example being the case where they are complements. We accomplish this by formulating a commonly used technology where effort and working conditions interact in a multiplicatively separable manner to generate cash flow. This mode of interaction is "neutral" in the sense that a working conditions function scales the marginal and average productivities of effort the same such that the elasticity of cash flow with respect to effort

[1] Because a two instrument hidden action agency problem introduces difficult second order complexities, we assume risk neutrality in order to make the analysis tractable.

is not affected by working conditions.[2] In the model, the greater the activity level of the firm, the greater the expected damages from a possible accident. We begin by examining the firm's optimal contract with consists of monetary compensation based on performance and an allocation of costly working conditions. While effort is unobservable, working conditions are observable and contractible. Our initial results show that, at the firm's second best optimum and given any damage share parameter, effort and working conditions are under supplied relative to the first best (total surplus optimum), assuming that effort and working conditions are strategic complements in total surplus, while allowing them to be strategic substitutes in cash flow.[3]

Next, we examine the comparative statics of the firm's second best equilibrium. Our first concern is with the effects of changes in the damage share parameter facing the firm. In the context of our model, we show that increases in this parameter reduce the firm's incentive payment to the agent, but such increases leave the level of optimal working conditions and the agent's effort unchanged. The reasoning behind this result is that increases in the firm's damage share result in damage savings to the agent which are exactly offset by a decrease in the firm's optimal payment to the agent. As a result, the marginal revenue to the agent for an extra unit of effort is unchanged and there is no change in effort and working conditions. Welfare of the agent stays the same as does profit and, of course total welfare, but damage costs are shifted from the agent to the firm.

A second set of comparative statics parameterizes the hazard level of the agent's job and the efficiency of the firm's working conditions in reducing damages and changing cash flow, by taking first order approximations of damage reduction and marginal cost. We show that a greater hazard level worsens working conditions, whereas greater efficiency betters working conditions. The effects

[2] We thereby consider the class of technologies wherein the elasticity of cash flow with respect to effort is unaffected by working conditions. This class includes the case where working conditions do not impact the productivity of effort.

[3] By strategic complements (substitutes) in total surplus, we mean that the marginal effect of effort on total surplus is increasing (decreasing) in working conditions. Strategic substitutes (complements) in cash flow means that the marginal effect of effort on the firm's revenue is decreasing (increasing) in working conditions.

of hazard level and efficiency level on the agent's pay depend on whether the firm or the agent bears a majority of the expected damages from accidents. When the employee bears a majority of the damages, rises in hazard increase the agent's pay, whereas rising hazard lowers pay, if the firm bears a majority of damages. If the firm bears a majority of damages, then greater efficiency of working conditions leads to greater pay, and the opposite occurs if the employee bears a majority of damages.

Our final set of results considers the regulation of working conditions, as with OSHA type regulation, and the direct regulation of damage shares, again in the neutral interaction mode. Direct regulation of working conditions results in increases in effort, working conditions, consumer welfare and total surplus, although profit decreases. Direct regulation of damage share placed on the firm, in addition to working conditions, does not affect welfare but does redistribute expected accident costs between the agent and the firm.

All of the above rather provocative results are derived under the fairly common assumption that effort and working conditions interact in a neutral fashion in producing cash flow for the firm. In Section 6, we present a classic example of non-neutral interaction, namely the case where effort and working conditions are perfect substitutes (additively separable with constant coefficients) in creation of cash flow in the firm.[4] Our results are shown to be subject to change in this non-neutral environment. This example points out the important policy lesson that the specific mode of interaction between effort and working conditions in producing cash flow must be identified empirically, before making accurate statements about the effects of regulatory policy.

Section 2 discusses related literature and Section 3 presents the model and the first best benchmark. Section 4 analyzes the second best equilibrium of the firm and Section 5 discusses regulation. Section 6 examines the implications of non-neutral interactions between effort and working condi-

[4] In this case, the marginal product of effort is unaffected by working conditions, but the average product of effort is increasing in working conditions. Whence, the elasticity of effort is affected.

tions, and Section 7 concludes.

2. Related Literature

Theoretical literature related to working conditions mostly consists of a set of papers examining in kind compensation. Jensen and Meckling (1976) permit perks to have a productivity use and to increase the firm's value. They focus primarily on the implications of the manager's ability to misuse perks. Marino and Zabojnik (2008a) place contractible perks into an optimal incentive contract and study the interaction between optimal in kind and monetary incentives. Their perks are work related perks with productive use and intrinsic value to the agent, as opposed to measures which prevent accidents/sickness and at the same time can affect productivity. Because Marino and Zabojnik (2008a) focuses on perks as opposed to measures taken by the firm to reduce damages, damage prevention and the effects of differential damage sharing on the parts of the firm and the employee are not considered. Moreover, unlike the present paper, Marino and Zabojnik (2008a) is not concerned at all with regulation of the non-monetary measures provided by the firm. Marino and Zabojnik (2008b) examine the optimal pricing and allocation of employee in kind benefits and discounts in a setting where there is no agency problem. Oyer (2008) considers a model of productivity enhancing benefits to show that a benefit will be supplied in greater amounts the more it reduces an employee's effort cost. However, Oyer does not consider formal incentive contracts. Weinschenk (2013) models contractible perks with moral hazard and limited liability. He shows that both under and oversupply are possible and finds cases where perks can harm the employee. Canidio and Gall (2012) show that perks can be optimally used to dampen inefficient but visible task choice due to career concerns. Our analysis of the trade off between the firm's damage share and the incentive payment to the agent is related to the large literature on compensating differences illustrated by Rosen (1986). Finally, there is a related empirical literature, exemplified by Fishback and Kantor (1995), which documents that U.S. workers' compensation law has had the effect of inducing employers to pass a significant part of the added cost of accident compensation onto workers in the form of lower wages. This empirical finding supports our theoretical result which states that increases in the damage share borne by the firm reduce the firm's optimal payment to the agent.

3. The Model

3.1. The Principal-Agent Problem

Consider a principal-agent situation where the agent produces cash flow for the principal through exertion of unobservable effort, denoted E. Work environment is controlled by the principal through an observable and contractible strategy variable y. This strategy variable is costly with cost function $g(y)$. We assume that $g', g'' > 0$ and $g(0) = 0$. Note that the agent is passive in terms of bettering work environment, so that we are emphasizing measures that are primarily unilaterally controlled by the firm, such as ventilation or other overall environmental and safety measures, and unlike wellness programs, wherein agent contributory harm reducing effort would be a joint input with y.[5] Better work environment has two key effects besides being costly to supply. First, it lowers the probability that an accident or health deterioration will occur. Second, it can affect the firm's expected cash flow by altering effort productivity.

Cash flow, denoted $\tilde{x}$, is such that $\tilde{x} \in \{0, x\}$, $x > 0$, where the probability of x is given by

$$Prob(\tilde{x} = x) = p(E)(\gamma + \sigma y) \in [0, 1).$$

[5] This analysis would be an interesting extension of the present study, but it is beyond our scope. If the agent could supply accident reducing effort (precautionary effort), then this would seem to increase the firm's provision of working conditions if cash flow effort and precautionary effort were complements and decrease the firm's provision of working conditions if they were substitutes.

The function p satisfies $p(0) = 0$, $p' > 0$, and $p'' < 0$. The parameters σ and γ satisfy $\gamma \in (0, 1)$, and $\sigma \in (-\gamma, 0) \cup (0, 1 - \gamma)$. The working conditions variable has upper and lower bounds as $y \in [0, 1]$. These restrictions keep the probability of a high output in the unit interval, make the marginal productivity of effort positive and diminishing, allow the marginal productivity of working conditions to be positive ($\sigma > 0$) or negative ($\sigma < 0$) in the production of cash flow alone. If σ is positive, then effort and working conditions are complements in cash flow ($\frac{\partial^2[p(E)(\gamma+\sigma y)]}{\partial e \partial y} > 0$), whereas they are substitutes if σ is negative. The agent's cost of effort is given by $\tilde{c}(E)$, where $\tilde{c}(0) = 0, \tilde{c}'$, and $\tilde{c}'' > 0$. Effort cost satisfies the usual conditions of being strictly convex and increasing in effort.

The high cash flow state brings with it a positive probability that the agent will be harmed on the job. Conditional on an accident occurring, the firm faces dollar damages of $L(x)$, with $L'(x) > 0$ and $L(0) = 0$. Conditional on an accident, damages are an increasing function of cash flow or the deepness of the pockets of the firm. The zero cash flow state is assumed to produce zero dollar damages. In the positive cash flow state, the probability of an accident is $h(y)$, so that the expectation of damages is $L(x)h(y)p(E)(\gamma+\sigma y)$. Conditional on effort, the probability of harm as a function of working conditions is $\phi(y) \equiv h(y)(\gamma + \sigma y)$. We assume that $\phi' < 0$ and $\phi'' > 0$.[6] Finally, the portion of damages borne by the firm is given by the exogenous institutional parameter $s \in (0, 1)$ such that the fraction $(1 - s)$ is internalized by the agent.

Damages are created by activity of the firm. The above model of liability ties damages to the activity of the firm in terms of cash flow. To generate the possibility of an accident, output or cash flow must be created by the agent. Examples might include a logging contractor/agent who bids on a job but does not get it will exert effort and reap no returns. If the bid is taken, both risk and returns will be generated. In construction or manufacturing, the generation of output creates risks

[6] These conditions are implied by $h'(y)(\gamma + \sigma y) + \sigma h < 0$ and $h''(y)(\gamma + \sigma y) + \sigma h'(y) > 0$.

of accidents, and inactivity in terms of output eliminates risk.

It is also possible to reinterpret the above model as one wherein the damages are tied to the effort activity of the agent (as opposed to the output of the firm). The variable p would be reinterpreted as the agent's effort, ϕ as the probability of harm per unit of effort, $L(x)$ as the damages per unit of effort, and $\tilde{c}$ as the cost of so defined effort. This rendition of the model would produce the same results as the one in which damages are tied to output.[7]

Both the principal and the agent are risk neutral and the agent has limited liability. The principal offers a flat wage α and an incentive payment $\beta\tilde{x}$.[8] Under the assumption of limited liability, we have that

$$\alpha, \beta \geq 0. \tag{LL}$$

It is assumed that the outside option of the agent is zero. The agent's objective function is given by

$$\alpha + [\beta x(\gamma + \sigma y) - (1 - s)L(x)\phi(y)]p(E) - \tilde{c}(E).$$

At this point, we will perform a change of variable in order to simplify the basic problem to an equivalent more manageable problem. Define $e \equiv p(E)$, so that $E = p^{-1}(e)$. Note that $p^{-1}(0) = 0$, $p^{-1\prime}(e) > 0$, and $p^{-1\prime\prime}(e) > 0$. Effort cost can be rewritten as $\tilde{c}(E) = \tilde{c}(p^{-1}(e)) \equiv c(e)$. From the above definitions, it follows that the function c satisfies $c(0) = 0$, c', and $c'' > 0$.[9]

A key feature of our cash flow technology is that working conditions impact the productivity of

[7] In this version, each unit of effort carries the same possible dollar damages. This makes sense, because effort is exerted to produce cash flow alone. Effort to reduce accident loss is not modeled., so that more or less cash flow effort does not change the possible damages per unit of effort. Each unit of effort (first or last unit) has the same damage attachment. That is, it is not possible for one to work one more hour and somehow be more or less negligent and contribute more or less to the possible damages for that hour. This type of analysis would be included in a model where the worker has precautionary care.

[8] Given the two outcome technology, this formulation is fully general.

[9] This is true, because the function c is a strictly convex and increasing composition of a strictly convex and increasing function.

effort in a (Hicks) neutral way.[10] By this we mean that the functional form $(\gamma+\sigma y)p(E)$ scales the average and the marginal productivities of E by the same function $(\gamma+\sigma y)$, so that the elasticity of $(\gamma+\sigma y)p$ with respect to E is not affected by y. More generally, all of the results of this paper hold if the function $(\gamma+\sigma y)$ is generalized to any regular concave function of y, say $f(y)$, with $f(y)\cdot p(E)$ defining the probability of the high outcome.[11] In this formulation, the functions $f(y)$ and $p(E)$ must be multiplicatively separable in y and e. While this neutrality interaction is common in the literature, violation of this assumption can overturn our key results. We show this in Section 6 where we provide a simple non-neutral case.

The agent's optimization problem can now be written as

$$\underset{\{e\}}{Max}\ \alpha + [\beta x(\gamma+\sigma y) - (1-s)L(x)\phi(y)]e - c(e),$$

such that equilibrium effort is described by

$$\beta x(\gamma+\sigma y) - (1-s)L(x)\phi(y) - c'(e) = 0.^{12}$$

Because c' is monotonically increasing, it has an inverse function which we write as

$$e = e(r_a) \equiv c'^{-1}(r_a), \tag{1}$$

where

$$r_a = \beta x(\gamma+\sigma y) - (1-s)L(x)\phi(y)$$

[10] This assumption is employed in Marino and Zabojnik (2008a).

[11] This class includes the case where f is a constant function of y so that effort productivity is not affected but damages are reduced by more y.

[12] Note that the agent's objective function has a negative second derivative in e, so that the second order condition is met.

is the marginal revenue of effort to the agent. We have that $e(r_a)$ is increasing and

$$\partial e/\partial\beta = x(\gamma+\sigma y)e' > 0, \partial e/\partial y = [\beta x\sigma - (1-s)L(x)\phi'(y)]e', \partial e/\partial s = L(x)\phi(y)e' > 0.$$

More effort is induced through a greater incentive share and a lesser share of expected damages placed on the agent. Better working conditions generate greater effort if $\sigma \geq 0$. This would be the leading example, as we would expect better working conditions to increase the marginal productivity of effort in cash flow production. That is, effort and working conditions are typically strategic complements in cash flow generation. However, if $\sigma < 0$, then it is possible for better working conditions to lower the marginal productivity of effort supplied by the agent.

The principal sets the flat salary, the incentive share and the agent's working conditions so as to maximize expected profit subject to the limited liability of the agent, (LL) and the agent's participation constraint,

$$\alpha + [\beta x(\gamma+\sigma y) - (1-s)L(x)\phi(y)]e - c(e) \geq 0, \tag{P}$$

where the agent's outside option is assumed to be zero. The principal's problem is given by

$$\underset{\{\alpha,\beta,y\}}{Max} \; -\alpha + [(1-\beta)x(\gamma+\sigma y) - s\phi(y)L(x)]e(r_a) - g(y),$$

subject to (LL), (P) and a non-negativity constraint on y. Before solving the principal's problem, we show in Lemma 1, that the principal's participation constraint is non-binding and that the principal's optimal flat salary is zero. All proofs are provided in the Appendix.

Lemma 1. At a solution to the principal's problem where $e > 0$, *the participation constraint is non-binding and the optimal flat salary is* $\alpha = 0$.

The flat salary does not affect the agent's incentive to supply effort, but the incentive payment does do so and, at the same time, it helps fulfill the participation constraint. Given the strict concavity of the agent's payoff and a zero outside option, a positive β generates effort and a sufficient surplus to make participation a non-binding constraint.

The principal's reduced form problem is now

$$\underset{\{\beta,y\}}{Max}\ [(1-\beta)x(\gamma+\sigma y)-s\phi(y)L(x)]e(r_a)-g(y).$$

Let π denote the principal's profit and let

$$r_f \equiv [(1-\beta)x(\gamma+\sigma y)-sL(x)\phi(y)]$$

denote the marginal revenue to the principal generated by the agent's effort. The first order conditions for an interior solution to the principal's problem are

$$\pi_y = [(1-\beta)x\sigma - sL(x)\phi'(y)]e(r_a)+r_f e'(r_a)[\beta x\sigma-(1-s)L(x)\phi'(y)]-g'=0, \tag{2}$$

$$\pi_\beta = -x(\gamma+\sigma y)e(r_a)+r_f e'(r_a)x(\gamma+\sigma y)=0. \tag{3}$$

Condition (2) equates the marginal cost of working conditions to the sum of two marginal terms, one of which must be positive for an interior solution. We can rewrite (2) as

$$\frac{\partial r_f}{\partial y}e(r_a)+r_f e'(r_a)\frac{\partial r_a}{\partial y}=g'. \tag{4}$$

The first term on the left side of (4) is the direct marginal effect of y on profit and the second is the indirect effect of y on profit, because y impacts profit through impacting effort. If σ is positive, then each of these terms is positive and each represents a marginal benefit of working conditions. If σ is negative such that the marginal effect of y on cash flow is negative, then at least one of the terms $\frac{\partial r_i}{\partial y}$, $i = a, f$, is positive for an interior solution. Thus, working conditions must raise the marginal revenue of effort in the firm's or in the agent's problem, in order for the firm to supply positive conditions. Condition (3) equates the direct cash loss of raising β to the indirect benefit which accounts for the extra net cash generated by the positive effect of raising β on effort. The choice variables at the principal's optimum, as determined by (2)-(3), will be denoted with an "o" superscript, so that β^o, y^o, and e^o characterize the principal's equilibrium. At a point where the first order conditions are met, we assume that the second order conditions are satisfied.[13]

3.2. The First Best Benchmark

As a benchmark, it is of interest to determine how a regulator who is interested in maximizing total surplus would set effort and the level of working conditions. Total surplus is given by $TS(e, y) = [x(\gamma + \sigma y) - L(x)\phi(y)]e - c(e) - g(y)$, so that the first best benchmark is characterized as,

$$TS_y = [x\sigma - L(x)\phi']e - g' = 0, \tag{5}$$

$$TS_e = x(\gamma + \sigma y) - L(x)\phi(y) - c' = 0. \tag{6}$$

Given that $g' > 0$, it must be that, for an interior solution in y, $x\sigma - L(x)\phi' > 0$. This condition is met if $\sigma > 0$, but if $\sigma < 0$ and $x\sigma - L(x)\phi' < 0$, for all y, then the socially optimal y would be zero and a firm supplying positive y would always oversupply working conditions. This is the case where y and e are substitutes in total surplus. If, on the other hand, we assume

A.1 If $\sigma < 0$, then $-(\frac{L(x)}{x})[\phi'(1)] > -\sigma$,

[13] The second order conditions are stated in the Appendix in the proof of Proposition 3. They are more likely to be met when $g'', \phi'' > 0$ are large, when $e'' \leq 0$ (the marginal cost of effort is convex and large in absolute value), and the direct effects $\pi_{\beta\beta}\pi_{yy}$ dominate the cross effects $(\pi_{y\beta})^2$.

then, for all y we have that $x\sigma - L(x)\phi'(y) > 0$. This says that while greater y decreases marginal product of effort in cash flow alone (strategic substitutes in cash flow), the damage savings due to y outweigh the decrease in cash flow, making effort and working conditions strategic complements in total surplus. An increase in working conditions raises the marginal social benefit of effort and vice versa.

The following second order conditions to the social optimum are met

$$TS_{ee} = -c'' < 0, \; TS_{yy} = -L(x)\phi''e - g'' < 0.$$

We assume that the remaining condition

$$TS_{ee}TS_{yy} - (TS_{ey})^2 = (-c'')[-L(x)\phi''e - g''] - [x\sigma - L(x)\phi'(y)]^2 > 0 \tag{7}$$

is met, so that the Hessian of TS is negative definite in its domain.[14] Denote the first best levels of working conditions and effort as (e^*, y^*).

At the first best, working conditions should be chosen such that its marginal social benefit in terms of net cash flow enhancement is equal to its marginal social cost. Likewise, for effort, the increment to cash flow produced by effort should equal its marginal cost in terms of direct effort cost and damage increment.

In what follows, we will consider a first order approximation of the accident probability, $\phi = a - by$, where $a, a/b < 1$ and $y \in [0, a/b)$. Under this formulation, the parameter a measures the innate hazard level of the job in that it characterizes the probability of harm with no care taken by the firm. The parameter b captures the efficiency of a better work environment in reducing potential damages. Using comparative static techniques, the effects of changes in the hazard level

[14] This requires that the direct effects dominate the cross effects again.

and efficiency parameters on the optimal level of working conditions and effort are summarized in

Proposition 1. Let the accident probability take the linear form $\phi = a - by$. At an interior first best, $\partial e^*/\partial b, \partial y^*/\partial b > 0$, whereas $\partial e^*/\partial a, \partial y^*/\partial a < 0$. Moreover, $\partial e^*/\partial i, \partial y^*/\partial i > 0$, for $i = \gamma, \sigma$.

At the first best, levels of working conditions and effort are positively related to the productivity of working conditions in reducing damages. Moreover, the more dangerous is the job, the lower are the first best levels of working conditions and effort. These results are expected, because, generically, greater efficiency represents a cost decrease whereas more danger represents a cost increase. Finally, if the productivity of working conditions in the production of cash flow alone become greater through increases in γ or σ, then both first best effort and working conditions are increased. Note in Proposition 1, by assuming an interior first best, we insure that e and y are local complements in total surplus.

4. Analysis of the Firm's Second Best Equilibrium

The first question of interest is how does the firm's second best selection of working conditions and effort (indirectly through choice of y^o and β^o) compare to those of the first best? First, consider the level of working conditions. From the optimality condition (2) for y^o, we can write

$$e(r_a)[(1-\beta)\sigma x - sL(x)\phi'] + e'(r_a)r_f[\beta x\sigma - (1-s)L(x)\phi] = g'. \tag{8}$$

From the condition for β^o, $e(r_a) = e'(r_a)r_f$, so that substituting into (8) we have

$$TS_y(e^o, y^o) = e(r_a^o)[\sigma x - L(x)\phi'(y^o)] - g'(y^o) = 0. \tag{9}$$

Comparing (9) to (5), it is clear that the firm's selection of working conditions is "pseudo efficient", in the sense that the firm chooses y^o and strategically moves the agent's effort to e^o

so that the gradient of total surplus is zero at the second best equilibrium. By pseudo efficiency, we mean that the partial derivative of total surplus with respect to a choice variable is zero at a point. Likewise, by pseudo under (over) supply we mean that the derivative of total surplus with respect to a choice variable is positive (negative) at a point. Pseudo efficiency of y^o does not itself determine the ranking of y^o and y^*.

Next, consider the firm's choice of e^o. The firm's condition $\pi_\beta = 0$ tells us that

$$e(r_a)/e'(r_a) = r_f. \tag{10}$$

However, r_f can be rewritten as $r_f = x(\gamma + \sigma y) - L(x)\phi - r_a$, with $r_a = c'(e)$, from the worker's incentive compatibility constraint. Thus, from (10),

$$TS_e(e^o, y^o) = x(\gamma + \sigma y^o) - L(x)\phi(y^o) - c'(e^o) = e(r_a^o)/e'(r_a^o) > 0. \tag{11}$$

Condition (11) then points out that, at the second best, effort's marginal social net benefit is positive so as to generate pseudo under supply of effort at the firm's equilibrium. Again, whether e is absolutely under supplied relative to e^* remains to be seen.

In the following proposition, we examine the absolute comparison of the first and second best.

Proposition 2. Suppose that condition (7) is met globally such that $TS(e, y)$ is strictly concave, and assume A.1. Then $e^* > e^o$ and $y^* > y^o$.

The solution to the firm's problem results in pseudo under supply of effort, given y, due to the moral hazard problem. With y pseudo efficient conditional on e, the complementarity of e and y in total surplus results in each being under supplied relative to the first best. These results hold, given any institutional sharing rule $s \in (0, 1)$.

Next, let us consider the comparative statics of the firm's equilibrium.

Proposition 3. Suppose that the local second order conditions to the firm's problem are met. Then, at an interior solution, $\frac{\partial y^o}{\partial s} = 0$ *and* $\frac{\partial \beta^o}{\partial s} = \frac{-L(x)\phi(y)}{x(\gamma+\sigma y)} < 0.$

If society changes institutions so as to shift more of the damages to the firm (s increases), then the firm will respond with a decrease in the optimal incentive payment β to the agent, regardless of the current level of damage sharing between the employee and the firm. Efforts to improve the agent's welfare by raising s are then countered by decreases in payments to the agent. The firm's response in terms of β will be such that the magnitude of r_a, the marginal revenue of effort going to the agent, will be unaffected in equilibrium. This in turn implies that the agent's equilibrium effort is unchanged:

$$\frac{\partial}{\partial s}e(r_a) = e'(r_a)\{x(\gamma+\sigma y)\frac{\partial \beta}{\partial s} + L(x)\phi + [\beta x\sigma - (1-s)L(x)\phi'(y)]\frac{\partial y}{\partial s}\} = 0.$$

The damage cost savings to the agent from an increase in s, given by $L\phi$, are offset exactly by the decrease in the optimal incentive payment to the agent, $x(\gamma + \sigma y)\partial\beta/\partial s$. With $\partial y/\partial s = 0$, r_a is rendered unchanged. From the fact that the firm's choice of y is determined through $e(r_a^o)[\sigma x - L(x)\phi'(y^o)] - g'(y^o) = 0$, it follows that y^o is unchanged in equilibrium by a change in the damage share. Further, because the agent's welfare is $r_a^o e(r_a^o) - c(r_a^o)$, the agent's welfare will be unaffected by an increase in s. Basically, while the agent's expected damages will decrease when s rises, there will be a decrease in the agent's incentive payment so as to result in no net change in welfare. Likewise, the firm's profit is unaffected, because $\partial\pi/\partial s = -x(\gamma+\sigma y)e\frac{\partial \beta}{\partial s} - L(x)e\phi = 0$. While the employee's welfare is unchanged, its composition changes with less incentive pay and less expected accident costs. Likewise the firm's surplus is constant, but retained cash flow increases while expected accident cost increases.

The policy impact of this result is interesting. In our model, attempts by policy makers to alter

s so as to produce some desired change in working conditions will be fruitless. In fact, an increase in s produces no change in working conditions or effort, no change in the agent's welfare, no change in the firm's profit, and, therefore, no change in total surplus. The only impact of an increase in s is to shift damage costs from the employee to the firm. This suggests that direct regulation of y might be necessary to affect changes in work environment and improved total welfare.

The comparative statics of the firm's equilibrium for the case of a linear ϕ function (first order approximation) could only be worked out for the case of a quadratic cost of effort (a first order approximation of the marginal cost of effort). Without this assumption, we would have to make sign and magnitude assumptions regarding third order derivatives. Further, unambiguous results could be achieved only under leading example where working conditions and effort are strategic complements in cash flow, $\sigma > 0$. For this quadratic case, we have

Proposition 4. Let the second order conditions to the firm's problem be met. Suppose that effort cost is quadratic, $c(e) = e^2/2$. *Then* $s \gtreqless \frac{1}{2}$ *implies* $\beta \lesseqgtr \frac{1}{2}$. *Further, if* ϕ *takes on the linear form* $\phi(y) = a - by$ and $\sigma > 0$, *then*

(i) $\frac{\partial y^o}{\partial a} < 0$, $\frac{\partial y^o}{\partial b} > 0$, $\frac{\partial y^o}{\partial \gamma} > 0$, and $\frac{\partial y^o}{\partial \sigma} > 0$, and

(ii) $\frac{\partial \beta^o}{\partial a} \lesseqgtr 0$ *as* $s \gtreqless \frac{1}{2}$, $\frac{\partial \beta^o}{\partial b} \gtreqless 0$ *as* $s \gtreqless \frac{1}{2}$, $\frac{\partial \beta^o}{\partial \gamma} \gtreqless 0$ as $s \gtreqless \frac{1}{2}$, and $\frac{\partial \beta^o}{\partial \sigma} \gtreqless 0$ as $s \gtreqless \frac{1}{2}$.

In the case of quadratic effort cost, if society imposes a majority of expected damages on the firm, then the firm will pay an incentive payment to the agent which is less than one half. If the converse holds, then the agent receives an incentive share greater than one half. In fact, this condition holds for a nonlinear general probability function ϕ, as all that is required for this property is that $r_a = r_f$. The latter is implied by quadratic effort cost.

The parameter a measures the hazard level of the job with no provisions for safety by the firm. Other things equal, greater hazard level will lead to a deterioration of working conditions, because damages become more expensive for the firm to ameliorate through greater y, at any s. The effect of the hazard level on the incentive payment to the agent depends on the share of damages borne by the firm. If a majority of the damages are shouldered by the firm, then greater hazard leads to a lesser incentive payment, whereas if the firm bears less than half of the damages, then the incentive payment increases with increases in the hazard level of the job. In economies where the firm bears most of the damages, the prediction is that greater hazard should lead to lower working conditions and lower incentive payments, whereas in economies where the agent bears most of the damages, greater hazard level should result in a greater incentive payment to the agent and lower working conditions.

The parameter b is a measure of the efficiency of working conditions in reducing damages. Greater b will raise the firm's optimal working conditions because y is more efficient in doing so, at any s. However, the impact on the incentive payment is positive when the firm bears a majority of the damages and negative when the converse holds. If economies with $s < 1/2$ experience increases in the efficiency of working conditions, then the prediction is that this should result in better working conditions and lesser incentive shares to the agent. In economies with $s > 1/2$, greater efficiency of working conditions should raise both the incentive share to the agent and working conditions.

The parameter $\sigma > 0$ is an interaction term which measures the change in the marginal product of effort in cash flow production with respect to a change in working conditions. The parameter γ measures the efficiency of effort alone in enhancing cash flow. Both σ and γ have the same effect on β^o and y^o as does the efficiency parameter b. An increases in each produces an increase in working conditions and an effect on β which is positive when s is large and negative when s is small.

5. Regulation

Consider a regulator with the goal of total surplus maximization and with the authority to set the firm's level of working conditions. The firm would retain control over the incentive payment, and the share of damages would again be parametric. We wish to characterize the regulator's equilibrium and compare it to both the first best and the firm's unregulated solution. In addition, we will ask whether it would be effective for the regulator to attempt to set the share of damages borne by the firm.

5.1. The Regulation of Working Conditions

The agent would optimize as before and set $e = e(r_a)$. The firm sets its contract as in the unregulated problem and chooses β according to

$$-x(\gamma + \sigma y)e + r_f e'(r_a)x(\gamma + \sigma y) = 0. \tag{12}$$

This choice again results in pseudo under supply of e, conditional on y. Condition (12) along with the agent's incentive compatibility constraint, $e = e(r_a)$, define β and e as functions of working conditions, y. We write $\beta = \beta(y)$. Given $e = e(r_a)$,

$$\frac{\partial e}{\partial y} = e'(r_a)[\beta'(y)(\gamma + \sigma y)x + \beta\sigma x - (1 - s)L(x)\phi']. \tag{13}$$

The sign of this derivative is characterized in

Lemma 2. Let A.1 hold. Then $\frac{\partial e}{\partial y} = [e'(r_a)]^2(\frac{\sigma x - L(x)\phi'}{2e'(r_a) - r_f e''(r_a)}) > 0.$

The regulator's problem can now be formulated as one of choosing y subject to the constraint that (12) hold, so that the agent's selection of e is determined by y. The regulator's problem is

$$\underset{\{y\}}{Max}\ [x(\gamma+\sigma y)-L(x)\phi]e-g(y)-c(e), \tag{14}$$

with first order condition

$$x\sigma e-L(x)\phi' e-g'+\ [x(\gamma+\sigma y)-L(x)\phi-c'(e)]\frac{\partial e}{\partial y}=0. \tag{15}$$

From the agent's selection of e and the firm's selection of β,we know that $TS_e=[x(\gamma+\sigma y)-L(x)\phi-c'(e)]>0$. Moreover, Lemma 2 tells us that $\partial e/\partial y>0$. From (15), it then follows that $TS_y=x\sigma e-L(x)\phi' e-g'<0$, at the regulator's optimum. That is, there is pseudo over supply of working conditions, conditional on effort, and there is pseudo under supply of effort, conditional on working conditions.

Let the y-regulated optimum be denoted $(\hat{e},\hat{y})$. We would like to compare this solution to the firm's free optimum. We have

Proposition 5. Suppose that condition (7) is met globally such that $TS(e,y)$ is strictly concave and that A.1 is met. Then $\hat{e}>e^o$ and $\hat{y}>y^o$.

The regulator with the goal of total surplus maximization raises the firm's level of working conditions above that which would be chosen at the firm's profit maximizing solution. The firm's profit maximizing solution, under a second best contract with moral hazard, results in under supply of effort and working conditions relative to the first best, when effort and working conditions are complements in total surplus. A regulator with the ability to impact working conditions will guide the (e,y) allocation through y choice so as to move both upward. The regulation of working conditions so as to maximize total surplus raises total surplus but lowers profit relative to the free firm optimum with moral hazard. Thus, consumer welfare and total surplus increase, while profit falls, as the result of regulating working conditions, in the context of the firm setting the monetary contract.

5.2. The Regulator Regulates Working Conditions and the Damage Share

Here we consider the case where the regulator can determine s as well as y. In this case, (12) will determine the firm's optimal β as a function $\beta(y, s)$. The regulator will now solve problem (14) over a selection of s and y. The first order condition for y is given by (15) and the first order condition for s is given by

$$[x(\gamma + \sigma y) - L(x)\phi - c']e'(r_a)[\frac{\partial \beta}{\partial s}x(\gamma + \sigma y) + L(x)\phi] = 0. \tag{16}$$

However, from the analysis of Section 4, $\frac{\partial e}{\partial s} = e'(r_a)[\frac{\partial \beta}{\partial s}x(\gamma + \sigma y) + L(x)\phi] = 0$, by $\frac{\partial \beta^o}{\partial s} = \frac{-L(x)\phi(y)}{x(\gamma+\sigma y)}$, so that the regulator is indifferent among all $s \in (0, 1)$. There is no unique optimal level of damage sharing. This result is not overturned if the regulator were to maximize a weighted average of consumer and producer surplus.[15]

6. Non-Neutral Interactions Between Working Conditions and Effort

Here we provide a simple illustration of the impact of non-neutral shifts and show that the implications of the model can be changed under this alteration. Suppose that the probability of a high outcome is $p(e, y) \equiv (ze + wy)$, given $e, y > 0$. This formulation does not have the intuitive appeal of the multiplicative interaction form (e is not essential for positive p, y perfectly substitutes for e in total p, and there is no interaction term), however, it presents a simple non-neutral shift of effort

[15] Let $\alpha_c, \alpha_p \in (0, 1)$ denote the exogenous weights applied to consumer and producer surplus, respectively, with $\alpha_c + \alpha_p = 1$. The regulator's objective function is $\tilde{TS} = \alpha_c[\beta xe(\gamma + \sigma y) - (1 - s)L(x)\phi e - c(e)] + \alpha_p[(1 - \beta)xe(\gamma + \sigma y) - sL(x)\phi e - g(y)]$. This objective function is equivalent to the original neutral case when $\alpha_c = \alpha_p = 1/2$. We have $\tilde{TS}_s = \alpha_c(r_a - c')\frac{\partial e}{\partial s} + \alpha_p r_f \frac{\partial e}{\partial s} + e(\alpha_c - \alpha_p)(\frac{\partial \beta}{\partial s}x(\gamma + \sigma y) + L(x)\phi)$. Again, $\frac{\partial e}{\partial s} = e'(r_a)[\frac{\partial \beta}{\partial s}x(\gamma + \sigma y) + L(x)\phi] = \frac{\partial \beta}{\partial s}x(\gamma + \sigma y) + L(x)\phi = 0$, so that $\tilde{TS}_s = 0$ and the optimal level of s is indeterminate.

productivity via y. The function ϕ is given by $\phi(y) = a - by$, and L is a constant function of x. Effort cost is $e^2/2$ and working conditions cost is $y^2/2$. In this case for the function p, the marginal product of e is a constant function of y and the average product of e is an increasing function of y, so that we have a non-neutral interaction.

The solutions for β^o, y^o, and e^o are closed form as are the first best solutions, e^* and y^*.[16] We can show that for the parameter values $a = .9$, $b = .38$, $z = .38$, $x = 4$, $L = 4.58$ and $w = .3$, we have that $y^* > y^o$ and $e^* > e^o$, if $s = .3$, and that $y^* < y^o$ and $e^* < e^o$, if $s = .4$. In this case, y^o and e^o are increasing in s and β^o is decreasing in s, for feasible parameter values of s. By adjusting the coefficient z to the value .55, with all other parameters the same as above, we have that both y^o and e^o are decreasing in s, for s close to one and that β^o is decreasing in s,for feasible parameter values of s close to one. Thus, in this non-neutral model, effort and working conditions can be under or over supplied at the second best depending on the magnitude of s, with under supply at small s and over supply at larger s. Moreover, y and e vary with changes in s, with each increasing in s for small effort productivity, $z = .38$, and each decreasing in s for larger effort productivity, $z = .55$, and values of s close to one. In these examples, the result that β decreases in s is maintained for feasible s, but all of the other results can vary form those of the neutral approach.[17]

This illustration points out the crucial importance of the mode of interaction of effort and working conditions for regulatory policy. The neutral results fail to hold generally in cases where working conditions affect the elasticity of cash flow with respect to effort. This occurs when working conditions differentially affect the average and marginal productivities of effort in the cash flow process. The key point of the example is that without neutrality, precise empirical knowledge

[16] See the Appendix for these solutions.

[17] In the fully general case where $p(e, y)$ is a concave function, we can show that there is pseudo under supply of effort at the firm's second best but that y might be pseudo over or under supplied. At this equilibrium, $TS_e = \frac{xp}{\partial e/\partial \beta} > 0$ whereas $TS_y = (\beta x p_y - (1-s)L\phi' e) + \frac{p}{p_e}(1-s)L\phi' - \frac{p}{p_e}\beta x p_{ey}$. The first term of TS_y is positive, the second term is negative and the third has the sigh of p_{ey}.Thus, TS_y is not generally signable. Further, the signs of $\partial y/\partial s$ and $\partial \beta/\partial s$ are indeterminate.

of the mode of interaction between e and y must be known in order to correctly inform and guide regulatory policy.

7. Conclusion

We study a hidden action agency problem in which the firm can contract on working conditions as well as the monetary contract and in which working conditions and effort interact in the cash flow process in a neutral way. At the firm's second best optimum, working conditions and effort are under supplied relative to the first best, in the case where effort and working conditions are complements in total surplus, no matter what the fraction of expected damages internalized by the firm. At this equilibrium, there is a trade-off between the share of damages and the incentive payment to the agent. Increases in the share of damages borne by the firm lead to decreases in the payment to the agent, so as to leave both working conditions and effort unchanged. The impact on welfare of the firm and the employee is null, but the share of damage costs are shifted from the employee to the firm.

Under first order approximations, if the job becomes more dangerous, then working conditions become worse and the incentive payment rises, in situations where the agent bears a majority of damage costs. If the firm bears a majority of the damage costs, more hazard lowers working conditions and the incentive payment. If working conditions become, more efficient, then both working conditions and payment to the agent rise in cases where the firm bears a majority of the damages. They both fall with more efficiency, if the agent bears a majority of the damages.

Regulation of working conditions can increase total welfare relative to the firm's free optimum by raising consumer surplus but lowering profit. Regulation of the damage share, in addition to working conditions, is ineffectual in changing total surplus and merely redistributes damage costs and the incentive payment for the firm and the agent.

Finally, we find that if effort and working conditions interact in a non-neutral manner, then all of the above results may be changed. Thus, we conclude with the point that, in the presence of neutral interactions between effort and working conditions, regulatory improvements in the allocation call for direct regulation of working conditions, while regulation of the damage share merely redistributes components of profit and employee surplus. Without neutrality, precise knowledge of these interactions must be known in order to correctly guide regulatory policy.

Appendix

Proof of Lemma 1: Divide (P) by $e > 0$

$$\beta x(\gamma + \sigma y) - (1 - s)L(x)\phi(y) - c(e)/e + \alpha/e \geq 0.$$

This condition is met if

$$\beta x(\gamma + \sigma y) - (1 - s)L(x)\phi(y) - c(e)/e \geq 0.$$

From (1), substitute $c' = \beta x(\gamma + \sigma y) - (1 - s)L(x)\phi(y)$, so that (P) is met if

$$c'(e) - c(e)/e \geq 0.$$

This expression is strictly positive because $c', c'' > 0$ and $c(0) = 0$.

The principal's Lagrangian can now be written as

$$L = -\alpha + [(1 - \beta)x(\gamma + \sigma y) - sL(x)\phi(y)]e(r_a) - g(y) + \lambda_\alpha \alpha + \lambda_\beta \beta + \lambda_y y,$$

where $\lambda_i \geq 0$, $i = \alpha, \beta, y$, are the multipliers for the non-negativity constraints on α, β and y, respectively. The first order condition for α is $-1+\lambda_\alpha = 0$. Thus, it follows that $\lambda_\alpha > 0$ and $\alpha = 0$.

■

Proof of Proposition 1: Let H_{TS} denote the Hessian of TS in (e, y). We have that $\partial e^*/\partial b = (1/|H_{TS}|)(L(x)g''y + L(x)e(x\sigma + bL(x))) > 0$ and $\partial y^*/\partial b = (1/|H_{TS}|)(L(x)ec'' + L(x)y(x\sigma + bL(x))) > 0$. Further, $\partial e^*/\partial a = -(1/|H_{TS}|)(L(x)g'') < 0$ and $\partial y^*/\partial a = -(1/|H_{TS}|)L(x)(\sigma x + L(x)b) < 0$. Finally, $\partial e^*/\partial \gamma = (1/|H_{TS}|)g''x > 0$, $\partial y^*/\partial \gamma = (1/|H_{TS}|)x(\sigma x + bL(x)) > 0$, $\partial e^*/\partial \sigma = (1/|H_{TS}|)[g'' \cdot xy + ex(x\sigma + bL(x))]$, and $\partial y^*/\partial \sigma = (1/|H_{TS}|)[exc'' + yx(\sigma x + bL(x))]$. ■

Proof of Proposition 2: Because TS is strictly concave and differentiable,

$$\sum_{i=e}^{y} \frac{\partial TS^o}{\partial i}(i^* - i^o) > TS(e^*, y^*) - TS(e^o, y^o) > 0.$$

Thus, $TS_e^o(e^* - e^o) + TS_y^o(y^* - y^o) > 0$. However, $TS_e^o > 0$ while $TS_y^o = 0$, so that $e^* > e^o$. Next note that $TS_{ey} = \sigma x - L(x)\phi' > 0$, by A.1. By $e^* > e^o$ and $TS_{ey} > 0$, it follows that

$$TS_y(e^*, y^*) - TS_y(e^o, y^*) > 0.$$

Because it is true that $TS_y(e^*, y^*) - TS_y(e^o, y^o) = 0$, it is, therefore, necessary that $y^o < y^*$, by $TS_{yy} < 0$. ■

Proof of Proposition 3: The second order conditions to the principal's problem include

$$\begin{aligned}\pi_{yy} &= -sL(x)e\phi'' - g'' + 2e'(\beta\sigma x - (1-s)L(x)\phi')((1-\beta)\sigma x - sL(x)\phi') \\ &\quad +(r_f)(e''(\beta\sigma x - (1-s)L\phi')^2 - e'((1-s)L(x)\phi'') < 0\end{aligned}$$

and

$$\pi_{\beta\beta} = -2e'x^2(\gamma + \sigma y)^2 + e''x^2(\gamma + \sigma y)^2 r_f < 0.$$

The second order cross partial $\pi_{y\beta}$ is given by

$$\begin{aligned}\pi_{y\beta} &= -x\sigma e - x(\gamma + \sigma y)e'[\sigma\beta x - (1-s)L(x)\phi'] + [(1-\beta)x\sigma - sL(x)\phi']e'x(\gamma + \sigma y) \\ &\quad + r_f e' x\sigma + r_f e'' x(\gamma + \sigma y)(\beta\sigma x - (1-s)L(x)\phi')].\end{aligned}$$

Using $\pi_\beta = 0$, we have that $e = r_f e'$, so that $\pi_{y\beta}$ can be rewritten as

$$\pi_{y\beta} = e'x(\gamma + \sigma y)[(1-2\beta)x\sigma + (1-2s)L(x)\phi'] + e''x(\gamma + \sigma y)r_f[\beta x\sigma - (1-s)\phi' L(x)].$$

We will make use of

$$\begin{aligned}\frac{\partial}{\partial s}\pi_y &\equiv \pi_{ys} = -L(x)e\phi' - L(x)\phi e'[\beta x\sigma - (1-s)\phi' L(x)] + [(1-\beta)x\sigma - sL(x)\phi']e'L(x)\phi \\ &\quad + r_f e'\phi' L(x) + r_f e'' L(x)\phi[\beta x\sigma - (1-s)\phi' L(x)].\end{aligned}$$

Employing $e = r_f e'$, this can be rewritten as

$$\pi_{ys} = e'L(x)\phi[(1-2\beta)x\sigma + (1-2s)L(x)\phi'] + e''L(x)\phi r_f[\beta x\sigma - (1-s)L(x)\phi'].$$

Moreover, we have

$$\frac{\partial}{\partial s}\pi_\beta \equiv \pi_{\beta s} = -2e'L(x)\phi x(\gamma + \sigma y) + e''L\phi x(\gamma + \sigma y)r_f.$$

The effect of s on y is given by

$$\partial y^o/\partial s = (1/H)(-\pi_{ys}\pi_{\beta\beta} + \pi_{\beta s}\pi_{y\beta}) = 0.$$

We will show that $[\pi_{ys}(-\pi_{\beta\beta}) + \pi_{\beta s}\pi_{y\beta}] = 0$. Let us use $Z \equiv [(1-2\beta)x\sigma + (1-2s)L(x)\phi']$ and $\partial r_a/\partial y = [\beta x\sigma - (1-s)L(x)\phi']$ to rewrite this expression:

$$\begin{aligned}
&[e'L\phi Z + e''L\phi r_f(\partial r_a/\partial y)][2e'x^2(\gamma+\sigma y)^2 - e''x^2(\gamma+\sigma y)^2 r_f] \\
&+[-2e'L\phi x(\gamma+\sigma y) + e''L\phi x(\gamma+\sigma y)r_f][e'x(\gamma+\sigma y)Z + e''x(\gamma+\sigma y)r_f(\partial r_a/\partial y)].
\end{aligned}$$

Multiplying terms

$$\begin{aligned}
&2(e')^2L\phi x^2(\gamma+\sigma y)^2Z - L\phi Z r_f x^2(\gamma+\sigma y)^2 e'e'' + 2L\phi r_f(\partial r_a/\partial y)x^2(\gamma+\sigma y)^2e'e'' \\
&-L\phi(r_f)^2(\partial r_a/\partial y)x^2(\gamma+\sigma y)^2(e'')^2 - 2(e')^2L\phi x^2(\gamma+\sigma y)^2Z + L\phi Z r_f x^2(\gamma+\sigma y)^2e'e'' \\
&-2L\phi r_f(\partial r_a/\partial y)x^2(\gamma+\sigma y)^2e'e'' + L\phi(r_f)^2x^2(\gamma+\sigma y)^2(\partial r_a/\partial y)(e'')^2 \\
=\;& 0.
\end{aligned}$$

Thus, $\partial y^o/\partial s = 0$.

Next consider

$$\partial \beta^o/\partial s = \frac{(-\pi_{yy}\pi_{\beta s} + \pi_{ys}\pi_{y\beta})}{\pi_{yy}\pi_{\beta\beta} - (\pi_{y\beta})^2}.$$

From above we know that $\pi_{ys} = (\pi_{\beta s}\pi_{y\beta})/\pi_{\beta\beta}$, so that

$$
\begin{aligned}
\partial\beta^o/\partial s &= \frac{(-\pi_{yy}\pi_{\beta s} + \frac{(\pi_{\beta s}\pi_{y\beta})}{\pi_{\beta\beta}}\pi_{y\beta})}{\pi_{yy}\pi_{\beta\beta} - (\pi_{y\beta})^2} = \frac{\frac{-\pi_{\beta s}}{\pi_{\beta\beta}}(\pi_{yy}\pi_{\beta\beta} - (\pi_{y\beta})^2)}{\pi_{yy}\pi_{\beta\beta} - (\pi_{y\beta})^2} \\
&= \frac{-\pi_{\beta s}}{\pi_{\beta\beta}} = \frac{2e'L(x)\phi x(\gamma+\sigma y) - e''L(x)\phi x(\gamma+\sigma y)r_f}{-2e'x^2(\gamma+\sigma y)^2 + e''x^2(\gamma+\sigma y)^2 r_f} \\
&= \frac{-L(x)\phi}{x(\gamma+\sigma y)}.
\end{aligned}
$$

This completes the proof. ■

Proof of Proposition 4: Note that, given a general ϕ, $e(r_a) = r_a, e(r_a) = e'(r_a)r_f$, $e' = 1$, so that $r_a = r_f$. It immediately follows that $s \gtreqless \frac{1}{2}$ implies $\beta \lesseqgtr \frac{1}{2}$. Now let $\phi = a - by$. For this case, at a point where the first order conditions are met,

$$
\begin{aligned}
\pi_{yy} &= 2[(1-\beta)\sigma x + sLb)(\beta\sigma x + (1-s)Lb] - g'', \\
\pi_{\beta\beta} &= -2x^2(\gamma+\sigma y)^2, \\
\pi_{y\beta} &= x(\gamma+\sigma y)[(1-2\beta)\sigma x - (1-2s)Lb].
\end{aligned}
$$

Whence, $\pi_{y\beta} \gtreqless 0$ as $s \gtreqless \frac{1}{2}$. Using the notation of the proof of Proposition 3,

$$
\begin{aligned}
\pi_{ya} &= -(1-s)L[(1-\beta)\sigma x + sLb) - sL(\beta\sigma x + (1-s)Lb] < 0, \\
\pi_{\beta a} &= x(\gamma+\sigma y)(1-2s)L.
\end{aligned}
$$

We have

$$
\partial y^o/\partial a = (1/H)(-\pi_{ya}\pi_{\beta\beta} + \pi_{\beta a}\pi_{y\beta}).
$$

With $|H| > 0$, the *sign* $\partial y^o/\partial a =$ *sign* $(-\pi_{ya}\pi_{\beta\beta} + \pi_{\beta a}\pi_{y\beta})$. Clearly, $-\pi_{ya}\pi_{\beta\beta} < 0$, $\pi_{\beta a} \gtreqless 0$ as $s \lesseqgtr \frac{1}{2}$, and $\pi_{y\beta} \gtreqless 0$ as $s \gtreqless \frac{1}{2}$. Thus, $\pi_{\beta a}\pi_{y\beta} < 0$ and $\partial y^o/\partial a < 0$.

Next note that

$$\partial \beta^o / \partial a = (1/H)(-\pi_{\beta a} \pi_{yy} + \pi_{ya} \pi_{y\beta}).$$

If $s \geq 1/2$, then $\pi_{\beta a} < 0$ and $-\pi_{\beta a}\pi_{yy} \leq 0$. Further, $s \geq 1/2$ implies $\pi_{y\beta} \geq 0$, with $\pi_{ya}\pi_{y\beta} \leq 0$. Whence, $s \geq 1/2$ implies $\partial \beta^o/\partial a \leq 0$. The converse argument for $s < 1/2$ implies $\partial \beta^o / \partial a > 0$.

Next consider a change in b. We have

$$\partial y^o / \partial b = (1/H)(-\pi_{yb} \pi_{\beta\beta} + \pi_{\beta b} \pi_{y\beta}).$$

Moreover,

$$\begin{aligned} \pi_{yb} &= sLr_a + [(1-\beta)\sigma x + sLb](1-s)Ly + [\beta \sigma x + (1-s)bL]sLy + r_f(1-s)L, \\ \pi_{\beta b} &= x(\gamma + \sigma y)yL(2s-1). \end{aligned}$$

Using the same arguments as above, $-\pi_{yb}\pi_{\beta\beta} > 0$ and $s \gtreqless \frac{1}{2}$ implies $\pi_{\beta b}\pi_{y\beta} \geq 0$. Thus, $\partial y^o/\partial b > 0$. Likewise,

$$\partial \beta^o / \partial b = (1/H)(-\pi_{\beta b} \pi_{yy} + \pi_{yb} \pi_{y\beta}).$$

We have $-\pi_{\beta b}\pi_{yy} \geq 0$ as $s \geq \frac{1}{2}$. Moreover, $s \geq \frac{1}{2}$ implies that $\pi_{yb}\pi_{y\beta} \geq 0$. Thus, $\partial \beta^o/\partial b \geq 0$. For $s < \frac{1}{2}$, the converse argument holds.

A change in γ will be considered next. We have

$$\begin{aligned} \pi_{y\gamma} &= [(1-\beta)\sigma x + sLb]\beta x + [\beta \sigma x + (1-s)Lb](1-\beta)x > 0, \\ \pi_{\beta\gamma} &= x^2(\gamma + \sigma y)(1-2\beta), \end{aligned}$$

The sign of $\partial y^o/\partial \gamma$ is that of $-\pi_{y\gamma}\pi_{\beta\beta} + \pi_{y\beta}\pi_{\beta\gamma}$. From above, $-\pi_{y\gamma}\pi_{\beta\beta} > 0$. Further, $\pi_{y\beta}, \pi_{\beta\gamma} \gtreqless 0$

as $s \gtreqless \frac{1}{2}$, so that $\pi_{y\gamma}\pi_{\beta\gamma} > 0$. Whence, $\partial y^o/\partial\gamma > 0$. The sign of $\partial\beta^o/\partial\gamma$ is that of $-\pi_{yy}\pi_{\beta\gamma}+\pi_{y\gamma}\pi_{\beta y}$. We have that $\pi_{\beta\gamma}, \pi_{\beta y} \gtreqless 0$ as $s \gtreqless \frac{1}{2}$ and $-\pi_{yy}\pi_{\beta\gamma} \gtreqless 0$ as $s \gtreqless \frac{1}{2}$. Thus, $\partial\beta^o/\partial\gamma \gtreqless 0$ as $s \gtreqless \frac{1}{2}$.

Finally, we examine a change in σ. For this case we have that

$$
\begin{aligned}
\pi_{y\sigma} &= r_a x + xy\{\beta[(1-\beta)\sigma x + sLb] + (1-\beta)[\beta\sigma x + (1-s)Lb]\} > 0, \\
\pi_{\beta\sigma} &= x^2(\gamma+\sigma y)(1-2\beta).
\end{aligned}
$$

The sign of $\partial y^o/\partial\sigma$ is that of $-\pi_{y\sigma}\pi_{\beta\beta} + \pi_{y\beta}\pi_{\beta\sigma}$. We have that $-\pi_{y\sigma}\pi_{\beta\beta} > 0$ and $\pi_{y\beta}, \pi_{\beta\sigma} \gtreqless 0$ as $s \gtreqless \frac{1}{2}$. It follows that $\partial y^o/\partial\sigma > 0$. The sign of $\partial\beta^o/\partial\sigma$ is that of $-\pi_{\beta\sigma}\pi_{yy} + \pi_{y\beta}\pi_{y\sigma}$. We have $-\pi_{\beta\sigma}\pi_{yy} \gtreqless 0$ as $s \gtreqless \frac{1}{2}$ and $\pi_{y\beta}\pi_{y\sigma} \gtreqless 0$ as $s \gtreqless \frac{1}{2}$, so that $\partial\beta^o/\partial\sigma \gtreqless 0$ as $s \gtreqless \frac{1}{2}$. ■

Proof of Lemma 2: From (12), we have that

$$
\beta'(y) = \frac{x(\gamma+\sigma y)e'(r_a)((1-2\beta)\sigma x + (1-2s)L\phi') + x(\gamma+\sigma y)e''(r_a)(\beta\sigma x - (1-s)L\phi')r_f}{2x^2(\gamma+\sigma y)^2 e'(r_a) - x^2(\gamma+\sigma y)^2 r_f e''(r_a)}.
$$

From (13), $\partial e/\partial y = e'(r_a)[\beta'(y)(\gamma+\sigma y)x + \beta\sigma x - (1-s)L(x)\phi']$. Substituting for $\beta'(y)$,

$$
\begin{aligned}
\partial e/\partial y &= \frac{x(\gamma+\sigma y)e'(r_a)\{x(\gamma+\sigma y)e'(r_a)[(1-2\beta)x + (1-2s)L\phi'] + x(\gamma+\sigma y)e''(r_a)[\beta\sigma x - (1-s)L\phi']r_f\}}{2x^2(\gamma+\sigma y)^2 e'(r_a) - x^2(\gamma+\sigma y)^2 r_f e''(r_a)} \\
&\quad + e'(r_a)[\beta\sigma x - (1-s)L\phi'].
\end{aligned}
$$

Placing this expression over a common denominator and simplifying we obtain

$$
\partial e/\partial y = \frac{e'(r_a)^2(\sigma x - L\phi')}{2e'(r_a) - r_f e''(r_a)}.
$$

The assumption that the second order condition $\pi_{\beta\beta} < 0$ holds implies that $2e'(r_a) - r_f e''(r_a) > 0$, and A.1 implies that $(\sigma x - L\phi') > 0$. Thus, $\frac{\partial e}{\partial y} > 0$. ■

Proof of Proposition 5: By strict concavity, $\sum_{i=e}^{y} \frac{\partial TS^o}{\partial i}(\hat{\imath} - i^o) > TS(\hat{e}, \hat{y}) - TS(e^o, y^o) > 0$. Thus, $TS_e^o(\hat{e} - e^o) - TS_y^o(\hat{y} - y^o) > 0$. We know that $TS_y^o = 0$ while $TS_e^o > 0$. It follows that $\hat{e} > e^o$. Next note that $TS_y(e^o, y^o) = 0$, while $TS_y(\hat{e}, y^o) > 0$, by $TS_{ye} > 0$. Given that $TS_y(\hat{e}, \hat{y}) < 0$, it must be that $\hat{y} > y^o$, because $TS_{yy} < 0$. ■

Non-Neutral Interactions Between Effort and Working Conditions: The second and first best solutions are given by

$$\begin{aligned}
\beta^o &= \frac{aLs[2bL(1-s)z^2 - w] + x[w + bL(1-s)z^2]}{x[w + bL(1-2s)z^2]}, \; y^o = \frac{z^2(aL - x)(w + bLz^2)}{[w + z^2 bL(1-2s)]^2}, \\
e^o &= \frac{z(aL - x)[-w^2(1-z) + 2z^4 b^2 L^2 (1-s)s - z^2 bwL(1 - 2s - z)]}{[w + z^2 bL(1-2s)]^2}, y^* = \frac{(aL - x)(w + bLz^2)}{(z^2 L^2 b^2 + 2bwL - 1)}, \text{ and} \\
e^* &= \frac{z(aL - x)(bwL - 1)}{(z^2 L^2 b^2 + 2bwL - 1)}.
\end{aligned}$$

References

[1] Canidio, Andrea and Thomas Gall. 2012. "Rewarding Idleness." Working paper.

[2] Fishback, Price V. and Shawn Everett Kantor. 1995. "Do Workers Pay for the Passage of Workers' Compensation Laws?" *Quarterly Journal of Economics* 110, August 1995: 713-742.

[3] Jensen, M. and Meckling, W.H. 1976. “Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure.” *Journal of Financial Economics* 3, October 1976: 305-360.

[4] Marino, Anthony M. and Jan Zabojnik. 2008a. “Work-Related Perks, Agency Problems, and Optimal Incentive Contracts,” *Rand Journal of Economics* 39, Summer 2008b: 565-585.

[5] Marino, Anthony M. and Jan Zabojnik. 2008b. “A Rent Extraction View of Employee Discounts and Benefits,” *Journal of Labor Economics* 26, July 2008: 485-518.

[6] Oyer, Paul. 2008. "Salary or Benefits?" *Research in Labor Economics* 28, 2008:429-467.

[7] Rosen, S. 1986. "Chapter 12 The Theory of Equalizing Differences," in Orley Ashenfelter and Richard Layard, eds., *Handbook of Labor Economics,* Volume 1, New York, Elsevier, 1986:641-692.

[8] Weinschenk, Philipp. 2013. "Compensation, Perks and Welfare." *Economic Letters* 120, July 2013: 67-70.