Equilibrium Theory under Ambiguity*

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Abstract

We extend the classical results on the Walras-core existence and equivalence to an ambiguous asymmetric information economy; that is, an economy where agents maximize Maximin Expected Utility (MEU). The interest of considering ambiguity arises from the fact that, in the presence of MEU decision making, there is no conflict between efficiency and incentive compatibility (contrary to the Bayesian decision making). Our new modeling of an ambiguous asymmetric information economy necessitates new equilibrium notions, which are always efficient and incentive compatible.

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1 Introduction

Modeling the market with uncertainty is of important academic significance and realistic value in economics as most decision making is made under uncertainty. Towards this direction, the Arrow-Debreu "state contingent model" allows the state of nature of the world to be involved in the initial endowments and payoff functions, which is an enhancement of the deterministic general equilibrium model of Arrow-Debreu-McKenzie. According to Arrow-Debreu, agents make contacts ex ante (in period one) before the state of nature is realized and once the state is realized (in period two) the contract is executed and consumption takes place. The issue of incentive compatibility doesn't arise in this model, as all the information is symmetric. However, for the state contingent model to make sense one must assume that there is an exogenous court or government that enforces the contract ex post, otherwise agents may find it beneficial to renege. Radner (1968, 1982) extended the analysis of Arrow and Debreu by introducing asymmetric (differential) information. In particular, each agent is now characterized by his own private information, a random initial endowment, a random utility function and a prior. The private information is modeled as a partition of a finite state space and the allocation of each agent is assumed to be measurable with respect to his own private information. This means that each agent only knows the atom of his partition including the true state, but cannot distinguish those states within the same atom when making decisions. The Walrasian equilibrium notion in this model is called 'Walrasian expectations equilibrium', or WEE in short. Along this line, Yannelis (1991) proposed a core concept, which is called private core.[1]

The Walrasian expectations equilibrium and private core share some interesting properties (in fact, the Walrasian expectations equilibrium is a strict subset of the private core): without the assumption of free disposal, whenever agents are Bayesian expected utility maximizers and allocations are private information measurable, the two above notions are both Bayesian incentive compatible and private information measurable efficient (see Koutsougeras and Yannelis (1993) and Krasa and Yannelis (1994)). However, these solution concepts are only efficient in the second best sense; that is, they are only private information measurable efficient

[1]For a recent treatment of general equilibrium with asymmetric information, see the books Glycopantis and Yannelis (2005) and Marakulin (2013a).

allocations and may result in a possible welfare loss (recall that from Holmstrom and Myerson (1983), we know that with the Bayesian expected utility it is not possible to have allocations which are both first best efficient and also incentive compatible). The existence of WEE in a free disposal economy can be found in Radner (1968, 1982). However, the free disposal WEE allocations may be not incentive compatible (see Glycopantis and Yannelis (2005)). Furthermore, if we require non-free disposal, then a WEE may not exist (see Einy and Shitovitz (2001)). Therefore, a natural question arises:

> Can one find an appropriate framework in the asymmetric information economy such that the existence of equilibrium and core notions continues to hold and furthermore, these notions are both incentive compatible and first best efficient?

A crucial assumption in the frameworks of Radner (1968, 1982) and Yannelis (1991) is that agents maximize Bayesian expected utilities. Nevertheless, from Ellsberg (1961) (see also de Castro and Yannelis (2014)), there is a huge literature which criticizes the Bayesian paradigm and explores the non-expected utility theory. The maximin expected utility of Gilboa and Schmeidler (1989) is one of the successful alternatives. Indeed, recently de Castro, Pesce and Yannelis (2011, 2014) and de Castro and Yannelis (2013) applied the maximin expected utility to an asymmetric information economy with a finite number of states of nature,[2] and introduced various core and Walrasian equilibrium notions. With the maximin expected utilities, agents take into account the worst possible state that can occur and choose the best possible allocations. de Castro, Pesce and Yannelis (2011) proved that the ex ante equilibrium and core notions based on the maximin expected utility, which are called maximin expectations equilibrium (MEE) and maximin core (MC) therein, are incentive compatible in the economy without free disposal. Moreover, it is noteworthy that since the allocations are not required to be measurable with respect to agents' private information, MEE and MC allocations are also first best efficient. Therefore, the conflict between efficiency and incentive compatibility is solved in this new approach. More importantly,

[2] MEU is first applied to a general equilibrium model of an asymmetric information economy by Correia-da-Silva and Hervés-Beloso (2009). They proved the existence of the ex ante Walrasian equilibrium in an asymmetric information economy with maximin preferences and a finite state space. However, their setup is different from ours and they do not consider the issue of incentive compatibility; see also Correia-da-Silva and Hervés-Beloso (2012, 2014).

de Castro and Yannelis (2013) showed that the conflict of incentive compatibility and first best efficiency is inherent in the standard expected utility decision making (Bayesian) and it is resolved only when agents maximize the maximin expected utility (MEU). In particular, they proved that the MEU is a necessary condition for efficient allocations to be incentive compatible. The above work implies the fact that one has to work with MEU if the first best efficiency is desirable. As a result, a natural question arises:

> Can one obtain the classical core-Walras existence and equivalence results for asymmetric information economies where agents are ambiguous (*i.e.*, MEU maximizers) and also the state space is not necessarily finite?

An affirmative answer to this question is of great importance because not only this way one develops a new equilibrium theory where there is no conflict between efficiency and incentive compatibility, but also such positive results could become the main tool for applications in other fields of economics.

The first aim of this paper is to prove the existence of the maximin expectations equilibrium and maximin core in a non-free disposal economy with countably many states of nature.[3] Since there is a countable number of states in the economy, the allocations are infinite dimensional. An advantage of the ambiguous economy modeling is that it allows us to view an asymmetric information economy as a deterministic economy with infinite dimensional commodity spaces. Thus, we can directly apply known results in the literature to obtain the existence of maximin expectations equilibrium.[4] As a corollary, we obtain that the consistency between incentive compatibility and efficiency also holds with a countable number of states.

The second aim of the current paper is to prove a core equivalence theorem for an economy with asymmetric information where agents are ambiguous (*i.e.*, maximize MEU). In a finite agent framework and complete information, Debreu and Scarf (1963) considered a sequence of replicated

[3]For a general equilibrium model with countably many states, see, for example, Hervés-Beloso, Martins-da-Rocha and Monteiro (2009).

[4]On the contrary, one can not readily convert an asymmetric information economy with Bayesian expected utility maximizers to an economy with infinite dimensional commodity spaces due to the restriction of the private information measurability requirement. For some papers with infinite dimensional commodity spaces, see, for example, Bewley (1972) and Podczeck and Yannelis (2008).

economy and showed that the set of non-blocked allocations in every replicated economy converges to the set of Walrasian equilibria. In Section 4, we follow the Debreu-Scarf approach and establish a similar equivalence result for an equal treatment economy with asymmetric information, a countable number of states and MEU preferences. In an atomless economy with complete information, Schmeidler (1972), Grodal (1972) and Vind (1972) improved the core-Walras equivalence theorem of Aumann (1964), by showing that if an allocation is not in the core, then it can be blocked by a non-negligible coalition with any given measure less than 1. Hervés-Beloso, Moreno-García and Yannelis (2005a,b) first extended this result to an asymmetric information economy with the equal treatment property and with an infinite dimensional commodity space by appealing to the finite dimensional Lyapunov's theorem. Bhowmik and Cao (2012, 2013a) obtained further extensions based on an infinite dimensional version of Lyapunov's theorem. All the above results rely on the Bayesian expected utility formulation and therefore the conflict of efficiency and incentive compatibility still holds despite the non atomic measure space of agents.[5] Our Theorem 6 is an extension of Vind's theorem to the asymmetric information economy with the equal treatment property and a countable number of states of nature, where agents behave as maximin expected utility maximizers. Thus, our new core equivalence theorem for the MEU framework, resolves the inconsistency of efficiency and incentive compatibility.

Finally, we provide two characterizations for maximin expectations equilibrium. In the complete information economy with finite agents, Aubin (1979) introduced a new approach that at a first glance seems to be different from the Debreu-Scarf; however one can show that they are essentially equivalent. Aubin considered a veto mechanism in the economy when a coalition is formed; in particular, agents are allowed to participate with any proportion of their endowments. The core notions defined by the veto mechanism, is called Aubin core and it coincides with the Walrasian equilibrium allocations. The approach of Aubin has been extended to an asymmetric information economy to characterize the Walrasian expectations equilibrium (see for example Graziano and Meo (2005), Hervés-Beloso, Moreno-García and Yannelis (2005b) and Bhowmik and Cao (2013a)). Another approach to characterize the Wal-

[5] As the work of Sun and Yannelis (2008) indicates, even with an atomless measure space of agents we cannot guarantee that WEE allocations are incentive compatible.

rasian expectations equilibrium is due to Hervés-Beloso, Moreno-García and Yannelis (2005a,b). They showed that the Walrasian expectations equilibrium allocation cannot be privately blocked by the grand coalition in any economy with the initial endowment redistributed along the direction of the allocation itself. This approach has been extended to a pure exchange economy with an atomless measure space of agents and finitely many commodities, and an asymmetric information economy with an infinite dimensional commodity space (e.g., see Hervés-Beloso and Moreno-García (2008), Bhowmik and Cao (2013a,b)). Our Theorem 2 and 3 extended these two characterizations to the asymmetric information economy with ambiguous agents and with countably many states of nature.

The paper is organized as follows. Section 2 states the model of ambiguous asymmetric information economies with a countable number of states and discusses main assumptions. Section 3 introduces the maximin expectations equilibrium and maximin core and proves their existence, and contains two different characterizations of maximin expectations equilibrium by using the maximin blocking power of the grand coalition. Section 4 extends the maximin expectations equilibrium and maximin core to an economy with a continuum of agents, and interprets the asymmetric information economy with finite agents as a continuum economy with finite types. In addition, two core-Walras equivalence theorems and an extension of Vind's result are given for an asymmetric information economy with a countable number of states. Section 5 shows that maximin efficient allocations are incentive compatible in economies with finite agents and atomless economies with the equal treatment property. Section 6 collects some concluding remarks and open questions. The appendix (Section 7) contains all the main proofs.

2 Ambiguous Asymmetric Information Economy

We define an exchange economy with uncertainty and asymmetric information. The **uncertainty** is represented by a measurable space $(\Omega, \mathcal{F})$, where $\Omega = \{\omega_n\}_{n\in\mathbb{N}}$ is a countable set and $\mathcal{F}$ is the power set of Ω. Let $\mathbb{R}^l_+$ be the commodity space, and $I = \{1, 2, \cdots, s\}$ the set of agents.

For each $i \in I$, $\mathcal{F}_i$ is the σ-algebra on Ω generated by the partition

Π_i of agent i, which represents the private information.[6] Let $\Pi_i(\omega)$ be the element in the partition Π_i which contains ω. Therefore, if any state $\omega \in \Omega$ is realized, then agent i can only observe the event $\Pi_i(\omega)$. The **prior** π_i of agent i is defined on $\mathcal{F}_i$ such that $\sum_{E \in \Pi_i} \pi_i(E) = 1$ and $\pi_i(E) > 0$ for every $E \in \Pi_i$. Notice that π_i is incomplete; that is, the probability of each element in the information partition Π_i is well defined, but not the probability of the event $\{\omega\}$ for every $\omega \in \Omega$. Let $u_i(\omega, \cdot)\colon \mathbb{R}^l_+ \to \mathbb{R}_+$ be the positive **ex post utility function** of agent i at state ω from the consumption space to the positive real line, and $e_i : \Omega \to \mathbb{R}^l_+$ be i's **random initial endowment**.

Let $\mathcal{E}$ be an **ambiguous asymmetric information economy**, where

$$\mathcal{E} = \{(\Omega, \mathcal{F}); (\mathcal{F}_i, u_i, e_i, \pi_i) : i \in I = \{1, \ldots, s\}\}.$$

A **price vector** p is a nonzero function from Ω to $\mathbb{R}^l$.[7] We assume that $\triangle$ denotes the set of all price vectors, where

$$\triangle = \{p \in (\mathbb{R}^l)^\infty : |\sum_{\omega \in \Omega} \sum_{j=1}^{l} p(\omega, j)| = 1\},$$

and $p(\omega, j)$ is the price of the commodity j at the state ω.

There are three stages in this economy: at the ex ante stage (t=0), the information partition and the economy structure are common knowledge; at the interim stage (t=1), each individual i learns his private information $\Pi_i(\omega)$ which includes the true state ω, and makes his consumption plan; at the ex post stage (t=2), agent i receives the endowment and consumes according to his plan.[8]

An **allocation** is a mapping x from $I \times \Omega$ to $\mathbb{R}^l_+$. For each $i \in I$, let

$$L_i = \{x_i : x_i(\omega) \in \mathbb{R}^l_+ \text{ and uniformly bounded for all } \omega \in \Omega\}$$

be the **set of all random allocations** of agent i.[9] If $x_i \in L_i$ and $p \in \triangle$,

[6] For more discussions on information partitions and σ-algebras, see, for example, Hervés-Beloso and Monteiro (2013).

[7] The vector p is said to be nonzero if p is not a constant function of value 0, but it is possible that $p(\omega) = 0$ for some ω.

[8] We consider a pure exchange economy and have no production in our model as for example in Marakulin (2013b). But the production sector can be included in the analysis and the results should still hold. For simplicity of the exposition, we have not included production.

[9] That is, $L_i = l^\infty_+$ for each $i \in I$.

we denote $\sum_{\omega\in\Omega} p(\omega)\cdot x_i(\omega)$ as $p\cdot x_i$.

Suppose that x is an allocation. Then $x_i(\omega)$ is a vector in $\mathbb{R}^l_+$ for each $i\in I$, which represents the allocation at the state ω. In addition, $x_i(\omega,j)$ denotes the allocation of commodity j at the state ω. An allocation x is said to be **feasible** if $\sum_{i\in I} x_i = \sum_{i\in I} e_i$. That is, for each $\omega\in\Omega$,

$$\sum_{i\in I} x_i(\omega) = \sum_{i\in I} e_i(\omega).$$

The feasibility here indicates that the economy has no free disposal.

Assumption (E)**.**

1. *For each $i\in I$, e_i is $\mathcal{F}_i$-measurable.*[10]
2. *There exists some $\beta>0$ such that for any $\omega\in\Omega$ and $1\le j\le l$, $e_i(\omega,j)\ge\beta$.*
3. *There exists some $\gamma>0$ such that for any $\omega\in\Omega$ and $1\le j\le l$, $\sum_{i\in I} e_i(\omega,j)\le\gamma$.*

Assumption (E) is about the endowment. Condition (1) says that each agent's endowment should be measurable with respect to his private information, otherwise the agent may disclose the true state from his endowment. Condition (2) implies that for every agent i, e_i is an interior point of $(\mathbb{R}^l_+)^\infty$ under the sup-norm topology. Condition (3) implies that $e_i\in L_i$; that is, the resource of the economy is limited no matter what the state is. This condition will be automatically satisfied if there are only finitely many states.[11]

Assumption (U)**.**

1. *For each $\omega\in\Omega$ and $i\in I$, $u_i(\omega,\cdot)$ is continuous, strictly increasing and concave.*
2. *For each $i\in I$ and $x\in\mathbb{R}^l_+$, $u_i(\cdot,x)$ is $\mathcal{F}_i$-measurable.*[12]
3. *For any $a\in\mathbb{R}^l_+$ and $K_0>0$ such that $a(j)\le K_0$ for $1\le j\le l$, there exists some $K>0$ such that $0\le u_i(\omega,a)\le K$ for any $i\in I$ and $\omega\in\Omega$. Let $u_i(\omega,0)=0$ for all $i\in I$ and $\omega\in\Omega$.*

Assumption (U) is about the utility. Conditions (1) and (2) are standard in the literature. Condition (3) basically says that agents' utility cannot be arbitrarily large with limited goods. This condition can be

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[10] Clearly, if e_i is independent of ω, then it is $\mathcal{F}_i$-measurable.

[11] Since the initial endowment is bounded, the value $p\cdot e_i$ of the initial endowment e_i is finite for any agent i and price p.

[12] If u_i is state independent, then it is automatically $\mathcal{F}_i$-measurable.

removed if Ω is finite: for each $i \in I$ and $\omega \in \Omega$, $u_i(\omega, a)$ is continuous at a, if a is bounded, then $u_i(\omega, \cdot)$ is bounded; since there are only finitely many states, $u_i(\omega, \cdot)$ is uniformly bounded among all ω. Moreover, the condition $u_i(\omega, 0) = 0$ means that agents have no payoff if they have no consumption.

For every agent i, his private prior may be incomplete and the allocation in L_i is not required to be $\mathcal{F}_i$-measurable. Thus, agents cannot evaluate the allocation based on the Bayesian expected utility. In the current paper, we will consider the maximin preference axiomatized by Gilboa and Schmeidler (1989).[13]

Let $\mathcal{M}_i$ be the set of all probability measures on $\mathcal{F}$ which agree with π_i on $\mathcal{F}_i$. That is,

$$\mathcal{M}_i = \{\mu : \mathcal{F} \to [0, 1] : \mu(E) = \pi_i(E), \forall E \in \mathcal{F}_i\}.$$

Let P_i be a nonempty and convex subset of $\mathcal{M}_i$, which is the set of priors of agent i.

We assume that agent i is ambiguous on the set P_i and will take the worst possible scenario when evaluating his payoff. In particular, for any two allocations $x_i, y_i \in L_i$, agent i prefers the allocation x_i to the allocation y_i if

$$\inf_{\mu \in P_i} \sum_{\omega \in \Omega} u_i(\omega, x_i(\omega))\mu(\omega) \geq \inf_{\mu \in P_i} \sum_{\omega \in \Omega} u_i(\omega, y_i(\omega))\mu(\omega).$$

For any allocation $\{x_i\}_{i \in I}$, the **maximin ex ante utility** of agent i is:

$$V_i(x_i) = \inf_{\mu \in P_i} \sum_{\omega \in \Omega} u_i(\omega, x_i(\omega))\mu(\omega).$$

The **maximin interim utility** of agent i with allocation x_i at the state ω is

$$v_i(\omega, x_i) = \frac{1}{\pi_i(\Pi_i(\omega))} \inf_{\mu \in P_i} \sum_{\omega_1 \in \Pi_i(\omega)} u_i(\omega_1, x_i(\omega_1))\mu(\omega_1).$$

We will slightly abuse the notations by writing $v_i(\omega, x_i) = v_i(E, x_i)$ for $\omega \in E \in \mathcal{F}_i$.

Remark 1. *If P_i is a singleton set for each agent i, then the maximin expected utility above reduces to the standard Bayesian expected utility. If $P_i = \mathcal{M}_i$, the set of all probability measures on $\mathcal{F}$ which agree with π_i*

[13] We can adopt the more general variational preferences axiomatized by Maccheroni, Marinacci and Rustichini (2006), and all the results in Sections 3 and 4 will still go through.

on $\mathcal{F}_i$, then it is the maximin expected utility considered in de Castro and Yannelis (2013). In the latter case, de Castro and Yannelis (2013) showed that for any two allocations $x_i, y_i \in L_i$, agent i prefers the allocation x_i to the allocation y_i if:

$$\sum_{E_i \in \Pi_i} [\inf_{\omega \in E_i} u_i(\omega, x_i(\omega))]\pi_i(E_i) \geq \sum_{E_i \in \Pi_i} [\inf_{\omega \in E_i} u_i(\omega, y_i(\omega))]\pi_i(E_i).^{14} \quad (1)$$

Remark 2. *It should be noted that the asymmetric information in a Bayesian model comes from the private information measurability of allocations. For example, if allocations are not required to be private information measurable, then the framework of Radner (1968) reduces to the standard Arrow-Debreu state-contingent model. In other words, the private information measurability of allocations captures the information asymmetry in a Bayesian model. Furthermore, despite the fact that the Walrasian expectations equilibrium is incentive compatible (see Koutsougeras and Yannelis (1993)), it may be only second best efficient due to the private information measurability requirement of the allocations, which is pointed out in the current paper (see Example 2 below) as well as de Castro and Yannelis (2013).*

In an ambiguity model, the information asymmetry is captured by the maximin expected utility itself. In particular, priors are defined on the information partition of each agent (while they are defined on the whole state space Ω in a Bayesian model). Thus, it is natural to relax the restriction of private information measurability of allocations in an ambiguity model. In addition, we show that the maximin expectations equilibrium is both first best efficient and incentive compatible.

The proposition below indicates that the maximin ex ante utility function satisfies several desirable properties.

Proposition 1. *If Assumption (U) holds, then V_i is increasing and concave, continuous in the sup-norm topology, and lower semicontinuous in the weak* topology.*

Proof. See appendix. □

[14] First, we use 'inf' in these two inequalities instead of 'min' used in de Castro and Yannelis (2013), since there are infinite states here. The existence of infimum is guaranteed since the ex post utility function is nonnegative. Thus the ex ante utility V_i is well defined. Second, although de Castro and Yannelis (2013) only argued that these two inequalities are equivalent when there are finitely many states, this observation is still true in our context.

3 Maximin Expectations Equilibrium and Maximin Core

3.1 Existence of MEE and MC

In this section, we define the notions of maximin core (MC) and maximin expectations equilibrium (MEE).

Given a price vector p, the budget set of agent i is defined as follows:

$$B_i(p) = \{x_i \in L_i : \sum_{\omega \in \Omega} p(\omega) \cdot x_i(\omega) \leq \sum_{\omega \in \Omega} p(\omega) \cdot e_i(\omega)\}.$$

Definition 1. *An allocation x is said to be a* ***maximin expectations equilibrium*** *allocation for the economy $\mathcal{E}$, if there exists a price vector p such that for any agent $i \in I$,*

1. *x_i maximizes $V_i(\cdot)$ subject to the budget set $B_i(p)$;*
2. *x is feasible.*

The following definition of a core concept in the current context implies that coalitions of agents cannot cooperate to become better off in terms of MEU.

Definition 2. *A feasible allocation x is said to be a* ***maximin core*** *allocation for the economy $\mathcal{E}$, if there do not exist a coalition $C \subseteq I$, $C \neq \emptyset$, and an allocation $\{y_i \in L_i\}_{i \in C}$ such that*
(i) $V_i(y_i) > V_i(x_i)$ for all $i \in C$;
(ii) $\sum_{i \in C} y_i(\omega) = \sum_{i \in C} e_i(\omega)$ for all $\omega \in \Omega$.

The allocation is said to be **maximin efficient** if $C = I$.

Remark 3. *The notions of maximin expectations equilibrium, maximin core and maximin efficiency in the above definitions correspond to the concepts of Walrasian equilibrium, core and efficiency in the standard model. If Bayesian expected utilities, instead of maximin expected utilities, are used in Definition 1, and the private information measurability assumption is imposed on allocations, then the solution concept is Walrasian expectations equilibrium defined in Radner (1968, 1982). In particular, the Walrasian expectations equilibrium is defined as follows: an allocation $x = (x_1, \ldots, x_s)$ is said to be a* ***Walrasian expectations equilibrium*** *allocation for the* *economy $\mathcal{E}$, if x_i is an $\mathcal{F}_i$-measurable mapping for each agent i and there exists a price vector p such that for any agent $i \in I$,*

1. *x_i maximizes agent i's expected utility subject to the budget set $B_i(p)$;*
2. *$\sum_{i\in I} x_i \leq \sum_{i\in I} e_i$.*

The following example shows that MEE provides strictly higher efficiency than the (free disposal) WEE allocations. Furthermore, we show that the MEE is incentive compatible.

Example 1. [15] *Consider the following economy with one commodity, the agent space is $I = \{1, 2\}$ and the state space is $\Omega = \{a, b, c\}$. The initial endowments and information partitions of agents are given by*

$$e_1 = (5, 5, 0), \Pi_1 = \{\{a, b\}, \{c\}\};$$

$$e_2 = (5, 0, 5), \Pi_2 = \{\{a, c\}, \{b\}\}.$$

It is also assumed that for $i \in I$, $u_i(\omega, x_i) = \sqrt{x_i}$, which is strictly concave and monotone in x_i, and the priors for both agents are the same: $\mu(\{\omega\}) = \frac{1}{3}$ for every $\omega \in \Omega$.

Suppose that agents are both Bayesian expected utility maximizers. It can be easily checked that there is no (non-free disposal) WEE. If we allow for free disposal, $x_1 = (4, 4, 1)$ and $x_2 = (4, 1, 4)$ is a (free disposal) WEE allocation with the equilibrium price $p(a) = 0$ and $p(b) = p(c) = \frac{1}{2}$. However, this allocation is not incentive compatible (see Example 2 in Section 5 for details).

If $P_i = \mathcal{M}_i$ for each i, and agents are maximin expected utility maximizers, then there exists an MEE (y, p), where $y_1 = (5, 4, 1)$, $y_2 = (5, 1, 4)$ and $p(a) = 0$, $p(b) = p(c) = \frac{1}{2}$.

If state b or c realizes, the ex post utility of agent 1 will be the same in both Bayesian preference setting and maximin preference setting, since $x_1(b) = y_1(b)$ and $x_1(c) = y_1(c)$. But if state a occurs, the ex post utility of agent 1 with maximin preference will be strictly higher than that in the Bayesian preference setting, since

$$x_1(a) = 4 < 5 = y_1(a).$$

[15] This example has been analyzed in Glycopantis and Yannelis (2005) in Bayesian preference setting for the existence and incentive compatibility of Walrasian expectations equilibrium and private core, and in Liu and Yannelis (2013) in maximin preference setting for the existence and incentive compatibility of maximin core. See also Bhowmik, Cao and Yannelis (2014).

Therefore, the maximin preference allows agents to reach higher efficiency.

The following lemma is standard, which shows that the set of maximin expectations equilibrium allocations is included in the set of maximin core allocations.

Lemma 1. *The set of MEE allocations is a subset of the MC allocations, and hence any maximin expectations equilibrium allocation is maximin efficient.*

This inclusion can be strict. It is clear that both the Arrow-Debreu 'state contingent model' and the deterministic general equilibrium model are special cases of our model: if $\mathcal{F}_i = \mathcal{F} = 2^\Omega$ for every $i \in I$, then the maximin expected utility coincides with the Bayesian expected utility and $\mathcal{E}$ is indeed the state contingent model; if Ω is a singleton, then $\mathcal{E}$ is the deterministic model. Moreover, it is well known that in those two models, the set of core allocations could strictly contain the set of Walrasian equilibrium allocations.

We now turn to the issue of the existence of MEE.

Theorem 1. *For an ambiguous asymmetric information economy $\mathcal{E}$, if Assumptions (E) and (U) hold, then there exists an MEE.*

Proof. See appendix. □

Based on Theorem 1 and Lemma 1, it is straightforward to show that the set of maximin core allocations is also nonempty.

Corollary 1. *Under the conditions of Theorem 1, a maximin core allocation exists.*

3.2 Equivalence Theorems

For the economy $\mathcal{E}$, Hervés-Beloso, Moreno-García and Yannelis (2005b) provided two equivalence results for the Walrasian expectations equilibrium in terms of the private blocking power of the grand coalition, and Bhowmik and Cao (2013a) extended this result to an asymmetric information economy whose commodity space is a Banach lattice. We will follow this approach and characterize the maximin expectations equilibrium. The two theorems below correspond to Theorem 4.1 and 4.2 of Hervés-Beloso, Moreno-García and Yannelis (2005b). The proofs are omitted since the same argument can be followed here.

For an allocation $x = \{x_i\}_{i \in I}$ and a vector $a = (a_1, \cdots, a_s) \in [0,1]^s$, consider the ambiguous asymmetric information economy $\mathcal{E}(a,x)$ which is identical with $\mathcal{E}$ except for the random initial endowment of each agent i given by the convex combination $e_i(a_i, x_i) = a_i e_i + (1 - a_i)x_i$.

Definition 3. *An allocation z is* ***maximin dominated*** *(or* ***maximin blocked*** *by the grand coalition) in the economy $\mathcal{E}(a,x)$ if there exists a feasible allocation y in $\mathcal{E}(a,x)$ such that $V_i(y_i) > V_i(z_i)$ for every $i \in I$.*

Theorem 2. *The allocation x is an MEE in $\mathcal{E}$ if and only if x is not a maximin dominated allocation in every economy $\mathcal{E}(a,x)$.*

Definition 4. *A coalition $S \subseteq I$ maximin blocks an allocation x in the sense of Aubin via $y = \{y_i\}_{i \in S}$ if for all $i \in S$, there is some $\alpha_i \in (0,1]$ such that $V_i(y_i) > V_i(x_i)$ and $\sum_{i \in S} \alpha_i y_i \leq \sum_{i \in S} \alpha_i e_i$. The* ***Aubin maximin core*** *is the set of all feasible allocations that cannot be maximin blocked by any coalition in the sense of Aubin. An allocation x is called Aubin non-dominated if x is not maximin blocked by the grand coalition in the sense of Aubin.*

Theorem 3. *The allocation x is an MEE in $\mathcal{E}$ if and only if x is not a maximin dominated allocation in the sense of Aubin in the economy $\mathcal{E}$.*

4 A Continuum Approach

4.1 Basics

In this section, we introduce the maximin expectations equilibrium and maximin core for an atomless economy. Let the atomless probability space $(T, \mathcal{T}, \lambda)$ denote the agent space. We define an **atomless ambiguous asymmetric information economy** as follows:

$$\mathcal{E}_0 = \{(\Omega, \mathcal{F}); (\mathcal{F}_t, u_t, e_t, \pi_t) : t \in T\}.$$

An **allocation** in the continuum economy $\mathcal{E}_0$ is a mapping f from $T \times \Omega$ to $\mathbb{R}^l_+$ such that $f(\cdot, \omega)$ is integrable for every $\omega \in \Omega$ and $f(t, \cdot) \in l^\infty_+$ for λ-almost all $t \in T$. The allocation is said to be **feasible** if $\int_T f(t,\omega)\, \mathrm{d}\lambda(t) = \int_T e(t,\omega)\, \mathrm{d}\lambda(t)$ for every $\omega \in \Omega$.

A coalition in T is a measurable set $S \in \mathcal{T}$ such that $\lambda(S) > 0$. An allocation f is **maximin blocked** by a coalition S in the economy

$\mathcal{E}_0$ if there exists some $g : S \times \Omega \to \mathbb{R}^l_+$ such that $\int_S g(t,\omega)\,\mathrm{d}\lambda(t) = \int_S e(t,\omega)\,\mathrm{d}\lambda(t)$ for every $\omega \in \Omega$, and $V_t(g(t)) > V_t(f(t))$ for λ-almost every $t \in S$.

Definition 5. *An allocation f is said to be the **maximin core** for the economy $\mathcal{E}_0$ if it is not maximin blocked by any coalition.*

Definition 6. *An allocation f is said to be a **maximin expectations equilibrium** allocation for the economy $\mathcal{E}_0$, if there exists a price vector p such that*

1. *f_t maximizes $V_t(\cdot)$ subject to the budget set $B_t(p)$ for λ-almost all $t \in T$;*
2. *f is feasible.*

4.2 A Continuum Interpretation of the Finite Economy

We associate an atomless economy $\mathcal{E}_c$ with the discrete economy $\mathcal{E}$ as in García-Cutrín and Hervés-Beloso (1993), Hervés-Beloso, Moreno-García and Yannelis (2005a,b) and Bhowmik and Cao (2013a). The space of agents in $\mathcal{E}_c$ is the Lebesgue unit interval $(T, \mathcal{T}, \mu)$ such that $T = \cup_{i=1}^s T_i$, where $T_i = [\frac{i-1}{s}, \frac{i}{s})$ for $i = 1, \cdots, s-1$ and $T_s = [\frac{s-1}{s}, 1]$. For each agent $t \in T_i$, set $\mathcal{F}_t = \mathcal{F}_i$, $\pi_t = \pi_i$, $u_t = u_i$ and $e_t = e_i$. Thus, the maximin ex ante utility V_t of agent t is V_i. We refer to T_i as the set of agents of type i, and

$$\mathcal{E}_c = \{(\Omega, \mathcal{F}); (T, \mathcal{F}_i, V_i, e_i, \pi_i) : i \in I = \{1, \cdots, s\}\}$$

is the **economy with the equal treatment property**. The allocations in $\mathcal{E}$ and $\mathcal{E}_c$ are closely related: for any allocation f in $\mathcal{E}_c$, there is an corresponding allocation x in $\mathcal{E}$, where $x_i(\omega) = \frac{1}{\mu(T_i)} \int_{T_i} f(t,\omega)\,\mathrm{d}\mu(t)$ for all $i \in I$ and $\omega \in \Omega$; conversely, an allocation x in $\mathcal{E}$ can be interpreted as an allocation f in $\mathcal{E}_c$, where $f(t,\omega) = x_i(\omega)$ for all $t \in T_i$, $\omega \in \Omega$ and $i \in I$. f is said to be a step allocation if $f(\cdot,\omega)$ is a constant function on T_i for any $\omega \in \Omega$ and $i \in I$.

Analogously to the theorems in Hervés-Beloso, Moreno-García and Yannelis (2005a,b), the next proposition shows that the maximin expectations equilibrium can be considered equivalent in discrete and continuum approaches.

Proposition 2. *Suppose that Assumption (U) holds. Then we have the following properties:*

- *If (x, p) is an MEE for the economy $\mathcal{E}$, then (f, p) is the MEE for the associated continuum economy $\mathcal{E}_c$, where $f(t, \omega) = x_i(\omega)$ if $t \in T_i$.*
- *If (f, p) is an MEE for the economy $\mathcal{E}_c$, then (x, p) is the MEE for the economy $\mathcal{E}$, where $x_i(\omega) = \frac{1}{\mu(T_i)} \int_{T_i} f(t, \omega) \, \mathrm{d}\mu$ for any $\omega \in \Omega$.*

The proof is straightforward, interested readers may refer to Theorem 3.1 of Hervés-Beloso, Moreno-García and Yannelis (2005b).

4.3 Core Equivalence with a Countable Number of States

The core-Walras equivalence theorem has been recently extended to a Bayesian asymmetric information economy. Specifically, Einy, Moreno and Shitovitz (2001) showed that the Walrasian expectations equilibrium is equivalent to the private core for atomless economies with a finite number of commodities in a free disposal setting, Angeloni and Martins-da-Rocha (2009) completed the discussion by proposing appropriate conditions which guarantees the core equivalence result in non-free disposal context. Hervés-Beloso, Moreno-García and Yannelis (2005a,b) and Bhowmik and Cao (2013a) followed the Debreu- Scarf approach and showed that the set of Walrasian expectations equilibrium allocations coincides with the private core in the asymmetric information economy with the equal treatment property, finitely many states and infinitely many commodities.

However, all these discussions focus on the asymmetric information economy with Bayesian expected utilities and a finite state space. Our aim here is to examine whether this result is still true when agents are ambiguous (have maximin expected utilities) and the state space is countable. The theorems below show that the core equivalence theorem holds with either of the following conditions:

1. Maximin expected utility and finitely many states;
2. Maximin expected utility, countably many states and the equal treatment property holds.

Theorem 4. *Let Ω be finite in the atomless economy $\mathcal{E}_0$. Assume that (E) and (U) hold. Then the set of MC allocations coincides with the set of MEE allocations.*

We omit the proof since it is standard, interested readers may check that the proof of the core equivalence theorem in Hildenbrand (1974) with minor modifications still holds.

Theorem 5. *Suppose Assumptions (E) and (U) hold. Let the step allocation f be feasible in the associated continuum economy $\mathcal{E}_c$. Then f is an MEE allocation if and only if f is an MC allocation.*

Proof. See appendix. □

4.4 An Extension of Vind's Theorem

Hervés-Beloso, Moreno-García and Yannelis (2005a,b) and Bhowmik and Cao (2013a) extended Vind's theorem to an asymmetric information economy with the equal treatment property. Sun and Yannelis (2007) established this theorem in an economy with a continuum of agents and negligible asymmetric information. Below, we extend this result to the atomless ambiguous asymmetric information economy with a countable number of states of nature.

Theorem 6. *Suppose that Assumptions (E) and (U) hold. If the feasible step allocation f is not in the MC of the associated continuum economy $\mathcal{E}_c$, then for any α, $0 < \alpha < 1$, there exists a coalition S such that $\mu(S) = \alpha$, which maximin blocks f.*

Proof. See appendix. □

5 Efficiency and Incentive Compatibility under Ambiguity

In this section, we will define a notion of maximin incentive compatibility, and then prove that any maximin efficient allocation is maximin incentive compatible.

First, we illustrate the incentive compatibility issue when agents adopt Bayesian preferences.

Example 2. *[Example 1 with Bayesian preference]*
Recall Example 1 in Section 3.1: the agent space is $I = \{1, 2\}$ and the state

space is $\Omega = \{a, b, c\}$. *The initial endowments and information partitions of agents are given by*

$$e_1 = (5, 5, 0), \Pi_1 = \{\{a, b\}, \{c\}\};$$

$$e_2 = (5, 0, 5), \Pi_2 = \{\{a, c\}, \{b\}\}.$$

It is also assumed that for $i \in I$, $u_i(\omega, x_i) = \sqrt{x_i}$, *which is strictly concave and monotone in* x_i, *and the priors for both agents are the same:* $\mu(\{\omega\}) = \frac{1}{3}$ *for every* $\omega \in \Omega$.

Suppose that agents are Bayesian expected utility maximizers, and all allocations are required to be private information measurable. The no-trade allocation $x_1 = (5, 5, 0)$ *and* $x_2 = (5, 0, 5)$ *is in the private core and it is incentive compatible. Indeed, it has been shown in Koutsougeras and Yannelis (1993) that private core allocations are always CBIC provided that the utility functions are monotone and continuous.*

This conclusion is not true in free disposal economies. Glycopantis and Yannelis (2005) pointed out that private core and Walrasian expectations equilibrium allocations need not be incentive compatible in an economy with free disposal. In this example, $x_1 = (4, 4, 1)$ *and* $x_2 = (4, 1, 4)$ *is a (free disposal) WEE allocation with the equilibrium price* $p(a) = 0$ *and* $p(b) = p(c) = \frac{1}{2}$, *and hence in the (free disposal) private core. However, this allocation is not incentive compatible. Indeed, if agent 1 observes* $\{a, b\}$, *he has an incentive to report state* c *to become better off. Note that agent 2 cannot distinguish the state* a *from the state* c. *In particular, if state* a *occurs, agent 1 has an incentive to report state* c *because his utility is* $u_1(e_1(a) + x_1(c) - e_1(c))$, *which is greater than the utility* $u_1(x_1(a))$ *when he truthfully reports state* a. *That is,*

$$u_1(e_1(a) + x_1(c) - e_1(c)) = u_1(5 + 1 - 0) = \sqrt{6} > \sqrt{4} = u_1(x_1(a)).$$

Hence, the free disposal WEE allocation is not incentive compatible.

Note that in the above example, when agent 1 reports $\{c\}$ and agent 2 reports $\{b\}$, there will be incompatible reports. To rule out such situations, we make the following assumption.

Assumption (R). *For any* $i \in I$ *and* $E_i \in \Pi_i$, $\cap_{i \in I} E_i = \{\omega\}$ *for some* $\omega \in \Omega$.

Remark 4. *This assumption is only needed in this section. Assump-*

tion (R) above guarantees that there are no incompatible reports. The assumption that the intersection is a singleton set is without loss of generality. If $\{a, b\} \subseteq \cap_{i \in I} E_i$ for two states a and b, then no one can distinguish these two states and hence they can be combined as one state.

de Castro and Yannelis (2013) showed that their choice of maximin expected utility is both sufficient and necessary for the incentive compatibility of maximin Pareto efficient allocations. In this section, we shall adopt the maximin expected utility considered in de Castro and Yannelis (2013). That is, as in Remark 1, for any two allocation $x_i, y_i \in L_i$, agent i prefers the allocation x_i to the allocation y_i if

$$\sum_{E_i \in \Pi_i} [\inf_{\omega \in E_i} u_i(\omega, x_i(\omega))]\pi_i(E_i) \geq \sum_{E_i \in \Pi_i} [\inf_{\omega \in E_i} u_i(\omega, y_i(\omega))]\pi_i(E_i).$$

Below, we propose a notion of maximin incentive compatibility.

Definition 7. *An allocation x is said to be* ***maximin incentive compatible*** *(MIC) if the following does not hold:*

1. *there exists an agent $i \in I$, and two events $E_i^1, E_i^2 \in \Pi_i$;*
2. *$e_i(\omega) + x_i(b(\omega)) - e_i(b(\omega)) \in \mathbb{R}^l_+$ for each $\omega \in E_i^1$ and $\{b(\omega)\} = (\cap_{j \neq i} \Pi_j(\omega)) \cap E_i^2$;*
3. $$\inf_{\omega_1 \in E_i^1} u_i(\omega_1, y_i(\omega_1)) > \inf_{\omega_1 \in E_i^1} u_i(\omega_1, x_i(\omega_1)),$$

   where

   $$y_i(\omega) = \begin{cases} e_i(\omega) + x_i(b(\omega)) - e_i(b(\omega)), & \text{if } \omega \in E_i^1; \\ x_i(\omega), & \text{otherwise.} \end{cases}$$

In other words, an allocation is maximin incentive compatible if it is impossible for any agent to misreport the realized event and become better off. That is, if the true event is E_i^1 and agent i reports E_i^2, then the allocation y_i under the misreported event E_i^2 will not make him better off.

In this paper, we consider a partition model for the information structure. Alternatively, one can also consider a type model.

Let $\Omega = \Theta = \prod_{i \in I} \Theta_i$, where Θ_i is the private information set of agent i. For any state $\omega \in \Omega$, $\omega = (\theta_1, \theta_2, \ldots, \theta_s)$, let $\Pi_i(\omega) = \{\theta_i\} \times \Theta_{-i}$, where Θ_{-i} is the set of states for all agents other than i. Then the maximin incentive compatibility can be described as follows, and Definitions 7 and 8 are equivalent.

Definition 8. *An allocation x is MIC if for every agent i and two distinct points $\tilde{\theta}_i, \hat{\theta}_i$ in Θ_i such that for every $\theta_{-i} \in \Theta_{-i}$,*

$$y_i^{\tilde{\theta}_i}(\hat{\theta}_i, \theta_{-i}) = e_i(\tilde{\theta}_i) + x_i(\hat{\theta}_i, \theta_{-i}) - e_i(\hat{\theta}_i) \in \mathbb{R}_+^l$$

and

$$\inf_{\theta_{-i} \in \Theta_{-i}} u_i(\tilde{\theta}_i, x_i(\tilde{\theta}_i, \theta_{-i})) \geq \inf_{\theta_{-i} \in \Theta_{-i}} u_i(\tilde{\theta}_i, y_i^{\tilde{\theta}_i}(\hat{\theta}_i, \theta_{-i})).$$

Thus, an agent i cannot become better off in terms of maximin expected utility by reporting $\hat{\theta}_i$ when his true state is $\tilde{\theta}_i$.

The following theorem shows that any maximin efficient allocation is maximin incentive compatible.

Theorem 7. *If Assumptions (E), (U) and (R) hold, then any maximin efficient allocation in $\mathcal{E}$ is MIC.*

Proof. See appendix. □

Corollary 2. *Under the conditions of Theorem 7, any MC or MEE allocation is maximin incentive compatible.*

Remark 5. *There is a substantial literature on the mechanism design under ambiguity; see, for example, Bodoh-Creed (2012), de Castro and Yannelis (2013) and Bose and Renou (2014). Bodoh-Creed (2012) considers a standard mechanism design environment except that agents are ambiguity averse with preferences of the maxmin expected utility. In particular, Bodoh-Creed (2012) assumes that each agent knows his valuation but has ambiguous beliefs about the distribution of valuations of the other agents which can be modeled by a convex set of priors, while we consider the particular case that this set contains all possible priors. There are significant differences between Bodoh-Creed's paper and ours. In particular, Bodoh-Creed (2012) focuses on the payoff equivalence theorem and characterizes the revenue maximizing mechanism, which could be constrained efficient (i.e., second best efficient). On the contrary, we study the issue between the first best efficiency and incentive compatibility.*

Remark 6. *One could extend the result of Angelopoulos and Koutsougeras (2015) on maximin value allocations to an ambiguous asymmetric information economy with countably many states. By standard arguments, one* *could show that the maximin value allocation is maximin efficient, and therefore, it is maximin incentive compatible by the above corollary.*

6 Concluding Remarks

We presented a new asymmetric information economy framework, where agents face ambiguity (*i.e.*, they are MEU maximizers) and also the state space is not necessarily finite. This new set up allowed us to derive new core -Walras existence and equivalence results. It should be noted that contrary to the Bayesian asymmetric information economy framework, our core and Walrasian equilibrium concepts formulated in an ambiguous asymmetric information economy framework are now incentive compatible and obviously efficient. For this reason, we believe that our new results will be useful to other fields in economics.

We would like to conclude by saying that the continuum of states and modeling perfect competition as in Sun and Yannelis (2007, 2008), Sun, Wu and Yannelis (2012, 2013) and Qiao, Sun and Zhang (2014), or modeling the idea of informational smallness (*i.e.*, approximate perfect competition) in countable replica economies as in McLean and Postlewaite (2003), or characterizing cores in economies where agents' information can be altered by coalitions as in Hervés-Beloso, Meo and Moreno-García (2014) in the presence of ambiguity remain open questions and further research in this direction seems to be needed.

7 Appendix

7.1 Proof of Proposition 1

It is clear that V_i is increasing and concave, we first show that it is weak* lower semicontinuous.

Suppose that the sequence $\{z^k\}_{k\geq 0} \subseteq L_i$, and $z^k \to z^0$ in the weak* topology as $k \to \infty$. Fix $\epsilon > 0$. Since $z^0 \in L_i = l_+^\infty$, there exists some positive number $K_0 > 0$ such that $z^0(\omega, j) < K_0$ for each $1 \leq j \leq l$ and $\omega \in \Omega$. By Assumption (U.3), there exists some $K > 0$ such that $u_i(\omega, z^0(\omega)) \leq K$ for any $\omega \in \Omega$.

Suppose that $\Pi_i = \{E_m\}_{m\in\mathbb{N}}$. Then there exists some m_0 sufficiently large such that $\pi_i(\cup_{1\leq m\leq m_0} E_m) > 1 - \frac{\epsilon}{2K}$. Let $\Omega^{m_0} = \cup_{1\leq m\leq m_0} E_m$. Then we have

$$\begin{aligned}V_i(z^k) - V_i(z^0) &= \inf_{\mu\in P_i}\sum_{\omega\in\Omega} u_i(\omega, z^k(\omega))\mu(\omega) - \inf_{\mu\in P_i}\sum_{\omega\in\Omega} u_i(\omega, z^0(\omega))\mu(\omega) \\ &\geq \inf_{\mu\in P_i}\sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^k(\omega))\mu(\omega) - \inf_{\mu\in P_i}\sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega) \\ &- \inf_{\mu\in P_i}\sum_{\omega\notin\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega).\end{aligned}$$

For the third term, we have

$$\inf_{\mu\in P_i}\sum_{\omega\notin\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega) \leq K\pi_i(\Omega\setminus\Omega^{m_0}) < \frac{\epsilon}{2}.$$

Since z^k weak* converges to z^0 and Ω^{m_0} is finite, $z^k(\omega)$ converges to $z^0(\omega)$ for each $\omega\in\Omega^{m_0}$. Thus, we have

$$\left|\inf_{\mu\in P_i}\sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^k(\omega))\mu(\omega) - \inf_{\mu\in P_i}\sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega)\right| < \frac{\epsilon}{2}$$

for k sufficiently large. As a result, $V_i(z^k) - V_i(z^0) > -\epsilon$ for k sufficiently large, which implies that $V_i(\cdot)$ is weak* lower semicontinuous.

Next we show that V_i is continuous in the sup-norm topology. The proof is similar as the argument above.

Suppose that the sequence $\{z^k\}_{k\geq 0}\subseteq L_i$, and $z^k\to z^0$ in the sup-norm topology. Then $\{z^k\}_{k\geq 0}$ is uniformly bounded by some K_0. By Assumption (U.3), there exists some $K>0$ such that $u_i(\omega, z^k(\omega))\leq K$ for any $k\geq 0$ and $\omega\in\Omega$. Following an analogous argument as in the proof of the weak* lower semicontinuity, one can obtain a finite subset Ω^{m_0} such that $\pi_i(\Omega^{m_0}) > 1-\frac{\epsilon}{2K}$. Then we have

$$\begin{aligned}\left|V_i(z^k) - V_i(z^0)\right| &= \left|\inf_{\mu\in P_i}\sum_{\omega\in\Omega} u_i(\omega, z^k(\omega))\mu(\omega) - \inf_{\mu\in P_i}\sum_{\omega\in\Omega} u_i(\omega, z^0(\omega))\mu(\omega)\right| \\ &\leq \left|\inf_{\mu\in P_i}\sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^k(\omega))\mu(\omega) - \inf_{\mu\in P_i}\sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega)\right| \\ &+ \sup_{\mu\in P_i}\sum_{\omega\notin\Omega^{m_0}} u_i(\omega, z^k(\omega))\mu(\omega) + \sup_{\mu\in P_i}\sum_{\omega\notin\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega).\end{aligned}$$

As in the above argument,

$$\sup_{\mu\in P_i}\sum_{\omega\notin\Omega^{m_0}} u_i(\omega, z^k(\omega))\mu(\omega),\ \sup_{\mu\in P_i}\sum_{\omega\notin\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega) < \frac{\epsilon}{2};$$

and

$$\left|\inf_{\mu\in P_i}\sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^k(\omega))\mu(\omega) - \inf_{\mu\in P_i}\sum_{\omega\in\Omega^{m_0}} u_i(\omega, z^0(\omega))\mu(\omega)\right| < \frac{\epsilon}{2}$$

for k sufficiently large. As a result, $\left|V_i(z^k) - V_i(z^0)\right| \leq \epsilon$ for k sufficiently large, which implies that $V_i(\cdot)$ is continuous in the sup-norm topology.

7.2 Proofs in Sections 3 and 4

One can view an ambiguous asymmetric information economy $\mathcal{E}$ as a complete information economy $\mathcal{E}_d = \{(l^\infty_+, V_i, e_i)\colon i \in I\}$ with the agent space I.[16] That is, each agent i has the utility function V_i and the infinite dimensional commodity space l^∞_+. Given the initial endowment $e_i\colon \Omega \to \mathbb{R}^l_+$ in the economy $\mathcal{E}$, since Ω is countable, e_i can be viewed as a point in the infinite dimensional commodity space l^∞_+ of the deterministic economy $\mathcal{E}_d$. By Proposition 1, the utility function V_i is increasing, concave and norm continuous, and lower semicontinuous in the weak* topology.

Given an allocation $x = (x_1, \ldots, x_s) \in l^\infty_+$ and a price $p \in (l^\infty)^\circ$, for any agent $i \in I$,

$$p \cdot x_i = \int_\Omega x_i(\omega) p(\mathrm{d}\omega).$$

An equilibrium in $\mathcal{E}_d$ is a pair $(x = (x_1, \ldots, x_s), p)$ with $x_i \in l^\infty_+$ for each $i \in I$ and $p \in (l^\infty)^\circ$ such that

1. $x_i \in B_i(p) = \{y \in l^\infty_+ \colon p \cdot y \leq p \cdot e_i\}$;
2. x_i maximizes $V_i(\cdot)$ on the budget set $B_i(p)$;
3. $\sum_{i\in I} x_i = \sum_{i\in I} e_i$.

It can be easily checked that if $p \in l^1$, then the equilibrium (x, p) in the economy $\mathcal{E}_d$ is also an MEE in the ambiguous asymmetric information economy $\mathcal{E}$.

[16] Let l^∞ and l^1 represent the spaces of all bounded sequences and all absolutely summable sequences, respectively. Denote by $(l^\infty)^\circ$ the topological dual space of l^∞.

Since V_i is norm continuous, it is Mackey continuous with respect to the Mackey topology $\tau(l^\infty, (l^\infty)^\circ)$ by Corollary 6.23 in Aliprantis and Border (2006). Then the economy $\mathcal{E}_d$ has a competitive equilibrium (x^*, p^*) by Propositions 5.2.3 and 5.3.1 in Florenzano (2003), where $p^* \in (l^\infty)^\circ$. Since V_i is lower semicontinuous in the weak* topology, it is also lower semicontinuous in the Mackey topology $\tau(l^\infty, l^1)$. By Theorem 2 in Bewley (1972), we know that p^* is indeed in l^1. One can then normalize p^* such that $\|p^*\|_1 = 1$. Then it is clear that (x^*, p^*) is also a maximin expectations equilibrium in the ambiguous asymmetric information economy $\mathcal{E}$, which proves Theorem 1.

If $\mathcal{E}_c$ is an atomless ambiguous asymmetric information economy, one can also view $\mathcal{E}_c$ as an atomless complete information economy $\mathcal{E}_{cd}$ as above. Then Theorems 5 and 6 follow from Theorems 3.2 and 3.3 in Hervés-Beloso, Moreno-García and Yannelis (2005b).

7.3 Proof of Theorem 7

Recall that for any agent i, allocation $z \in L_i$ and event $E \in \Pi_i$, $v_i(E, z) = \inf_{\omega \in E} u_i(\omega, z(\omega))$. Let $\{x_i\}_{i\in I}$ be a maximin efficient allocation, and assume that it is not maximin incentive compatible. Then there exist an agent $i \in I$, and two events $E_i^1, E_i^2 \in \Pi_i$ such that

$$v_i(E_i^1, y_i) > v_i(E_i^1, x_i),$$

where

$$y_i(\omega) = \begin{cases} e_i(\omega) + x_i(b) - e_i(b), & \text{if } \omega \in E_i^1, \{b\} = (\cap_{j\neq i}\Pi_j(\omega)) \cap E_i^2; \\ x_i(\omega), & \text{otherwise.} \end{cases}$$

For each $j \neq i$, define $y_j \colon \Omega \to \mathbb{R}_+^l$ as follows:

$$y_j(\omega) = \begin{cases} e_j(\omega) + x_j(b) - e_j(b), & \text{if } \omega \in E_i^1, \{b\} = (\cap_{j\neq i}\Pi_j(\omega)) \cap E_i^2; \\ x_j(\omega), & \text{otherwise.} \end{cases}$$

It can be easily checked that $\{y_i\}_{i\in I}$ is feasible:

1. If $\omega \in E_i^1$ and $\{b\} = (\cap_{j\neq i}\Pi_j(\omega)) \cap E_i^2$, then $\sum_{j\in I} y_j(\omega) = \sum_{j\in I} e_j(\omega) + \sum_{j\in I} x_j(b) - \sum_{j\in I} e_j(b) = \sum_{j\in I} e_j(\omega)$, since $\sum_{j\in I} e_j(b) =$

$\sum_{j\in I} x_j(b)$.

2. If $\omega \notin E_i^1$, then $\sum_{j\in I} y_j(\omega) = \sum_{j\in I} x_j(\omega) = \sum_{j\in I} e_j(\omega)$.

We now show that agent i is better off and all other agents are not worse off if considering the allocation y instead of x.

For agent i, if $\omega \notin E_i^1$, then $v_i(\omega, y_i) = v_i(\omega, x_i)$. In addition, $v_i(E_i^1, y_i) > v_i(E_i^1, x_i)$. Therefore, $V_i(y_i) = \sum_{E_i\in\Pi_i} v_i(E_i, y_i)\pi_i(E_i) > \sum_{E_i\in\Pi_i} v_i(E_i, x_i)\pi_i(E_i) = V_i(x_i)$.

For $j \neq i$ and event E_j, if $\omega \in E_i^1$, then there exists a point $b(\omega) \in E_j \cap E_i^2$ such that $e_j(b(\omega)) = e_j(\omega)$ and $y_j(\omega) = e_j(\omega) + x_j(b(\omega)) - e_j(b(\omega)) = x_j(b(\omega))$. Notice that $u_j(\omega, y_j(\omega)) = u_j(\omega, x_j(b(\omega))) = u_j(b(\omega), x_j(b(\omega)))$. If $\omega \notin E_i^1$, then $y_j(\omega) = x_j(\omega)$. Thus, we have

$$\begin{aligned}
v_j(E_j, y_j) &= \min\left(\inf_{\omega\in E_j, \omega\in E_i^1} u_j(\omega, y_j(\omega)), \inf_{\omega\in E_j, \omega\notin E_i^1} u_j(\omega, y_j(\omega))\right) \\
&= \min\left(\inf_{\omega\in E_j, \omega\in E_i^1} u_j(b(\omega), x_j(b(\omega))), \inf_{\omega\in E_j, \omega\notin E_i^1} u_j(\omega, x_j(\omega))\right) \\
&= \inf_{\omega\in E_j, \omega\notin E_i^1} u_j(\omega, x_j(\omega)) \\
&\geq \inf_{\omega\in E_j} u_j(\omega, x_j(\omega)) \\
&= v_j(E_j, x_j).
\end{aligned}$$

Then $V_j(y_j) = \sum_{E_j\in\Pi_j} v_j(E_j, y_j)\pi_j(E_j) \geq \sum_{E_j\in\Pi_j} v_j(E_j, x_j)\pi_j(E_j) = V_j(x_j)$ for all $j \neq i$.

Since $\epsilon y_i \to y_i$ as $\epsilon \to 1$ in $(\mathbb{R}_+^l)^\infty$ and V_i is continuous, there exists $\epsilon \in (0, 1)$ such that

$$V_i(\epsilon y_i) > V_i(x_i) \text{ for all } i \in C.$$

For all $\omega \in \Omega$, define

$$z_j(\omega) = \begin{cases} \epsilon y_j(\omega) & \text{if } j = i; \\ y_j(\omega) + \frac{1-\epsilon}{\|I-1\|} y_i(\omega) & \text{if } j \neq i. \end{cases}$$

Then $V_i(z_i) = V_i(\epsilon y_i) > V_i(x_i)$. Moreover, since $u_i(\omega, \cdot)$ is strongly monotone, for all $j \neq i$

$$V_j(z_j) = V_j(y_j + \frac{1-\epsilon}{\|I-1\|} y_i) > V_j(y_j) \geq V_j(x_j). \tag{2}$$

Notice that for every $\omega \in \Omega$,

$$\sum_{i \in I} z_i(\omega) = \epsilon y_i(\omega) + \sum_{j \neq i} y_j(\omega) + (1-\epsilon) y_i(\omega)$$
$$= \sum_{i \in I} y_i(\omega) = \sum_{i \in I} e_i(\omega).$$

That is, z is feasible and by (2), $V_i(z_i) > V_i(x_i)$ for any i. Thus, $\{x_i\}_{i \in I}$ is not maximin efficient, a contradiction.

References

C. D. Aliprantis and K. C. Border, Infinite Dimensional Analysis: A Hitchhiker's Guide, Springer, Berlin, 2006.

L. Angeloni and V. F. Martins-da-Rocha, Large economies with differential information and without free disposal, *Economic Theory* **38** (2009), 263–286.

A. Angelopoulos and L. C. Koutsougeras, Value allocation under ambiguity, *Economic Theory* **59** (2015), 147–167.

J. P. Aubin, Mathematical Methods of Game and Economic Theory, North-Holland, Amsterdam, 1979.

R. J. Aumann, Markets with a continuum of traders, *Econometrica* **32** (1964), 39–50.

T. F. Bewley, Existence of equilibria in economies with infinitely many commodities, *Journal of Economic Theory* **4** (1972), 514–540.

A. Bhowmik and J. Cao, Blocking efficiency in an economy with asymmetric information, *Journal of Mathematical Economics* **48** (2012), 396–403.

A. Bhowmik and J. Cao, On the core and Walrasian expectations equilibrium in infinite dimensional commodity spaces, *Economic Theory* **53** (2013a), 537–560.

A. Bhowmik and J. Cao, Robust efficiency in mixed economies with asymmetric information, *Journal of Mathematical Economics* **49** (2013b), 49–57.

A. Bhowmik, J. Cao and N. C. Yannelis, Aggregate preferred correspondence and the existence of a maximin REE, *Journal of Mathematical Analysis and Applications* **414** (2014), 29–45.

A. L. Bodoh-Creed, Ambiguous beliefs and mechanism design, *Games and Economic Behavior* **75** (2012), 518–537.

S. Bose and L. Renou, Mechanism design with ambiguous communication devices, *Econometrica* **82** (2014), 1853–1872.

J. Correia-da-Silva and C. Hervés-Beloso, Prudent expectations equilibrium in economies with uncertain delivery, *Economic Theory* **39** (2009), 67–92.

J. Correia-da-Silva and C. Hervés-Beloso, General equilibrium with uncertain delivery, *Economic Theory* **51** (2012), 729–755.

J. Correia-da-Silva and C. Hervés-Beloso, Irrelevance of private information in two-period economies with more goods than states of nature, *Economic Theory* **55** (2014), 439–455.

L. I. de Castro, M. Pesce and N. C. Yannelis, Core and equilibria under ambiguity, *Economic Theory* **48** (2011), 519–548.

L. I. de Castro, M. Pesce and N. C. Yannelis, A new perspective to rational expectations, working paper (2014).

L. I. de Castro and N. C. Yannelis, Ambiguity aversion solves the conflict between efficiency and incentive compatibility, working paper (2013).

L. I. de Castro and N. C. Yannelis, An interpretation of Ellsbergs paradox based on information and incompleteness, *Economic Theory Bulletin* **1** (2013), 139–144.

G. Debreu and H. Scarf, A limit theorem on the core of an economy, *International Economic Review* **4** (1963), 235–246.

E. Einy, D. Moreno and B. Shitovitz, Competitive and core allocations in large economies with differentiated information, *Economic Theory* **18** (2001), 321–332.

E. Einy and B. Shitovitz, Private value allocations in large economies with differential information, *Games and Economic Behavior* **34** (2001), 287–311.

D. Ellsberg, Risk, ambiguity, and the savage axioms, *Quarterly Journal of Economics* **75** (1961), 643–669.

M. Florenzano, General equilibrium analysis: existence and optimality properties of equilibria, Springer Science and Business Media, 2003.

J. García-Cutrín and C. Hervés-Beloso, A discrete approach to continuum economies, *Economic Theory* **3** (1993), 577–583.

I. Gilboa and D. Schmeidler, Maximin expected utility with non-unique prior, *Journal of Mathematical Economics* **18** (1989), 141–153.

D. Glycopantis and N. C. Yannelis, Differential information economies, Springer, 2005.

M. G. Graziano and C. Meo, The Aubin private core of differential information economies, *Decisions in Economics and Finance* **28** (2005), 9–31.

B. Grodal, A second remark on the core of an atomless economy, *Econometrica* **40** (1972), 581–583.

C. Hervés-Beloso, E. Moreno-García and N. C. Yannelis, An equivalence theorem for differential information economy, *Journal of Mathematical Economics* **41** (2005a), 844–856.

C. Hervés-Beloso, E. Moreno-García and N. C. Yannelis, Characterization and incentive compatibility of Walrasian expectations equilibrium in infinite dimensional commodity spaces, *Economic Theory* **26** (2005b), 361–381.

C. Hervés-Beloso and E. Moreno-García, Competitive equilibria and the grand coalition, *Journal of Mathematical Economics* **44** (2008), 697–706.

C. Hervés-Beloso, V. F. Martins-da-Rocha and P. K. Monteiro, Equilibrium theory with asymmetric information and infinitely many states, *Economic Theory* **38** (2009), 295–320.

C. Hervés-Beloso and P. K. Monteiro, Information and σ-algebras, *Economic Theory* **54** (2013), 405–418.

C. Hervés-Beloso, C. Meo and E. Moreno-García, Information and size of coalitions, *Economic Theory* **55** (2014), 545–563.

W. Hildenbrand, Core and Equilibria of a Large Economy, Princeton University Press, Princeton, NJ, 1974.

B. Holmstrom and R. B. Myerson, Efficient and durable decision rules with incomplete information, *Econometrica* **51** (1983), 1799–1819.

S. Krasa and N. C. Yannelis, The value allocation of an economy with differential information, *Econometrica* **62** (1994), 881–900.

L. C. Koutsougeras and N. C. Yannelis, Incentive compatibility and information superiority of the core of an economy with differential information, *Economic Theory* **3** (1993), 195–216.

Z. Liu and N. C. Yannelis, Implementation under ambiguity, working paper (2013).

F. Maccheroni, M. Marinacci and A. Rustichini, Ambiguity aversion, robustness, and the variational representation of preferences, *Econometrica* **74** (2006), 1447–1498.

V. M. Marakulin, Equilibria and core as an essence of economic contractual interactions: mathematical foundations, Novosibirsk, 320 p., monograph under work, 2013a.

V. M. Marakulin, On the Edgeworth conjecture for production economies with public goods: A contract-based approach, *Journal of Mathematical Economics* **49** (2013b), 189–200.

R. McLean and A. Postlewaite, Informational size, incentive compatibility, and the core of a game with incomplete information, *Games and Economic Behavior* **45** (2003), 222–241.

K. Podczeck and N. C. Yannelis, Equilibrium theory with asymmetric information and with infinitely many commodities. *Journal of Economic Theory* **141** (2008), 152–183.

L. Qiao, Y. N. Sun and Z. Zhang, Conditional exact law of large numbers and asymmetric information economies with aggregate uncertainty, *Economic Theory* (2014) forthcoming. DOI: 10.1007/s00199-014-0855-6

R. Radner, Competitive equilibrium under uncertainty, *Econometrica* **36** (1968), 31–58.

R. Radner, Equilibrium under uncertainty, in Handbook of mathematical economics, vol. II. (K. J. Arrow and M. D. Intriligator eds.) North Holland, Amsterdam, 1982.

D. Schmeidler, A remark on the core of an atomless economy, *Econometrica* **40** (1972), 579–580.

Y. N. Sun, L. Wu and N. C. Yannelis, Existence, incentive compatibility and efficiency of the rational expectations equilibrium, *Games and Economic Behavior* **76** (2012), 329–339.

Y. N. Sun, L. Wu and N. C. Yannelis, Incentive compatibility of rational expectations equilibrium in large economies: A counterexample, *Economic Theory Bulletin* **1** (2013), 3–10.

Y. N. Sun and N. C. Yannelis, Core, equilibria and incentives in large asymmetric information economies, *Games and Economic Behavior* **61** (2007), 131–155.

Y. N. Sun and N. C. Yannelis, Ex ante efficiency implies incentive compatibility, *Economic Theory* **36** (2008), 35–55.

K. Vind, A third remark on the core of an atomless economy, *Econometrica* **40** (1972), 585–586.

N. C. Yannelis, The core of an economy with differential information, *Economic Theory* **1** (1991), 183–196.

Rational Expectations Equilibrium Under Ambiguity*

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Abstract: Rational expectations equilibrium (REE) seeks a proper treatment of behavior under private information by assuming that the information revealed by prices is taken into account by agents in their decisions. Despite being an important advancement in economic theory, REE has been the subject of intense criticism since its inception. The criticisms usually focus on the unrealistic behavior of agents in face of uncertainty. Among those criticisms, rarely appears the fact that REE may fail to exist Kreps (1977). This paper suggests that the two kind of problems are connected and a deeper understanding of the existence problem may lead to advances in the other. Why sometimes a REE does not exist? Essentially, there may be an inconsistency between the behavior with the information provided by prices and the information that this behavior actually reveals. Since the 80's, the usual reaction to this problem is to dismiss it, since existence may be guaranteed generically Radner (1979); Allen (1981). However, lack of equilibrium existence may be a sign of the need of an alternative. In this paper, we present such an alternative, i.e., we explore the implications of introducing ambiguity aversion instead of the usual Bayesian approach in rational expectations. We show that if individuals' preferences are variational (i.e., satisfy a general form of ambiguity aversion), then a REE exists *universally* and not only generically. If we particularize the preferences to a particular form of the maximin expected utility (MEU) model introduced in Gilboa and Schmeidler (1989), then we are able to prove efficiency and incentive compatibility, properties that do not hold for Bayesian Rational Expectation Equilibria.

Keywords: Rational Expectations, Ambiguity Aversion, Maximin Expected Utility.

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1 Introduction

Some of the most important developments in economics were related to modeling of information and the study of its use by agents in certain economic situations. The introduction of the rational expectations paradigm is a good example of such breakthroughs. Rational expectation equilibrium (REE) theory offers a rigorous conceptual framework to modeling the information conveyed by prices into the decision of economic agents. The fact that prices may convey useful information to market participants is well known at least since Hayek (1945). The main feature of REE is the requirement of consistency of the optimal actions of economic agents and the information that those optimal actions reveal through prices.

Since its introduction, REE played a central role in the analysis of financial markets and markets of contracts, more broadly. However, it has also attracted many criticisms, especially related to its unrealistic requirements on the knowledge and sophistication of economic agents; it is frequently argued that REE requires just too much rationality to be credible. Despite the longevity of this conundrum, we still lack of an alternative.

Besides the unrealistic requirements on knowledge and sophistication of individuals, a much less debated problem of REE is the fact that sometimes it may fail to exist. This fact was established by Kreps (1977), through an influential and well-known example, which we revisit in Section 1.1 below.[4] Many economists do not see lack of existence as a legitimate economic problem. However, we are used to expect our models to be internally consistent. A model without equilibrium indicates a basic failure of consistency, that hampers all analysis based on it. This problem of lack of existence led to considerable efforts to overcome it. In seminal papers, Radner (1979) and Allen (1981) prove the *generic* existence of REE when individuals are Bayesians. For decades, these generic existence results seemed to be satisfactory enough to more or less ignore the problem.[5][6]

But if an equilibrium may fail to exist, isn't this a sign that there are some problems with the formulation of the concept? Connecting this with the unrealistic treatment of information and rationality, isn't it the case that we are simply not using the more suitable model for information and behavior under uncertainty? This paper explores this possible alternative.

As it turns out, the criticism of the usual model of behavior assumed for REE—expected utility maximization—is even older and broader than the criticism to REE itself. Indeed, dissatisfaction with the reigning Bayesian paradigm is as old as the paradigm itself. Important criticisms of Savage (1954)'s expected utility theory go back to Allais (1953), Ellsberg (1961) and others. However, no criticism has substantial impact without an alternative. To this date, the most successful alternative to Bayesianism is the Maximin Expected Utility (MEU) model introduced by Gilboa and Schmeidler (1989), which generated a huge literature on ambiguity aversion. In recent years, ambiguity aversion models have led to interesting applications in finance, macroeconomics, game theory and mechanism design, as we briefly discuss in Section

[4] Green (1977) also presented a different non-existence example of the rational expectations equilibrium.

[5] For a history of rational expectations equilibrium, see Grossman (1981).

[6] Generic results are not satisfactory (not robust) because if one perturbs the utility functions or the initial endowments, the equilibrium can fail.

1.2 below. One of the purpose of this paper is to show that ambiguity aversion also has an important impact on the way we understand *rational expectations equilibrium (REE)*.

We prove that a REE exists *universally* (not generically) for a general class of preferences with ambiguity. Particularizing the preferences to MEU, we prove that the REE is also efficient and further particularizing to a special kind of MEU, we show that it is incentive compatible.[7] For this, we present an alternative to the standard rational expectations equilibrium of Radner (1979) and Allen (1981) where individuals are ambiguous. Specifically, in our setup, agents maximize their maximin expected utility conditioning on their own private information and also on the information the equilibrium prices have generated.

The following reexamination of the financial example introduced by Kreps (1977) clarifies our results.

1.1 Kreps' example

Kreps (1977) provides a simple financial example that allows us to understand the heart of our contribution. He assumes that there are two assets: a riskless asset that costs and pays 1 and a risky asset that is sold at period $t = 1$ by the price $p(\omega) \in \mathbb{R}_+$ and pays $V(\omega)$ in period $t = 2$, where ω denotes the state of the world. There are two individuals, both with utility $U(c) = -e^{-c}$ for the consumption of c units at $t = 2$. Individual 1 knows whether $V(\omega)$ is distributed according to a normal with mean m_1 and variance σ^2 or according to a normal with mean m_2 and variance σ^2. Let s_1 denote the first distribution and s_2, the second. That is, individual 1 knows which distribution s_j $(j = 1, 2)$ governs $V(\omega)$. On the other hand, individual 2 only knows that the distribution governing $V(\omega)$ is in the set $S \equiv \{s_1, s_2\}$, but he can infer s once he observes the prices.[8] To complete the description, assume that individual i is endowed with k_{ij} units of the risky asset if s_j occurs, for $i, j \in \{1, 2\}$. Endowments of the riskless asset are constant and, therefore, ignored.

Now if an individual knows s and buys q units of the risky asset, his consumption will be $x(\omega) = -p(\omega) \cdot q + (q + k_{i\cdot}) \cdot V(\omega)$, leading to the expected utility:

$$u_i(s, x) = E_s \{- \exp [- (-p \cdot q + (q + k_{i\cdot}) \cdot V)]\}, \tag{1}$$

where E_s denotes expectation with respect to $s \in \{s_1, s_2\}$. As natural, we assume that the price $p(\omega)$ depends only on s and write $p(\omega) = p_j$ if $s = s_j$, $j = 1, 2$. Given the normality of the risky asset returns, we have for $j = 1, 2$:

$$u_i(s_j, x) = - \exp \left[pq - m_j (q + k_{ij}) + \frac{\sigma^2}{2} (q + k_{ij})^2 \right], \tag{2}$$

which leads to the following optimal quantity if the individual knows which s obtains:

$$q_{ij} = \frac{m_j - p_j}{\sigma^2} - k_{ij}, \quad \text{for } i = 1, 2 \text{ and } s = s_j, j = 1, 2. \tag{3}$$

[7] See the preference definition in section 2.3 and a discussion about it in subsection 7.2.

[8] Nothing changes in the analysis if we assume that individual 2 considers all convex combinations of s_1 and s_2 as possible.

Let us consider the case in which *both individuals are Bayesian*. If individual 2 is uniformed, that is, $p_1 = p_2$, then he considers a mixture of normals (s_1 and s_2). In any case, his optimal choice, although not given by (3), is a *single quantity* $q_{21} = q_{22}$. Kreps first observes that if $m_1 \neq m_2$ and $k_{1j} = 0$, for $j = 1, 2$ then prices cannot be uninformative, that is, we cannot have $p_1 = p_2$. Indeed, in this case $q_{11} \neq q_{12}$, but since $q_{2j} = -q_{1j}$, this would imply $q_{21} \neq q_{22}$, contradicting the previous observation.[9]

Thus, assume that $p_1 \neq p_2$ and individual 2 is informed, that is, all choices are given by (3). Kreps notes that if $m_1 = 4$, $m_2 = 5$, $k_{21} = 2$, $k_{22} = 4$ and $\sigma^2 = 1$, then $p_1 = p_2 = 3$, which contradicts $p_1 \neq p_2$. This contradiction shows that no rational expectations equilibrium exists.

Let us now observe what happens with our MEU formulation. Under full information, there is no ambiguity and the individuals' behaviors are exactly as above. However, in the case that 2 is uniformed ($p_1 = p_2$), then he faces ambiguity and takes the worst-case scenario in his evaluation. He is, therefore, indifferent among a set of different quantities q_{ij}—in particular, he is indifferent among quantities that promises utilities above the minimum between the two states.[10] Which among his equally good quantities will be selected? It is standard to think that a Walrasian auctioneer selects the quantity that clears the market, but the information about the quantity chosen by the Walrasian auctioneer is available to the individual only after all choices are made and, therefore, cannot affect his behavior.[11] This means that the restriction $q_{21} = q_{22}$ used above no longer holds. He could receive different quantities on different states. For example, an equilibrium with the above parameters would be $p_1 = p_2 = 3$ and $q_{21} = -1$ and $q_{22} = -2$.

As the reader has noticed, the crucial property is the individual's indifference among many bundles. This indifference between allocations leads to an important departure from the Bayesian case. Early works, such as Dow and Werlang (1992), have explored this indifference. Note also that the property described that the individual does not know how many units he will actually receive is not an artifact of our model. In real world markets, this is almost always true. Once a trader submits an order, especially big ones, he does not know how many actual units will be traded and, when he learns that, the trade is already completed. In *dark pools*, this separation between the price and the volume information is even more pronounced, and our model and above discussion seems even more relevant.

Remark 1.1 Notice that in the original Kreps' model, private information measurability of the quantities play a crucial role in the failure of existence, as we pointed out in footnote 9. Basically, the problem is that if prices do not reveal information, we may end up requiring that quantities bought in different states be different, but this is possible only if prices do reveal information. The requirement of measurability makes sense if each individual is buying the quantity itself, but it is natural to dispense with

[9] Another way of describing the same problem is to think that the decision on quantities is measurable with respect to the information partition that the individual has after observing prices. We comment more on this in Remark 1.1 below.

[10] Note that he is indifferent taking in account the information that he has when making decisions. Obviously, he is *not* indifferent ex post.

[11] Agents are not allowed to retrade after receiving their quantities delivered by the auctioneer.

this restriction if we see this as a negotiation of contracts in the interim period, whose quantities are finally determined in the ex post period. Relaxation of private information measurability is also an important ingredient in our theory (please see discussion in Sections 2.5 and 7.1).

1.2 Relevant literature

Our paper belongs to the growing literature that applies ambiguity aversion to revisit old puzzles and facts that were not well understood under the Bayesian framework, but could be successfully explained using ambiguity aversion. Our contribution shows a new feature of ambiguity aversion that highlights the usefulness of these models established by previous papers in many different applications.

Hansen and Sargent (2001) establishes a link between the maximin expected utility and the robust control theory and opens the avenue for applications of the MEU theory in macroeconomics issues.

Caballero and Krishnamurthy (2008) used a MEU model to study flight to quality episodes, which are an important source of financial and macroeconomic instability. Given the repeated occurrence of such crises and their economic impact, this is an important topic of investigation. Their MEU model is able to explain crisis regularities such as market-wide capital immobility, liquidity hoarding and agents' disengagement from risk.

Epstein and Schneider (2007) consider portfolio choices and the effect of changes in confidence due to learning for Bayesian and ambiguity averse agents. They show that ambiguity aversion induces more stock market participation and investment in comparison to Bayesian individuals. A variation on this topic is pursued in Epstein and Schneider (2008), that assumes that investors perceive a range of signal precisions, and evaluate financial decisions with respect to worst-case scenarios. As a result, good news is discounted, while bad news is taken seriously. This implies that expected excess returns are thus higher when information quality is more uncertain. They are also able to provide an explanation to the classic question in finance of why stock prices are so much more volatile than measures of the expected present value of dividends. The recent work by Epstein and Schneider (2010) discusses how ambiguity aversion models have implications for portfolio choice and asset pricing that are very different from those of the standard Bayesian model. They also show how this can explain otherwise puzzling features of the data.

Condie and Ganguli (2011a) studied rational expectations equilibria with ambiguity averse decision makers. They show that partial revelation can be robust in a MEU model, while Condie and Ganguli (2011b) established full revelation for almost all sets of beliefs for Choquet expected utility with convex capacities.

Perhaps one of the more interesting set of implications of ambiguity aversion models has been obtained by Ju and Miao (2012). They calibrated a smooth ambiguity model that matches the mean equity premium, the mean risk-free rate and the volatility of the equity premium. Their model also allows to explain many curious facts previously observed in the data, such as the procyclical variation of price-dividend ratios, the countercyclical variation of equity premia and equity volatility, the leverage effect and the mean reversion of excess returns. All these results hinge crucially on the pessimistic behavior of ambiguity averse agents.

Ilut and Schneider (2012) use ambiguity aversion to study business cycle fluctuations in a DSGE model. They show that a loss of confidence about productivity works like "unrealized" bad news. This time-varying confidence can explain much of business cycle fluctuations.

Hansen and Sargent (2012) define three types of ambiguity, depending on how the models of a planner and agents differ. All these variations are departures from the standard rational expectations Bayesian model, where planner and agents share exactly the same model. They compute a robust Ramsey plan and an associated worst-case probability model for each of three types of ambiguity and examine distinctive implications of these models.

With respect to the standard REE literature, it is well known by now that the Bayesian REE as formulated by Radner (1979), Allen (1981) and Grossman (1981) exists only generically.

The description of differential information via a partition of the state space was used by Radner (1968) and Allen (1981). In contrast, Radner (1979) and Condie and Ganguli (2011a) use a model based on signals, similar to the one described above. (Allen and Jordan, 1998, p. 7-8) discuss the reinterpretation of this kind of model in Allen (1981)'s partition model. In particular, Radner (1979) and Condie and Ganguli (2011a) fix a "state-dependent utility" in the terminology of Allen (1981) and specify various economies by the appropriate notion of conditional beliefs. Radner (1979) describes signals as providing information on the conditional probability distribution over a set of states. All information in Radner (1979) is obtained by knowing everyone's joint signal.

As such, the partitions observable by traders are over the space of joint signals as opposed to the state space over which consumption occurs. Radner calls these consumption states the "payoff-relevant part of the environment" (top of page 659). If an individual receives signal t_i then he knows that the joint signal is in the set of joint signals for which he receives the signal t_i. This imposes additional structure on the types of partitions over the signal space that agents observe.

In a series of papers, Correia-da Silva and Hervés-Beloso (2008, 2009, 2012, 2014); Angelopoulos and Koutsougeras (2015); Zhiwei (2014, 2015); de Castro, Yannelis, and Zhiwei (2015); Bhowmik, Cao, and Yannelis (2014) introduced economies with uncertain delivery, where agents negotiate contracts that are not measurable with respect to their information. As in our paper, agents may receive bundles that were supposed to be delivered in different states of nature. Those papers are perhaps the closest to our research. In particular, Correia-da Silva and Hervés-Beloso (2009) proved an existence theorem for a Walrasian equilibrium for an economy with asymmetric information, where agents' preferences are represented by maximin expected utility functions. Their MEU formulation is in the ex-ante sense. This seems to be the first application of the MEU to the general equilibrium existence problem with asymmetric information. Since they work with the ex-ante maximin expected utility formulation, their results have no bearing on ours. Finally, there are several game theoretical applications, e.g. Bodoh-Creed (2012) and Aryal and Stauber (2014) where it is shown that ambiguity provides new insights.

1.3 Organization of the paper

The paper is organized as follows: in Section 2 we describe the economic model and define the two sets of preferences that we consider in our paper. In Section 3 we define and compare the standard Bayesian REE and our maximin REE (MREE). Section 4 establishes the existence of MREE. Sections 5 and 6 deal respectively with the efficiency and incentive compatibility of maximin REE. Some concluding remarks and open questions are collected in Section 7. The appendix (Section 8) collects longer proofs.

2 Model—Differential Information Economy

2.1 Differential information economy

We consider an exchange economy under uncertainty and asymmetrically informed agents. The uncertainty is represented by a measurable space $(S, \mathcal{F})$, where S is a finite set of possible states of nature and $\mathcal{F}$ is the algebra of all the events, i.e., $\mathcal{F}$ is S's power set. Let $\mathbb{R}^{\ell}_{+}$ be the commodity space and I be a set of n agents, i.e., $I = \{1, \ldots, n\}$. **A differential information exchange economy** $\mathcal{E}$ is the following collection:

$$\mathcal{E} = \{(S, \mathcal{F});\ (\mathcal{F}_i, u_i, e_i)_{i \in I}\},$$

where for all $i \in I$

- $\mathcal{F}_i$ is a partition of S, representing the **private information** of agent i. The interpretation is as usual: if $s \in S$ is the state of nature that is going to be realized, agent i observes $\mathcal{F}_i(s)$, the unique element of $\mathcal{F}_i$ containing s. By an abuse of notation, we still denote by $\mathcal{F}_i$ the algebra generated by the partition $\mathcal{F}_i$.

- a **random utility function** (or state dependent utility) representing his (ex post) preferences:

$$\begin{array}{rccc} u_i: & S \times \mathbb{R}^{\ell}_{+} & \to & \mathbb{R} \\ & (s, x) & \to & u_i(s, x). \end{array}$$

  We assume that for all $s \in S$, $u_i(s, \cdot)$ is continuous and monotone.

- a **random initial endowment** of physical resources represented by a function $e_i : S \to \mathbb{R}^{\ell}_{+}$.

For some results (but not for our existence Theorem 4.1), we will need the following:

Assumption 2.1 *For each $i \in I$, $e_i(\cdot)$ is $\mathcal{F}_i$-measurable.*

We discuss this assumption, the interpretation of the above economy and its timing in Section 2.4 below.

We use the following notations. For two vectors $x = (x^1, \dots, x^\ell)$ and $y = (y^1, \dots, y^\ell)$ in $\mathbb{R}^\ell$, we write $x \geq y$ when $x^k \geq y^k$ for all $k \in \{1, \dots, \ell\}$; $x > y$ when $x \geq y$ and $x \neq y$; and $x \gg y$ when $x^k > y^k$ for all $k \in \{1, \dots, \ell\}$. A function $u : \mathbb{R}^\ell_+ \to \mathbb{R}$ is (strictly) monotone if for all $x, y \in \mathbb{R}^\ell_+$, $(x > y)$ $x \gg y$ implies that $u(x) > u(y)$; and it is (strictly) quasi-concave if for all $x, y \in \mathbb{R}^\ell_+$ and all $\alpha \in (0, 1)$ we have that $(u(\alpha x + (1-\alpha)y) > \min\{u(x), u(y)\})$ $u(\alpha x + (1-\alpha)y) \geq \min\{u(x), u(y)\}$. Given two sets A and B, the notation $A \setminus B$ refers to the set-theoretic difference, i.e., $A \setminus B = \{a : a \in A \, \text{and} \, a \notin B\}$. Finally $\sigma(u_i) \subseteq \mathcal{F}_i$ refers to $u_i(\cdot, t)$ is $\mathcal{F}_i$-measurable for all $t \in \mathbb{R}^\ell_+$; and $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ means that $u_i(\cdot, t)$ and $e_i(\cdot)$ are $\mathcal{F}_i$-measurable for all $t \in \mathbb{R}^\ell_+$. Notice that if $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for all $i \in I$, in particular Assumption 2.1 holds.

A **price** p is a function from S to $\mathbb{R}^\ell_+$. In some references, a price is defined as a non-zero function from S to $\mathbb{R}^\ell_+$, meaning that for some s (not necessarily for all s) $p(s) > 0$. However, with standard arguments it can be proved that if there is at least one agent i such that $u_i(s, \cdot)$ is monotone for all $s \in S$, then the equilibrium price p is positive in any state (i.e., $p(s) > 0$ for any $s \in S$). Moreover, if $p : S \to \Delta$, where Δ is the $(\ell-1)$-dimensional unit simplex in $\mathbb{R}^\ell_+$, (as defined for example in Allen (1981)) then in particular $p(s) > 0$ for any $s \in S$. Thus, since throughout the paper we assume that $u_i(s, \cdot)$ is monotone for all $s \in S$ and all $i \in I$, the equilibrium price p is positive in each state, i.e., $p : S \to R^\ell_+ \setminus \{0\}$.

In order to introduce the rational expectation notions in Section 3, we need the following notation. Let $\sigma(p)$ be the smallest sub-algebra of $\mathcal{F}$ for which $p(\cdot)$ is measurable and let $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$ denote the smallest algebra containing both $\mathcal{F}_i$ and $\sigma(p)$.

A function $x : I \times S \to \mathbb{R}^\ell_+$ is said to be a **random consumption vector or allocation**.

For each i, the function $x_i : S \to \mathbb{R}^\ell_+$ is said to be an allocation[12] of agent i, while for each s, the vector $x_i(s) \in \mathbb{R}^\ell_+$ is a bundle of agent i in state s. We denote by L_i the set of all i's allocations, moreover let $\bar{L}_i$ and L_i^{REE} be the following sets:

$$\begin{aligned} \bar{L}_i &= \{x_i \in L_i : \ x_i(\cdot) \text{ is } \mathcal{F}_i\text{−measurable}\}. && (4) \\ L_i^{REE} &= \{x_i \in L_i : \ x_i(\cdot) \text{ is } \mathcal{G}_i\text{−measurable}\}. && (5) \end{aligned}$$

Clearly, for each agent $i \in I$, since any $\mathcal{F}_i$-measurable allocation is also $\mathcal{G}_i$-measurable, it follows that $\bar{L}_i \subseteq L_i^{REE} \subseteq L_i$, and hence $\bar{L} \subseteq L^{REE} \subseteq L$, where $L = \prod_{i \in I} L_i$, $\bar{L} = \prod_{i \in I} \bar{L}_i$ and $L^{REE} = \prod_{i \in I} L_i^{REE}$.

An allocation x (i.e., $x \in L$) is said to be **feasible**[13] if

$$\sum_{i \in I} x_i(s) = \sum_{i \in I} e_i(s) \quad \text{for all } s \in S.$$

[12] For simplicity, we will often use the symbol $x_i(s) \in \mathbb{R}^\ell_+$ to denote $x(i, s) \in \mathbb{R}^\ell_+$. Similarly, $x_i(\cdot)$ refers to the function $x(i, \cdot) : S \to \mathbb{R}^\ell_+$. Finally, $x(s)$ refers to the function $x(\cdot, s) : I \to \mathbb{R}^\ell_+$.

[13] We assume the so-called exact feasibility because it is well known that incentive compatibility may fail under the free disposal condition.

We will describe the agents' preferences below. The above structure, including each agent's preference, is common knowledge for all agents.

2.2 Bayesian Expected utility

We define now the interim expected utility. To this end, we assume that each individual $i \in I$ has a known probability π_i on $\mathcal{F}$, such that $\pi_i(s) > 0$ for any $s \in S$. For any partition $\Pi \subset \mathcal{F}$ of S and any allocation $x_i : S \to \mathbb{R}^\ell_+$, agent i's **interim expected utility** function with respect to Π at x_i in state s is given by

$$v_i(x_i|\Pi)(s) = \sum_{s' \in S} u_i(s', x_i(s'))\pi_i(s'|s),$$

where

$$\pi_i(s'|s) = \begin{cases} 0 & \text{for} \quad s' \notin \Pi(s) \\ \frac{\pi_i(s')}{\pi_i\big(\Pi(s)\big)} & \text{for} \quad s' \in \Pi(s). \end{cases}$$

We can also express the interim expected utility as follows

$$v_i(x_i|\Pi)(s) = \sum_{s' \in \Pi(s)} u_i(s', x_i(s'))\frac{\pi_i(s')}{\pi_i\big(\Pi(s)\big)}. \tag{6}$$

Notice that the interim expected utility function v_i is well defined since we have assumed that for each $i \in I$ and $s \in S$, $\pi_i(s) > 0$, therefore $\pi_i(\Pi(s)) > 0$.

In the applications below, the partition Π will be agent-dependent, being the original private information partition $\mathcal{F}_i$ or, more frequently, the partition generated also by the prices, $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$.

2.3 Preferences that allow for ambiguity

In this section, we discuss preferences which allow for ambiguity. The most general case just requires continuity and monotonicity. To define this, let $\Pi_i \subset \mathcal{F}$ be a partition of S representing the information available to individual i. The preferences of each individual i, with private information Π_i in state s are represented by $V_i(\cdot, \Pi_i, s)$ such that given $x_i, y_i \in L_i$

$(*)$ If $u_i(s', x_i(s')) \geq u_i(s', y_i(s'))$ for all $s' \in \Pi_i(s)$ then $V_i(x_i, \Pi_i, s) \geq V_i(y_i, \Pi_i, s)$

$(**)$ If $\Pi_i(s) = \{s\}$, then $V_i(x_i, \Pi_i, s) = u_i(s, x_i(s))$.

The above consistency requirements between interim and ex-post preferences[14]

[14] Notice that condition $(*)$ implies Axiom 4 in de Castro, Pesce, and Yannelis (2011). imply the existence of a continuous and monotonic function $A : \mathbb{R}^{|\Pi_i(s)|} \to \mathbb{R}$ such that:

$$V_i(x_i, \Pi_i, s) = A((u_i(s', x_i(s'))_{s' \in \Pi_i(s)}). \tag{7}$$

(see Proposition 2.2 in de Castro, Pesce, and Yannelis (2011)).

Note that standard Bayesian preferences will satisfy (7). Therefore, (7) is not restricted to ambiguous preferences. We will further particularize the preferences to include that of Gilboa and Schmeidler (1989).

Let $\mathcal{C}_i^s$ be the set of all probabilities with support contained on $\Pi_i(s)$.[15] Let $\mathcal{M}_i^s$ be a non empty, closed and convex subset of $\mathcal{C}_i^s$ which is the set of priors for agent i. We assume that agent i is ambiguous on the set $\mathcal{M}_i^s$ and he will take the worst possible scenario when evaluating his payoff. For any two allocations $x_i, y_i \in L_i$; agent i prefers x_i to y_i if

$$\inf_{\mu \in \mathcal{M}_i^s} E_\mu \left[u_i(\cdot, x_i(\cdot)) \right] \geq \inf_{\mu \in \mathcal{M}_i^s} E_\mu \left[u_i(\cdot, y_i(\cdot)) \right].$$

Thus, for any allocation $x_i \in L_i$ the utility of agent i with respect to the information Π_i in state s is:

$$\underline{u}_i^{\Pi_i}(s, x_i) = \inf_{\mu \in \mathcal{M}_i^s} E_\mu \left[u_i(\cdot, x_i(\cdot)) \right]. \tag{8}$$

In the case of state independent utility, (8) represents the seminal conditional preferences in the Gilboa-Schmeidler form. For this reason we call it **maximin expected utility (MEU)**.

Remark 2.1 If $\mathcal{M}_i^s$ is a singleton set then the maximin expected utility (MEU) reduces to the standard Bayesian expected utility. If $\mathcal{M}_i^s = \mathcal{C}_i^s$ then it is the maximin expected utility considered in de Castro and Yannelis (2011) where it is shown that

$$\inf_{\mu \in \mathcal{C}_i^s} E_\mu \left[u_i(\cdot, x_i(\cdot)) \right] = \min_{s' \in \Pi_i(s)} u_i(s', x_i(s')).$$

We will adopt the model of de Castro and Yannelis (2011) so that the utility of each agent i with respect to Π_i at the allocation x_i in state s is:

$$\underline{u}_i^{\Pi_i}(s, x_i) = \min_{s' \in \Pi_i(s)} u_i(s', x_i(s')). \tag{9}$$

It is proved in de Castro and Yannelis (2011) that efficient allocations are incentive compatible if and only if individuals' preferences are represented by (9). Moreover, as we note in Subsection 2.5, if agents' utility is given by (9), the requirement that allocations may not be private information measurable is justified. This is crucial in proving the existence of a rational expectations equilibrium (see Example 3.3 and Theorem 4.1). However, all results, except incentive compatibility, holds true for the general MEU formulation (8).

Whenever for each agent i the partition Π_i is his private information partition $\mathcal{F}_i$, then we do not use the superscript, i.e.,

$$\underline{u}_i(s, x_i) = \min_{s' \in \mathcal{F}_i(s)} u_i(s', x_i(s')).$$

[15] In particular this means that for any $\mu \in \mathcal{C}_i^s$, $\mu(s') = 0$ for any $s' \notin \Pi_i(s)$ and $\sum_{s' \in \Pi_i(s)} \mu(s') = 1$.

On the other hand, when we deal with the notion of rational expectations equilibrium (according to which agents take into account also the information that the equilibrium prices generate), then for each agent i the partition Π_i is $\mathcal{G}_i$ and the maximin utility is defined as

$$\underline{u}_i^{REE}(s, x_i) = \min_{s' \in \mathcal{G}_i(s)} u_i(s', x_i(s')), \text{ where } \mathcal{G}_i = \mathcal{F}_i \vee \sigma(p).$$

2.4 Timing and Budget Sets

We can specify the timing of the economy as follows. There are three periods: ex ante ($t = 0$), interim ($t = 1$) and ex post ($t = 2$). Although consumption takes place only at the ex post stage, the other events occur as follows:

- At $t = 0$, the state space, the partitions, the structure of the economy and the price functional $p : S \to \mathbb{R}_+^\ell$ are common knowledge. This stage does not play any role in our analysis and it is assumed just for a matter of clarity.
- At $t = 1$, each individual learns his private information signal $\mathcal{F}_i(s)$ and the prevailing price $p(s) \in \mathbb{R}_+^\ell$. Therefore, he learns $\mathcal{G}_i(s)$, where $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. With this information, the individual plans how much he will consume, $x_i(s)$. Note, however, that his actual consumption (as his endowment) may be contingent to the final state of the world, not yet known by the individual. The agent only knows that one of the states $s' \in \mathcal{G}_i(s)$ obtains, but not exactly which. Therefore, he needs to make sure that he will be able to pay his consumption plan $x_i(s')$ for all $s' \in \mathcal{G}_i(s)$, that is, $p(s') \cdot x_i(s') \leq p(s') \cdot e_i(s')$ for all $s' \in \mathcal{G}_i(s)$.
- At $t = 2$, individual i receives and consumes his entitlement $x_i(s)$.

The interpretation of this model is that the plan that the individual makes at the interim stage ($t = 1$) serves as the channel through which his information is passed to the system, or to the "Walrasian auctioneer," if one prefers. This is necessary for the purpose of aggregation of information among the individuals and to guarantee the feasibility of the final allocations.

Note that the above discussion leads to the following budget set:

$$B_i(s, p) = \{y_i \in L_i \ : \ p(s') \cdot y_i(s') \leq p(s') \cdot e_i(s') \ \text{ for all } s' \in \mathcal{G}_i(s)\}. \tag{10}$$

An important departure of the above model from Radner (1979) is that the private information measurability condition needs not to be imposed. We discuss this issue in greater detail in the sequel.

2.5 Private Information Measurability

A particular case of the above specification is the model in which endowments are private information measurable (the individual knows his endowment), as in (Allen, 1981, p. 1179). Our model certainly allows this case, but our main result does not require it. Therefore, we refrain from imposing this condition in our general framework. This allows us to cover situations in which is more natural to assume that individuals *do not* know their endowments. For instance, in labor markets, workers may fail to be completely informed of their abilities. Another example: someone has stored corn in a barn, but does not know how much of it survived the appetite of the barn's rats.

Note also that the consumption plan $x_i(\cdot)$ does not need to be private information measurable, as it is usually assumed in these models (see Radner (1979), Radner (1982)). We have already discussed this in the context of financial markets (see the end of Section 1.1), but it is also reasonable in many other situations. For example, assume that you visit a restaurant in an exotic country for the first time. Although you know how much you have in cash and the price that you will have to pay for your meal, you will not know exactly what you will eat (or its quality) until the meal is actually served to you. Yet another example: you may know what you contracted and how much is the premium for your insurance, but do not know how good their services will be in the event that you file a claim.

In Radner (1979) and Allen (1981) all market participants have preferences represented by the interim expected utility function given by (6), where for any $i \in I$, $\Pi_i = \mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$, since according to the notion of rational expectations equilibrium, agents make their consumption decision taking into account not only their private information, but also the information generated by the equilibrium price. Thus, if s is the realized state of nature, each agent i receives the information signal $\mathcal{G}_i(s)$, the unique element of the partition $\mathcal{G}_i$ containing s. With this information, agent trades. In the second period, once consumption takes place, the state of nature is only incompletely and differently observed by agents. Indeed, if s occurs, each individual i does not know which state belonging in the event $\mathcal{G}_i(s)$ has occurred. Hence, i asks to consume the same bundle in those states he is not able to distinguish, which means that allocations are required to be $\mathcal{G}_i$-measurable. Therefore, the consumption set of each individual i is L_i^{REE} given by (5).

This measurability requirement on allocations is not needed in differential information economies in which all market participants have preferences represented by the maximin utility function given by (9), where for any $i \in I$, $\Pi_i = \mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. In what follows we explain why. Again, in the second period, the state of nature is only incompletely and differently observed by agents, with the usual interpretation: if s occurs, each individual i does not know which state belonging in the event $\mathcal{G}_i(s)$ has occurred. Suppose, for example, that $S = \{a, b, c\}$ and for some $i \in I$, $\mathcal{G}_i = \{\{a, b\}, \{c\}\}$ and $x_i : S \to \mathbb{R}^\ell_+$ is an allocation for i. If a is the realized state of nature, agent i receives the informational signal $\{a, b\}$, meaning that he is not able to understand which states between a and b is realized. Since, according to the maximin expected utility, $i \in I$ considers the worst possible scenario, he expects to receive the bundle $x_i(s)$ such that $u_i(s, x_i(s)) = \min\{u_i(a, x_i(a)); u_i(b, x_i(b))\}$. Therefore, he is indifferent between $x_i(a)$ and $x_i(b)$ because, whatever he will receive ex-post, he is sure to obtain something ensuring him the lowest possible bound of happiness. Moreover, if we impose allocations to be private information measurable, in the event $\{a, b\}$ agent i is obligated to ask the same bundle in states a and b meaning that $x_i(a) = x_i(b)$. But since he always considers the worst possible scenario, nothing really changes because from the maximin point of view the private information measurability makes just a meaningless restriction. For this reason, at the time of contracting, agent i asks to consume ex-post one bundle between $x_i(a)$ and $x_i(b)$, and not necessarily the same bundle in states a and b. In other words, for this reason in economies in which agents' preferences are represented by (9), allocations need not be private information measurable. Clearly, this does not hold in economies in which agents' preferences are represented by (6) and hence allocations are required to be private information measurable. A similar assumption is made in Correia-da Silva and Hervés-Beloso (2009), who allow agents to choose a plan of lists of bundles and to consume one of the bundles in the list, where agents' lists are merely non empty finite subsets of R^{ℓ}_{+}.

3 Maximin REE vs the Bayesian REE

This section defines the standard Bayesian rational expectations equilibrium (REE), followed by the maximin REE (MREE). We compare the two notions in Section 3.3 and establish further properties of the MREE in Section 3.4.

3.1 Bayesian rational expectations equilibrium (REE)

In this section, we consider a differential information economy in which all market participants have preferences represented by the Bayesian interim expected utility function given by (6).

According to the notion of rational expectations equilibrium, agents make their consumption decision taking into account not only their private information, but also the information generated by the equilibrium price. Thus, agents' preferences are represented by (6) where for any $i \in I$, $\Pi_i = \mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$.

The notion below is due to Radner (1979) and Allen (1981) (see also Einy, Moreno, and Shitovitz (2000)).

Definition 3.1 *A price p and a feasible allocation x are said to be a* **Bayesian rational expectations equilibrium** *(REE) for the economy $\mathcal{E}$ if*

(i) *for all $i \in I$, the allocation $x_i(\cdot)$ is $\mathcal{G}_i$-measurable;*

(ii) *for all $i \in I$ and for all $s \in S$, $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$;*

(iii) *for all $i \in I$ and for all $s \in S$,*

$$v_i(x_i|\mathcal{G}_i)(s) = \max_{y_i \in B_i(s,p) \cap L_i^{REE}} v_i(y_i|\mathcal{G}_i)(s),$$

where $B_i(s,p)$ was defined by (10) and L_i^{REE} was defined by (5).

Note that the maximization is done over a budget set that is more restricted than $B_i(s,p)$, because we require that the acts are $\mathcal{G}_i$-measurable. This definition does not seem to be exactly the one given by Radner (1968), who requires that the sum of prices are not exceeded, that is[16]

$$B_i^*(s,p) = \left\{ y_i \in L_i \; : \; \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot y_i(s') \leq \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot e_i(s') \right\}. \tag{11}$$

This difference is only superficial, because the above definition is equivalent to Radner (1968)'s, as the following lemma establishes.

Lemma 3.1 *Let assumption 2.1 hold. Given $i \in I$ and $s \in S$, the following conditions are equivalent:*

(i) $y_i \in B_i(s,p) \cap L_i^{REE}$;

(ii) $y_i \in B_i^*(s,p) \cap L_i^{REE}$;

(iii) y_i *is* $\mathcal{G}_i$*-measurable and* $p(s) \cdot y_i(s) \leq p(s) \cdot e_i(s)$.

Proof: See Appendix.

The Bayesian REE is an interim concept since agents maximize conditional expected utility based on their own private information and also on the information that equilibrium prices have generated. The resulting allocation clears the market for every state of nature.

It is by now well known that a Bayesian rational expectations equilibrium (REE), as introduced in Allen (1981), may not exist. It only exists in a generic sense and not universally. Moreover, it fails to be fully Pareto optimal and incentive compatible and it is not implementable as a perfect Bayesian equilibrium of an extensive form game Glycopantis, Muir, and Yannelis (2005). This is not the case for the MREE as we will see in the sequel.

3.2 Maximin REE

In this section, we consider a differential information economy in which all market participants have preferences represented by the maximin utility function given by (9), where for any $i \in I$, $\Pi_i = \mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Again, in the second period, the state of nature is only incompletely and differently observed by agents, with the usual interpretation: if s occurs, each individual i does not know which state belonging in the event $\mathcal{G}_i(s)$ has occurred. According to the maximin expected utility, any individual $i \in I$ considers the worst possible scenario, that is the lowest possible bound of happiness. Thus, he does not ask to consume the same bundle in those states he is not able to distinguish, but to consume the bundle in the event $\mathcal{G}_i(s)$ that maximizes his lowest bound of happiness (i.e., his maximin expected utility). Formally, if $s \in S$ is realized,

[16] Actually in Radner (1968) the budget set condition of agent i is given by

$$\sum_{s' \in S} p(s') \cdot y_i(s') \leq \sum_{s' \in S} p(s') \cdot e_i(s'),$$

which can be viewd as a particular case of (11) once $\mathcal{G}_i(s) = S$ for any $s \in S$.

each agent $i \in I$ maximizes $\underline{u}_i^{REE}(s, x_i)$ subject[17] to $p(s') \cdot x_i(s') \leq p(s') \cdot e_i(s')$ for all $s' \in \mathcal{G}_i(s)$ (which implies that $\sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot x_i(s') \leq \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot e_i(s')$). We are now able to define the notion of a maximin rational expectations equilibrium (MREE).

Definition 3.2 *A price p and a feasible allocation x are said to be a* **maximin rational expectations equilibrium** *(MREE) for the economy $\mathcal{E}$ if:*

(i) *for all $i \in I$ and for all $s \in S$, $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$;*

(ii) *for all $i \in I$ and for all $s \in S$, $\underline{u}_i^{REE}(s, x_i) = \max_{y_i \in B_i(s,p)} \underline{u}_i^{REE}(s, y_i)$.*

Condition (ii) indicates that each individual maximizes his maximin utility conditioned on his private information and the information the equilibrium prices have generated, subject to the budget constraint.

Either a Bayesian REE or a MREE are said to be (i) *fully revealing* if the equilibrium price reveals to each agent all states of nature, i.e., $\sigma(p) = \mathcal{F}$; (ii) *non revealing* if the equilibrium price reveals nothing, that is $\mathcal{G}_i = \mathcal{F}_i$ for all $i \in I$ or, equivalently, if $\sigma(p) \subseteq \bigwedge_{i \in I} \mathcal{F}_i$; finally (iii) *partially revealing* if the equilibrium price reveals some but not all states of nature, i.e., $\bigwedge_{i \in I} \mathcal{F}_i \subset \sigma(p) \subset \mathcal{F}$.
Provided that $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for all $i \in I$, if (x, p) is a maximin REE, then for all $i \in I$ and all $s \in S$, $x_i \in B_i^*(s, p)$ and $\underline{u}_i^{REE}(s, x_i) = \max_{y_i \in B_i^*(s,p)} \underline{u}_i^{REE}(s, y_i)$, where $B_i^*(s, p)$ is defined as in (11). The converse is also true if in addition $u_i(s, \cdot)$ is strictly quasi-concave. Such an equivalence holds true even if the equilibrium price p is fully revevaling because $B_i(p, s) = B_i^*(p, s)$ for any $i \in I$ and $s \in S$.

3.3 Relationship between the Bayesian REE and the maximin REE

We denote by $REE(\mathcal{E})$ and $MREE(\mathcal{E})$ respectively the set of Bayesian rational expectations equilibrium allocations and the set of maximin rational expectations equilibrium allocations of the economy $\mathcal{E}$.

We first notice that whenever the equilibrium price p is fully revealing, i.e., $\sigma(p) = \mathcal{F}$, since $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$, it follows that $\mathcal{G}_i = \mathcal{F}$ for each agent $i \in I$. Thus, for each state $s \in S$ and each agent $i \in I$, $\mathcal{G}_i(s) = \{s\}$, and hence $v_i(x_i|\mathcal{G}_i)(s) = u_i(s, x_i(s))$ as well as $\underline{u}_i^{REE}(s, x_i) = u_i(s, x_i(s))$. Moreover, the $\mathcal{G}_i$-measurability assumption on Bayesian REE allocations plays no role, i.e., $L = L^{REE}$. Therefore, fully revealing Bayesian REE and fully revealing maximin REE coincide, i.e., $REE_{FR}(\mathcal{E}) = MREE_{FR}(\mathcal{E})$[18]. Such an equivalence is not true in general as shown below in Kreps' example (see Kreps (1977)). More is true: if (x, p) is a fully revealing maximin REE,

[17] Notice that if Assumption 2.1 holds, since $p(\cdot)$ is $\mathcal{G}_i$-measurable for any $i \in I$, it follows that for all $s \in S$ and any allocation $x_i \in L_i$

$$p(s') \cdot x_i(s') \leq p(s') \cdot e_i(s') \text{ for all } s' \in \mathcal{G}_i(s) \Leftrightarrow \max_{s' \in \mathcal{G}_i(s)} p(s') \cdot x_i(s') \leq \max_{s' \in \mathcal{G}_i(s)} p(s') \cdot e_i(s').$$

This means that any agent pays the highest value to achieve the lowest possible bound of his satisfaction.

[18] The subscript "FR" means that we are considering only fully revealing equilibria. Thus, $REE_{FR}(\mathcal{E})$ and $MREE_{FR}(\mathcal{E})$ are respectively the set of fully revealing Bayesian REE and fully revealing maximin REE of the economy $\mathcal{E}$. In the fully revealing case, $B_i(s, p) = B_i^*(s, p)$ for any i, p and s.

then (x, p) is an ex-post Walrasian equilibrium (see Section 4). Therefore, fully revealing Bayesian REE and fully revealing maximin REE coincide and they are both ex-post Walrasian equilibria. The converse is not true as shown in the Kreps's example below where an ex-post Walrasian equilibrium exists and coincides with the unique non-revealing maximin REE, but the set of fully revealing maximin REE, as well as of Bayesian REE, is empty. However, the set of non-revealing MREE is non empty. This is consistent with Lemma 8.1 in the Appendix.

Consider an economy with two states, two agents and two goods. Endowments are identical and positive. Preferences are state-dependent and such that in state one (two), the agent type one (two) prefers good one relatively more. In a differential information economy in which the preferences of all agents are represented by Bayesian expected utility function (see (6)), since the setup is symmetric, the full information equilibrium price is the same in both states.

Now suppose that agent one can distinguish the states but agent two cannot. There cannot be a fully revealing Bayesian REE: it would have to coincide with the full information equilibrium, and that equilibrium has a constant price across states, which is not compatible with revelation. Also, there cannot be a non revealing equilibrium. In a non revealing equilibrium with equal prices across states, demand of the uninformed agent would have to be the same across states. But demand of the informed agent would be different across states, and therefore there will not be market clearing. Note that a key reason for the nonexistence of a non revealing equilibrium is that the demand of the uninformed agent is measurable with respect to his private information.

On the other hand, if we impose maximin evaluation of plans, then we can have a non revealing equilibrium. In such an equilibrium, the uninformed agent two puts probability one on the worse of the two states, and zero on the better one. Thus he is indifferent between any two consumption bundles in the better state - his optimal demand is a correspondence. Therefore, we can select an element from the correspondence to clear the market. Note that the allocation is then typically not measurable with respect to the uninformed agent's information and this overcomes the non-existence problem.

Below, we explicitly consider again Kreps' example and show that while the Bayesian REE does not exist, a maximin rational expectations equilibrium does exist. From this we can conclude that the sets of MREE and REE may not coincide. This form of the example can be found in (Mas-Colell, Whinston, and Green, 1995, p. 722, Example 19.H.3).

Example 3.3 (Kreps[19]) There are two agents, two commodities and two equally probable states of nature $S = \{s_1, s_2\}$. The primitives of the economy are:

$$\begin{aligned} e_1 &= \left(\left(\frac{3}{2}, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{3}{2}\right)\right) \quad \mathcal{F}_1 = \{\{s_1\}, \{s_2\}\}; \\ e_2 &= \left(\left(\frac{3}{2}, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{3}{2}\right)\right) \quad \mathcal{F}_2 = \{\{s_1, s_2\}\}. \end{aligned}$$

[19] We are grateful to T. Liu and L. Sun for having checked the computations in Example 3.3.

The utility functions of agents 1 and 2 in states s_1 and s_2 are given as follows

$$\begin{aligned} u_1(s_1, x_1, y_1) &= \log x_1 + y_1 \qquad u_1(s_2, x_1, y_1) = 2\log x_1 + y_1 \\ u_2(s_1, x_2, y_2) &= 2\log x_2 + y_2 \qquad u_2(s_2, x_2, y_2) = \log x_2 + y_2. \end{aligned}$$

It is well known that for the above economy, a Bayesian rational expectations equilibrium does not exist (see Kreps (1977)). However we will show below that a maximin rational expectations equilibrium does exist.

The information generated by the equilibrium price can be either $\{\{s_1\}, \{s_2\}\}$ or $\{\{s_1, s_2\}\}$. In the first case, the MREE coincides with the Bayesian REE, therefore it does not exist. Thus, let us consider the case $\sigma(p) = \{\emptyset, S\}$, i.e., $p^1(s_1) = p^1(s_2) = p$ and $p^2(s_1) = p^2(s_2) = q$.

Since for each s, $\mathcal{G}_1(s) = \{s\}$, agent one solves the following constraint maximization problems:

Agent 1 in state s_1:

$$\max_{x_1(s_1), y_1(s_1)} \log x_1(s_1) + y_1(s_1) \quad \text{subject to}$$

$$px_1(s_1) + qy_1(s_1) \leq \frac{3}{2}(p+q).$$

Thus,

$$x_1(s_1) = \frac{q}{p} \quad y_1(s_1) = \frac{3}{2}\frac{p}{q} + \frac{1}{2}.$$

Agent 1 in state s_2:

$$\max_{x_1(s_2), y_1(s_2)} 2\log x_1(s_2) + y_1(s_2) \quad \text{subject to}$$

$$px_1(s_2) + qy_1(s_2) \leq \frac{3}{2}(p+q).$$

Thus,

$$x_1(s_2) = \frac{2q}{p} \quad y_1(s_2) = \frac{3}{2}\frac{p}{q} - \frac{1}{2}.$$

Agent 2 in the event $\{s_1, s_2\}$ maximizes

$$\min\{2logx_2(s_1) + y_2(s_1); logx_2(s_2) + y_2(s_2)\}.$$

Therefore, we can distinguish three cases:

I Case: $2logx_2(s_1) + y_2(s_1) > logx_2(s_2) + y_2(s_2)$. In this case, agent 2 solves the following constraint maximization problem:

$\max\ logx_2(s_2) + y_2(s_2)$ subject to $px_2(s_1) + qy_2(s_1) \leq \frac{3}{2}(p+q)$ and $px_2(s_2) + qy_2(s_2) \leq \frac{3}{2}(p+q)$. Thus,

$$x_2(s_2) = \frac{q}{p} \quad y_2(s_2) = \frac{3}{2}\frac{p}{q} + \frac{1}{2}.$$

From feasibility it follows that $p = q$, and

$$(x_1(s_1), y_1(s_1)) = (1, 2) \quad (x_1(s_2), y_1(s_2)) = (2, 1)$$

$$(x_2(s_1), y_2(s_1)) = (2,1) \quad (x_2(s_2), y_2(s_2)) = (1,2).$$

Notice that $2logx_2(s_1) + y_2(s_1) = 2log2 + 1 > log1 + 2 = logx_2(s_2) + y_2(s_2)$.

II Case: $2logx_2(s_1) + y_2(s_1) < logx_2(s_2) + y_2(s_2)$. In this case, agent 2 solves the following constraint maximization problem:

$\max\ 2logx_2(s_1)+y_2(s_1)$ subject to $px_2(s_1)+qy_2(s_1) \leq \frac{3}{2}(p+q)$ and $px_2(s_2)+qy_2(s_2) \leq \frac{3}{2}(p+q)$ Thus,

$$x_2(s_1) = \frac{2q}{p} \quad y_2(s_1) = \frac{3}{2}\frac{p}{q} - \frac{1}{2}.$$

From feasibility it follows that $p = q$, and

$$(x_1(s_1), y_1(s_1)) = (1,2) \quad (x_1(s_2), y_1(s_2)) = (2,1)$$

$$(x_2(s_1), y_2(s_1)) = (2,1) \quad (x_2(s_2), y_2(s_2)) = (1,2).$$

Clearly, as noticed above, $2log2 + 1 > log1 + 2$. Therefore, in the second case there is no maximin rational expectations equilibrium.

III Case: $2logx_2(s_1) + y_2(s_1) = logx_2(s_2) + y_2(s_2)$. In this case, agent 2 solves one of the following two constraint maximization problems:

$\max\ logx_2(s_2)+y_2(s_2)$ or $\max\ 2logx_2(s_1)+y_2(s_1)$ subject to $px_2(s_1)+qy_2(s_1) \leq \frac{3}{2}(p+q)$ and $px_2(s_2) + qy_2(s_2) \leq \frac{3}{2}(p+q)$. In both cases, from feasibility it follows that $p = q$, and

$$(x_1(s_1), y_1(s_1)) = (1,2) \quad (x_1(s_2), y_1(s_2)) = (2,1)$$

$$(x_2(s_1), y_2(s_1)) = (2,1) \quad (x_2(s_2), y_2(s_2)) = (1,2).$$

Hence, since $2logx_2(s_1) + y_2(s_1) = 2log2+1 > log1+2 = logx_2(s_2)+y_2(s_2)$, there is no maximin rational expectations equilibrium in the third case.

Therefore, we can conclude that the *unique* maximin REE allocation is given by

$$(x_1(s_1), y_1(s_1)) = (1,2) \quad (x_1(s_2), y_1(s_2)) = (2,1)$$

$$(x_2(s_1), y_2(s_1)) = (2,1) \quad (x_2(s_2), y_2(s_2)) = (1,2).$$

Observe that the maximin REE bundles are not $\mathcal{F}_i$-measurable. We will show in Section 4 that the non-existence problem of the REE is deeply linked with the $\mathcal{G}_i$-measurability of allocations.

Remark 3.2 It should be noted that in the above example, whenever agents maximize a Bayesian (subjective) expected utility as Kreps showed, the Bayesian REE either revealing or non revealing does not exist. However, allowing agents to maximize a non expected utility (i.e., the maximin utility), and choose a non measurable allocations, we showed that a maximin rational expectations equilibrium exists. For this result, the assumption that a MREE allocation may not be private information measurable is crucial. Indeed, in the above example, the unique MREE allocation is not $\mathcal{G}_i$-measurable. This outcome is also incentive compatible and efficient (see Sections 5 and 6).

Remark 3.3 As we have already observed, the maximin rational expectations equilibrium allocations may not be $\mathcal{G}_i$-measurable. However, if we assume strict quasi-concavity and $\mathcal{F}_i$-measurability of the random utility function of each agent, then the resulting maximin REE allocations will be $\mathcal{G}_i$-measurable, as the following proposition indicates.

Proposition 3.4 *Assume that for all i and for all $s \in S$, $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ and $u_i(s, \cdot)$ is strictly quasi concave. If (p, x) be a maximin REE, then $x_i(\cdot)$ is $\mathcal{G}_i$-measurable for all $i \in I$, where $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$.*

Proof: See Appendix.

A similar proposition can be proved for the Bayesian rational expectations equilibrium, that is whenever the utility functions are private information measurable and strictly quasi concave, from the uniqueness of the maximizer, we obtain that the equilibrium allocations must be private information measurable. In other words, if the utility functions are private information measurable and strictly quasi-concave, condition (i) in Definition 3.1 is automatically satisfied. Moreover, the same holds true if in Definition 3.2 the budget set is defined by (11) as well as with the general MEU formulation (8) provided that for any agent i and state s the set $\mathcal{M}_i^s$ contains only positive priors (i.e., $\mu(s') > 0$ for any $s' \in \Pi_i(s)$ and $\sum_{s' \in \Pi_i(s)} \mu(s') = 1$). See section 8.5 in the appendix for more details.

It was shown in Example 3.3 that the maximin and the Bayesian REE are not comparable. We have already observed that in the special case of fully revealing equilibrium prices, both concepts coincide. We show below that the same holds whenever $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for all $i \in I$. Note that in Example 3.3, utility functions are not $\mathcal{F}_i$-measurable and therefore Example 3.3 does not fulfill the assumptions of Proposition 3.5 below.

Proposition 3.5 *Assume that $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for all $i \in I$. If (p, x) is a Bayesian REE, then (p, x) is a MREE. The converse is also true if $x_i(\cdot)$ is $\mathcal{G}_i$-measurable for all $i \in I$.*

Proof: See Appendix.

Remark 3.4 The above proposition holds with the general MEU formulation (8) and it remains true if we replace the $\mathcal{G}_i$-measurability of the allocations by the strict quasi concavity of the random utility functions. This follows by combining Propositions 3.4 and 3.5.

3.4 Properties of a maximin rational expectations equilibrium

In this section we investigate some basic properties of a maximin rational expectations equilibrium.

The first property of a MREE regards the equilibrium price p. We show that under certain assumptions the equilibrium price is strictly positive in each state of nature, i.e., $p(s) \gg 0$ for all $s \in S$.

Remark 3.5 Recall that in a complete information economy, if the utility function of at least one agent is strictly monotone, the equilibrium price is strictly positive. We prove the same for the MREE prices. Notice that, typically in differential information economies an additional assumption is needed: for each state $s \in S$, there exists an agent $i \in I$ such that $\{s\} \in \mathcal{F}_i$. It implies that $\bigvee_{i \in I} \mathcal{F}_i = \mathcal{F} = 2^S$ which is used in Allen (1981) and Einy, Moreno, and Shitovitz (2000). The converse is not true: in particular in a differential information economy with three states of nature $S = \{a, b, c\}$ and two agents $I = \{1, 2\}$, with $\mathcal{F}_1 = \{\{a, b\}, \{c\}\}$ and $\mathcal{F}_2 = \{\{a, c\}, \{b\}\}$, it is true that $\mathcal{F}_1 \vee \mathcal{F}_2 = \{\{a\}, \{b\}, \{c\}\}$, but $\{a\} \notin \mathcal{F}_i$ for any $i \in \{1, 2\}$. Although this assumption is quite common in the literature (see for example Angeloni and Martins-da Rocha (2009) and Correia-da Silva and Hervés-Beloso (2012)), we can prove that MREE prices are strictly positive by dispensing with it.

Proposition 3.6 *Assume that there is at least one agent $i \in I$ such that $u_i(s, \cdot)$ is strictly monotone for any $s \in S$. If (p, x) is a maximin rational expectations equilibrium, then $p(s) \gg 0$ for any $s \in S$.*

Proof: See Appendix.

The above results holds true even with the general MEU formulation (8). We now show a second property of a MREE: if the utility functions are private information measurable, then for each agent $i \in I$, the maximin utility at any MREE allocation is constant in each event of the partition $\mathcal{G}_i$.

Proposition 3.7 *Assume that $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for all $i \in I$. If (p, x) is a maximin rational expectations equilibrium, then for all i and s, $\underline{u}_i^{REE}(s, x_i) = u_i(s', x_i(s'))$ for all $s' \in \mathcal{G}_i(s)$, that is the minimum in the event $\mathcal{G}_i(s)$ is obtained in each state s' of the event.*

Proof: See Appendix.

Notice that if (p, x) is a fully revealing Maximin REE, Proposition 3.7 is trivially satisfied even if the utility functions and the initial endowments are not private information measurable. Moreover Proposition 3.7 holds true even with the general MEU formulation (8) provided that for any agent i and state s, the set $\mathcal{M}_i^s$ contains only positive priors. See section 8.5 in the appendix for more details.

4 Existence of a maximin rational expectations equilibrium

In this section, we prove the existence of a maximin rational expectations equilibrium. It should be noted that under the assumptions, which guarantee that a maximin rational expectations equilibrium exists, the Bayesian REE need not exist. In studies of rational expectations equilibria, it is common to appeal to an artificial family of complete information economies (see e.g., Radner (1979); Allen (1981); Einy, Moreno, and Shitovitz (2000), De Simone and Tarantino (2010)). Given a differential information economy $\mathcal{E}$ described in Section 2, since S is finite, there is a finite number of complete information economies $\{\mathcal{E}(s)\}_{s\in S}$. For each fixed s in S, the complete information economy $\mathcal{E}(s)$ is given as follows:

$$\mathcal{E}(s) = \left\{I, \mathbb{R}^{\ell}_{+}, (u_i(s), e_i(s))_{i\in I}\right\},$$

where $I = \{1, \ldots, n\}$ is still the set of n agents, and for each $i \in I$, $u_i(s) = u_i(s, \cdot) : \mathbb{R}^{\ell}_{+} \to \mathbb{R}$ and $e_i(s) \in \mathbb{R}^{\ell}_{+}$ represent respectively the utility function and the initial endowment of agent i. Let $W(\mathcal{E}(s))$ be the set of *Walrasian equilibrium allocations* of $\mathcal{E}(s)$.

We prove that the set of maximin REE allocations contains all the selections from the Walrasian equilibrium correspondence of the associated family of complete information economies. From the existence of a Walrasian equilibrium in each complete information economy $\mathcal{E}(s)$, we deduce the existence of a maximin REE. A related result has been shown by Einy, Moreno, and Shitovitz (2000) and De Simone and Tarantino (2010) but under the additional private information measurability assumption on the utility functions and the initial endowments (see also Theorem 4.3).

Theorem 4.1 (Existence) *If for any $i \in I$ and $s \in S$ the function $u_i(s, \cdot)$ is quasi-concave and $e_i(s) \gg 0$, then there exists a maximin rational expectations equilibrium in $\mathcal{E}$, i.e., $MREE(\mathcal{E}) \neq \emptyset$.*

Proof: See Appendix.

Remark 4.1 The sum of quasi-concave functions is not quasi-concave and thus in the Bayesian model one has to assume concavity. However the maximin utility is quasi-concave if the ex post utility is quasi-concave. Thus, the MEU allows us to use the more general assumption of quasi-concavity instead of concavity. In order to prove the existence of a maximin rational expectations equilibrium, we show that it contains the nonempty set of ex post Walrasian equilibria (see Lemma 8.1 in the Appendix). In the example below, we show that such an inclusion is strict, that is, there may exist a maximin rational expectations equilibrium which is not a Walrasian equilibrium in some complete information economy $\mathcal{E}(s)$.

Example 4.2 Consider a differential information economy with three states of nature, $S = \{a, b, c\}$, two goods, $\ell = 2$ (the first good is considered as numerarie) and two agents, $I = \{1, 2\}$ whose characteristics are given as follows:

$$\begin{array}{cccc} u_i(a,x,y) = \sqrt{xy} & u_i(b,x,y) = \sqrt{xy} & u_i(c,x,y) = log(xy) & \text{for all } i = 1,2 \\ e_1(a) = e_1(b) = (2,1) & e_1(c) = (1,2) & e_2(a) = e_2(c) = (1,2) & e_2(b) = (2,1) \\ \mathcal{F}_1 = \{\{a,b\};\{c\}\} & & \mathcal{F}_2 = \{\{a,c\};\{b\}\}. & \end{array}$$

Notice that the initial endowment is private information measurable, while the utility functions are not. Hence this example does not contradict Lemma 8.2 in the Appendix.

The set W of ex post Walrasian equilibrium has only one element, i.e.,

$$\begin{array}{lll} (p(a),q(a)) = (1,1) & (x_1(a),y_1(a)) = \left(\frac{3}{2},\frac{3}{2}\right) & (x_2(a),y_2(a)) = \left(\frac{3}{2},\frac{3}{2}\right) \\ (p(b),q(b)) = (1,2) & (x_1(b),y_1(b)) = (2,1) & (x_2(b),y_2(b)) = (2,1) \\ (p(c),q(c)) = \left(1,\frac{1}{2}\right) & (x_1(c),y_1(c)) = (1,2) & (x_2(c),y_2(c)) = (1,2)\,. \end{array}$$

Clearly, this equilibrium is also a fully revealing maximin rational equilibrium, since $(p(a),q(a)) \neq (p(b),q(b)) \neq (p(c),q(c))$ and hence $\mathcal{G}_i = \sigma(p,q) = \{\{a\},\{b\},\{c\}\}$ for any $i = 1,2$. However, it is not unique. Indeed, the set $MREE(\mathcal{E})$ contains the following further element:

$$\begin{array}{lll} (p(a),q(a)) = \left(1,\frac{1}{2}\right) & (x_1(a),y_1(a)) = \left(\frac{5}{4},\frac{5}{2}\right) & (x_2(a),y_2(a)) = \left(\frac{7}{4},\frac{1}{2}\right) \\ (p(b),q(b)) = (1,2) & (x_1(b),y_1(b)) = (2,1) & (x_2(b),y_2(b)) = (2,1) \\ (p(c),q(c)) = \left(1,\frac{1}{2}\right) & (x_1(c),y_1(c)) = (1,2) & (x_2(c),y_2(c)) = (1,2)\,. \end{array}$$

This is a partially revealing equilibrium, since $(p(a),q(a)) = (p(c),q(c)) \neq (p(b),q(b))$ and hence $\sigma(p,q) = \{\{a,c\},\{b\}\}$, that is $\mathcal{G}_1 = \{\{a\},\{b\},\{c\}\}$, while $\mathcal{G}_2 = \mathcal{F}_2$. Notice that the equilibrium allocations are not $\mathcal{G}_i$-measurable.

Remark 4.2 If for any $i \in I$ and $s \in S$ $e_i(s) \gg 0$, $\sigma(u_i,e_i) \subseteq \mathcal{F}_i$, and the function $u_i(s,\cdot)$ is strictly quasi-concave, then from Remark 3.4 and Theorem 4.1 it follows that there exists a Bayesian REE in $\mathcal{E}$.

Remark 4.3 Notice that in Example 3.3, where the Bayesian REE does not exist, not all the above assumptions of Remark 4.2 are satisfied. In particular, the random utility functions are not $\mathcal{F}_i$-measurable. Hence, the Kreps's example of the nonexistence of a Bayesian REE does not contradict Remark 4.2.

Theorem 4.3 *Assume that for any $i \in I$ and $s \in S$ the function $u_i(s,\cdot)$ is strictly quasi-concave and $\sigma(u_i,e_i) \subseteq \mathcal{F}_i$. Let x be a feasible allocation. The following statements are equivalent:*

(1) x is a maximin REE[20]*. ;*

(2) x is a Bayesian REE;

(3) x is an ex-post Walrasian equilibrium allocation.

Proof: See Appendix.

Remark 4.4 Observe that the non-existence problem of a Bayesian REE is deeply linked to the private information measurability of the allocations. Moreover, if we consider a Bayesian REE (p,x) but removing from Definition 3.1 the private information

[20] We can consider also the general MEU formulation (8) provided that for all agent i and state s, the set $\mathcal{M}_i^s$ contains only positive priors (see section 8.5.)

measurability of allocations (i.e., condition (i)) and also consider in the optimization problem the budget set defined as (10), then we end up with the following notion which coincides in the Kreps's example 3.3 with the maximin REE:

1. for all i and for all s, $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$;
2. for all i and for all s, $v_i(x_i|\mathcal{G}_i)(s) = \max_{y_i \in B_i(s,p)} v_i(y_i|\mathcal{G}_i)(s)$; where

$$B_i(s,p) = \{y_i \in L_i \ : \ p(s') \cdot y_i(s') \leq p(s') \cdot e_i(s') \text{ for all } s' \in \mathcal{G}_i(s)\}.$$

3. $\sum_{i \in I} x_i(s) = \sum_{i \in I} e_i(s)$ for all $s \in S$.

It is easy to show that the above REE notion coincides with the ex post Walrasian equilibrium, and therefore it exists under suitable assumptions. However, the above notion does not provide any new insights, since it is "equivalent" with the ex post Walrasian equilibrium. Moreover, from Theorem 4.1 and Example 4.2, one can easily deduce that any Bayesian REE allocation not private information measurable is a maximin REE but the reverse is not true. Furthermore, we show in the next section, that whenever we drop the private information measurability constraint, the REE exists, but it may not be incentive compatible (see Glycopantis, Muir, and Yannelis (2005)). *This conflict does not arise anymore with the maximin utility functions. In fact, a maximin rational expectations equilibrium exists and it is incentive compatible.*

4.1 General Case (Existence)

In this section we show that the non-existence problem of a REE is deeply linked to the private information measurability of the allocations. To this end, assume that the preferences of each individual i, with private information Π_i in state s are represented by $V_i(\cdot, \Pi_i, s)$ (see (7)).

Definition 4.4 *A price p and a feasible allocation x are said to be a* **V-rational expectations equilibrium** *(V-REE) for the economy $\mathcal{E}$ if:*

(i) *for all $i \in I$ and for all $s \in S$, $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$;*

(ii) *for all $i \in I$ and for all $s \in S$, $V_i(x_i, \mathcal{G}_i, s) = \max_{y_i \in B_i(s,p)} V_i(y_i, \mathcal{G}_i, s)$; where for any $i \in I$, $\mathcal{G}_i = \sigma(p) \vee \mathcal{F}_i$.*

Theorem 4.5 *If for any $i \in I$ and $s \in S$ the function $u_i(s, \cdot)$ is quasi-concave and $e_i(s) \gg 0$, then there exists a V-rational expectations equilibrium in $\mathcal{E}$, i.e., $V - REE(\mathcal{E}) \neq \emptyset$.*

Proof: See Appendix.

Remark 4.5 Notice that the utility

$$\underline{u}_i^{\Pi_i}(s, x_i) = \min_{\mu \in \mathcal{M}_i^s} E_\mu[u_i(\cdot, x_i(\cdot))], \tag{12}$$

where $\mathcal{M}_i^s$ is a non-empty, closed and convex set of probabilities with support contained on $\Pi_i(s)$, is a particular case of $V_i(\cdot, \Pi_i, s)$. Clearly if $\mathcal{M}_i^s$ is a singleton then (12) is the (Bayesian) expected utility function (6). On the other hand, if $\mathcal{M}_i^s$ is the set $\mathcal{C}_i^s$ of all probabilities with support contained on $\Pi_i(s)$ then (12) coincides with (9). This in particular means that a maximin REE, as well as the equilibrium defined in Remark 4.4, is a V-REE and hence Theorem 4.1 is a mere consequence of Theorem 4.5. It is worth noting that except for the fully revealing case, the Bayesian REE is not a particular case of the V-REE since according to Definition 3.1, allocations are required to be $\mathcal{G}_i$-measurable. Moreover, according to Definition 4.4 we may also consider the more realistic situation in which different agents have different attitude toward ambiguity. Consequently Theorem 4.5 guarantees the existence of the equilibrium in economies where some agents' preferences are represented by $v_i(x_i|\Pi_i)(s) = \sum_{s' \in \Pi_i(s)} u_i(s', x_i(s'))$ (i.e., (6)) some by $\underline{u_i}^{\Pi_i}(s, x_i) = \min_{s' \in \Pi_i(s)} u_i(s', x_i(s'))$ (i.e., (9)) and some other by the intermediate case (12).

We now observe in some cases, the set of V-REE coincides with the set of ex-post Walrasian equilibria. Precisely, the following holds.

Proposition 4.6 *If (x, p) is an ex-post Walrasian equilibrium, then (x, p) is a V-REE. Conversely, let (x, p) be a V-REE and $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$ for all $i \in I$. Assume that for any $i \in I$ and any $s \in S$, $V_i(\cdot, \mathcal{G}_i, s)$ is such that*

$$(\ast\ast\ast)\ u_i(s', x_i(s')) > u_i(s', y_i(s'))\ in\ some\ s' \in \mathcal{G}_i(s), \Rightarrow\ V_i(x_i, \mathcal{G}_i, s) > V_i(z_i, \mathcal{G}_i, s),$$

where $z_i(s') = y_i(s')$ and $z_i(s) = x_i(s)$ for all $s \in \mathcal{G}_i(s) \setminus \{s'\}$.
Then, (x, p) is an ex-post Walrasian equilibrium.

Proof: See Appendix.

Notice that condition $(\ast\ast\ast)$ is satisfied by the Bayesian expected utility, provided that $\pi_i(s) > 0$ for any agent i and any state s, while the maximin expected utility may violate it. This is consistent with Remark 4.4 and Example 4.2. However, according to Theorem 4.3, whenever agents' initial endowment and utility are private information measurable, the maximin REE also coincides with the ex-post Walrasian equilibria.

5 Efficiency of the maximin REE

We now define the notion of maximin and ex post Pareto optimality and we will exhibit conditions which guarantee that any maximin REE is maximin efficient and ex post Pareto optimal. The results illustrated in this section also holds for the general MEU formulation.[21].

[21]Notice that to prove the statements of Theorems 5.4 and 5.8 under the first condition (i.e., $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for all $i \in I$), for any agent i and state s, the set $\mathcal{M}_i^s$ must contain only positive priors (see section 8.5)

Definition 5.1 *A feasible allocation x is said to be ex post efficient (or ex post Pareto optimal) if there does not exist an allocation $y \in L$ such that*

$$\begin{aligned}
(i) \quad & u_i(s, y_i(s)) \geq u_i(s, x_i(s)) \quad for\ all\ i \in I\ and\ for\ all\ s \in S, \\
& with\ at\ least\ a\ strict\ inequality. \\
(ii) \quad & \sum_{i \in I} y_i(s) = \sum_{i \in I} e_i(s) \quad for\ all\ s \in S.
\end{aligned}$$

Definition 5.2 *A feasible allocation x is said to be maximin efficient (or maximin Pareto optimal) with respect to information structure*[22] Π*, if there does not exist an allocation $y \in L$ such that*

$$\begin{aligned}
(i) \quad & \underline{u}_i^{\Pi_i}(s, y_i) \geq \underline{u}_i^{\Pi_i}(s, x_i) \quad for\ all\ i \in I\ and\ for\ all\ s \in S, \\
& with\ at\ least\ a\ strict\ inequality. \\
(ii) \quad & \sum_{i \in I} y_i(s) = \sum_{i \in I} e_i(s) \quad for\ all\ s \in S.
\end{aligned}$$

Proposition 5.3 *Let Π an information structure such that*

1. *for any state s there exists an agent i such that $\Pi_i(s) = \{s\}$*[23]*;*
2. *$u_i(s, \cdot)$ is strict monotone for any $i \in I$ and any $s \in S$.*

Then any maximin efficient allocation x with respect to the information structure Π is ex post Pareto optimal. The converse may not be true.

Proof: See Appendix.

The assumption that for any state s there exists an agent i such that $\Pi_i(s) = \{s\}$ is fundamental for Proposition 5.3 as shown by Example 8.1 in the Appendix.

We are now ready to exhibit the conditions under which any MREE is maximin efficient and also ex post Pareto optimal.

Theorem 5.4 *Let (p, x) be a maximin rational expectations equilibrium. If one of the following conditions holds true:*

1. *$\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for each $i \in I$;*
2. *p is fully revealing, i.e., $\sigma(p) = \mathcal{F}$;*

[22] An information structure Π is simply a vector $(\Pi_1, \ldots, \Pi_i, \ldots, \Pi_n)$, where for each $i \in \{1, \ldots, n\} = I$, Π_i is a partition of S. If $\Pi_i = \mathcal{F}_i$ for each $i \in I$, then the information structure is the initial private information.

[23] This assumption is quite common in the literature of asymmetric information economies (see for example Angeloni and Martins-da Rocha (2009) and Correia-da Silva and Hervés-Beloso (2012)) (see remark 3.5 in section 3.4)

then x is ex-post efficient and maximin Pareto optimal with respect to the information structure $\mathcal{G} = (\mathcal{G}_i)_{i \in I}$, where $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$ for any $i \in I$.

Moreover, if none of the above conditions is satisfied, a maximin REE may not be maximin efficient.

Proof: See Appendix.

Remark 5.1 The ex-post Pareto optimality does not follows from Proposition 5.3 because we do not require that the information structure $\mathcal{G}_i$ is such that for any state s there exists an agent i such that $\mathcal{G}_i(s) = \{s\}$, neither that $u_i(s, \cdot)$ is strict monotone for any $s \in I$ and any $i \in I$.

According to the efficiency concept (Definitions 5.1 and 5.2), an improvement requires a strict utility increase for some pair $(j, \bar{s}) \in I \times S$ and no utility decreases for all $(i, s) \in I \times S$. A weaker notion defined below would require strict utility increases for all agents in all states of nature. Clearly, any maximin Pareto optimal allocation is weak maximin efficient. The converse may not be true (see Examples 8.3 and 8.4 and Remark 5.3).

Definition 5.5 *A feasible allocation x is said to be weak maximin efficient (or weak maximin Pareto optimal) with respect to information structure Π, if there does not exist an allocation $y \in L$ such that*

$$\begin{aligned} &(i) \qquad \underline{u}_i^{\Pi_i}(s, y_i) > \underline{u}_i^{\Pi_i}(s, x_i) \quad \textit{for all } i \in I \textit{ and for all } s \in S, \\ &(ii) \qquad \sum_{i \in I} y_i(s) = \sum_{i \in I} e_i(s) \quad \textit{for all } s \in S. \end{aligned}$$

Similarly the notion of weak ex-post efficiency is given as follows.

Definition 5.6 *A feasible allocation x is said to be weak ex post efficient (or weak ex post Pareto optimal) if there does not exist an allocation $y \in L$ such that*

$$\begin{aligned} &(i) \qquad u_i(s, y_i(s)) > u_i(s, x_i(s)) \quad \textit{for all } i \in I \textit{ and for all } s \in S, \\ &(ii) \qquad \sum_{i \in I} y_i(s) = \sum_{i \in I} e_i(s) \quad \textit{for all } s \in S. \end{aligned}$$

Proposition 5.7 Any weak maximin efficient allocation x (with respect to any information structure) is weak ex post Pareto optimal. The converse may not be true.

Proof: See Appendix.

Notice that, contrary to Proposition 5.3, we need no further assumptions on the information structure neither on agents' utility functions.

We now list the conditions guaranteeing that a maximin REE is weak maximin efficient and a fortiori weak ex-post Pareto optimal (see Proposition 5.7).

Theorem 5.8 *Let (p, x) be a maximin rational expectations equilibrium. If one of the following conditions holds true:*

(i) $\sigma(u_i, e_i) \subseteq F_i$ *for each* $i \in I$*;*

(ii) p *is fully revealing, i.e.,* $\sigma(p) = \mathcal{F}$*;*

(iii) there exists a state of nature $\bar{s} \in S$*, such that* $\{\bar{s}\} = \mathcal{G}_i(\bar{s})$ *for all* $i \in I$*;*

(iv) the $n-1$ *agents are fully informed.*

then x *is weak maximin Pareto optimal with respect to the information structure* $\mathcal{G} = (\mathcal{G}_i)_{i \in I}$*, where* $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$ *for any* $i \in I$*, and hence weak ex post efficient.*

Moreover if none of the above conditions is satisfied, then a maximin REE may not be weak maximin efficient (and a fortiori it may not be maximin Pareto optimal).

Proof: See Appendix.

Remark 5.2 Notice that in the first two cases (i.e., under conditions (i) or (ii)), the result easily follows from Theorem 5.4 and from the observation that any allocation maximin efficient with respect to Π is weak maximin Pareto optimal with respect to Π. On the other hand, it can be shown that under condition (iii) or (iv) a maximin REE allocation is weak maximin Pareto optimal but it may not be maximin efficient (see Examples 8.3 and 8.4 in the Appendix).

Remark 5.3 Notice that in Kreps's example (Example 3.3), one of the two agents is fully informed, hence condition (iv) of Theorem 5.8 is satisfied. This guarantees that the unique maximin rational expectations equilibrium (MREE) is weak maximin Pareto optimal and hence weak ex post efficient. On the other hand, no condition of Theorem 5.4 is verified and the unique maximin REE is not maximin efficient. Indeed consider the following feasible allocation

$$\begin{aligned}
(t_1(s_1), z_1(s_1)) &= \left(\frac{5}{4}, 2\right) \quad (t_1(s_2), z_1(s_2)) = (x_1(s_2), y_1(s_2)) = (2, 1) \\
(t_2(s_1), z_2(s_1)) &= \left(\frac{7}{4}, 1\right) \quad (t_2(s_2), z_2(s_2)) = (x_2(s_2), y_2(s_2)) = (1, 2),
\end{aligned}$$

and notice that

$$\begin{aligned}
\underline{u}_1^{REE}(s_1, t_1, z_1) &= log\frac{5}{4} + 2 > 2 = \underline{u}_1^{REE}(s_1, x_1, y_1) \\
\underline{u}_1^{REE}(s_2, t_1, z_1) &= 2log2 + 1 = \underline{u}_1^{REE}(s_2, x_1, y_1) \\
\underline{u}_2^{REE}(s_1, t_2, z_2) &= \underline{u}_2^{REE}(s_2, t_2, z_2) = \min\left\{2log\frac{7}{4} + 1; 2\right\} = 2 \\
&= \min\{2log2 + 1; 2\} = \underline{u}_2^{REE}(s_2, x_2, y_2) = \underline{u}_2^{REE}(s_1, x_2, y_2).
\end{aligned}$$

Thus, the unique maximin REE is weak maximin efficient but not maximin Pareto optimal with respect to the information structure $\mathcal{G} = (\mathcal{G}_1, \mathcal{G}_2)$ neither with $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2)$ since the equilibrium is non revealing and $\mathcal{G} = \mathcal{F}$. On the other hand, the unique non revealing maximin REE is an ex-post Walrasian equilibrium and hence it is ex-post efficient. Indeed assume on the contrary that there exists an alternative feasible allocation (t, z) such that

$$\begin{array}{ll} (i) & log(3 - t_2(s_1)) + (3 - z_2(s_1)) \geq 2 \\ (ii) & 2log(t_2(s_1)) + z_2(s_1) \geq 2log2 + 1 \\ (iii) & 2log(t_1(s_2)) + z_1(s_2) \geq 2log2 + 1 \\ (iv) & log(3 - t_1(s_2)) + (3 - z_1(s_2)) \geq 2, \end{array}$$

with at least one strict inequality. If one of (i) and (ii) is strict, then $(3-t_2(s_1))t_2^2(s_1) > 4$ or equivalently that $(t_2(s_1)+1)(t_2(s_1)-2)^2 < 0$ which is a contradiction. Similarly if one of (iii) and (iv) is strict.

Therefore, Kreps's example can also be used to show that a weak maximin efficient allocation may not be maximin Pareto optimal and an ex-post efficient allocation may not be maximin efficient. Moreover, an ex-post Walrasian equilibrium allocation, which is always ex-post efficient, may not be maximin Pareto optimal.

5.1 Further remarks on the efficiency of maximin REE

Someone could debate the fact that we have considered the algebra $\mathcal{G}_i$ and not $\mathcal{F}_i$. What is the correct definition? It seems to us that it depends on what kind of interpretation or story one has in mind. For example one may say that the notions of efficiency and incentive compatibility are independent of prices and as a consequence agents have to condition their expectations on $\mathcal{F}_i$. This view however can be challenged because at REE each agent in the interim stage behaves like having observed the equilibrium price and conditions herself on the information $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Thus the relevant information for each agent is $\mathcal{G}_i$ and not $\mathcal{F}_i$. For this reason we chose to present the definitions of efficiency and incentive compatibility considering the two different private information sets, $\mathcal{F}_i$ and $\mathcal{G}_i$.

We now investigate the efficiency of maximin REE with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_i)_{i \in I}$. We do the same for the incentive compatibility (see subsection 6.2).

Remark 5.4 Clearly, for any non revealing maximin rational expectations equilibrium the results of Section 5 still hold simply because $\mathcal{G}_i = \mathcal{F}_i$ for all $i \in I$. In particular notice that the equilibrium in Example 8.2 is non-revealing. On the other hand, any fully revealing maximin REE is maximin efficient with respect to $\mathcal{G}_i$ and also ex-post efficient (see Theorem 5.4), but it may not be maximin efficient with respect to $\mathcal{F}_i$ as the following example shows.

Example 5.9 Consider a differential information economy with two states of nature, $S = \{a, b\}$, two goods, $\ell = 2$ (the first good is considered as numerarie) and three agents, $I = \{1, 2, 3\}$ whose characteristics are given as follows:

$$\begin{array}{lll} e_1(a) = (2,1) & e_1(b) = (1,2) & \mathcal{F}_1 = \{\{a\};\{b\}\} \\ e_2(a) = (1,2) & e_2(b) = (1,2) & \mathcal{F}_2 = \{\{a,b\}\} \\ e_3(a) = (2,1) & e_3(b) = (2,1) & \mathcal{F}_3 = \{\{a,b\}\}. \end{array}$$
$$u_i(a,x,y) = \sqrt{xy} \quad u_i(b,x,y) = x^2 y \quad \text{for any } i \in \{1,2\} \quad u_3(a,x,y) = u_3(b,x,y) = logxy.$$

Consider the following fully revealing maximin rational expectations equilibrium

$$\begin{array}{llll} (p(a), q(a)) = \left(1, \frac{5}{4}\right) & (x_1(a), y_1(a)) = \left(\frac{13}{8}, \frac{13}{10}\right) & (x_2(a), y_2(a)) = \left(\frac{7}{4}, \frac{7}{5}\right) & (x_3(a), y_3(a)) = \left(\frac{13}{8}, \frac{13}{10}\right) \\ (p(b), q(b)) = \left(1, \frac{10}{19}\right) & (x_1(b), y_1(b)) = \left(\frac{26}{19}, \frac{13}{10}\right) & (x_2(b), y_2(b)) = \left(\frac{26}{19}, \frac{13}{10}\right) & (x_3(b), y_3(b)) = \left(\frac{24}{19}, \frac{12}{5}\right). \end{array}$$

The above fully revaling maximin REE is of course ex post efficient since it coincides with an ex post Walrasian equilibrium. On the other hand, we now show that it is not maximin efficient with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_i)_{i \in I}$. To this end, consider the following feasible allocation (t, z)

$$\begin{array}{rcl} (t_i(a), z_i(a)) & = & (x_i(a), y_i(a)) \quad \text{for all } i \in I, \\ (t_1(b), z_1(b)) & = & \left(\frac{31}{19}, \frac{7}{5}\right) \\ (t_2(b), z_2(b)) & = & \left(\frac{25}{19}, \frac{6}{5}\right) \\ (t_3(b), z_3(b)) & = & \left(\frac{20}{19}, \frac{12}{5}\right), \end{array}$$

and notice that,

$$\begin{array}{rcl} \underline{u}_1^{\mathcal{F}_1}(a, t_1, z_1) & = & u_1(a, t_1(a), z_1(a)) = u_1(a, x_1(a), y_1(a)) = \underline{u}_1^{\mathcal{F}_1}(a, x_1, y_1) \\ \underline{u}_1^{\mathcal{F}_1}(b, t_1, z_1) & = & u_1(b, t_1(b), z_1(b)) = \left(\frac{31}{19}\right)^2 \frac{7}{5} > \left(\frac{26}{19}\right)^2 \frac{13}{10} = u_1(b, x_1(b), y_1(b)) = \underline{u}_1^{\mathcal{F}_1}(b, x_1, y_1) \\ \underline{u}_2^{\mathcal{F}_2}(a, t_2, z_2) & = & \underline{u}_2^{\mathcal{F}_2}(b, t_2, z_2) = \min\left\{\sqrt{\frac{49}{20}}, \left(\frac{25}{19}\right)^2 \frac{6}{5}\right\} = \sqrt{\frac{49}{20}} \\ & = & \min\left\{\sqrt{\frac{49}{20}}, \left(\frac{26}{19}\right)^2 \frac{13}{10}\right\} = \underline{u}_2^{\mathcal{F}_2}(b, x_2, y_2) = \underline{u}_2^{\mathcal{F}_2}(a, x_2, y_2) \\ \underline{u}_3^{\mathcal{F}_3}(a, t_3, z_3) & = & \underline{u}_3^{\mathcal{F}_3}(b, t_3, z_3) = \min\left\{log\frac{169}{80}, log\frac{240}{95}\right\} = log\frac{169}{80} \\ & = & \min\left\{log\frac{169}{80}, log\frac{288}{95}\right\} = \underline{u}_3^{\mathcal{F}_3}(b, x_3, y_3) = \underline{u}_3^{\mathcal{F}_3}(a, x_3, y_3). \end{array}$$

Hence, the equilibrium allocation (x, y) is not maximin Pareto optimal with respect to the information structure $\mathcal{F} = (\mathcal{F}_i)_{i \in I}$.

We now list the conditions under which a maximin REE fully revealing or not is maximin efficient with respect to the initial private information structure $(\mathcal{F}_i)_{i \in I}$.

Theorem 5.10 *Let (p, x) be a maximin rational expectations equilibrium. If one of the following conditions holds true:*

(a) there exists a state of nature $\bar{s} \in S$, such that $\{\bar{s}\} = \mathcal{F}_i(\bar{s})$ for all $i \in I$;

(b) the $n - 1$ agents are fully informed.

then x is weak maximin Pareto optimal with respect to $\mathcal{F} = (\mathcal{F}_i)_{i \in I}$ and with respect to $\mathcal{G} = (\mathcal{G}_i)_{i \in I}$.

Moreover if none of the above conditions is satisfied, then a maximin REE may not be weak maximin efficient with respect to the information structure $\mathcal{F} = (\mathcal{F}_i)_{i \in I}$ and a fortiori maximin Pareto optimal.

6 Incentive compatibility of rational expectations equilibrium

We now recall the notion of coalitional incentive compatibility in Krasa and Yannelis (1994).

Definition 6.1 *An allocation x is said to be coalitional incentive compatible (CIC) with respect to the information structure $\Pi = (\Pi_i)_{i \in I}$ if the following does not hold: there exists a coalition C and two states a and b such that*

$$\begin{array}{lll} (i) & \Pi_i(a) = \Pi_i(b) & for\ all\ i \notin C, \\ (ii) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^\ell_+ & for\ all\ i \in C,\ and \\ (iii) & u_i(a, e_i(a) + x_i(b) - e_i(b)) > u_i(a, x_i(a)) & for\ all\ i \in C. \end{array}$$

In order to explain what incentive compatibility means in an asymmetric information economy, let us consider the following two examples[24].

Example 6.2 Consider an economy with two agents, three equally probable states of nature, denoted by a, b and c, and one good per state denoted by x. The primitives of the economy are given as follows:

$$\begin{array}{lll} u_1(\cdot, x_1) = \sqrt{x_1}; & e_1(a, b, c) = (20, 20, 0); & \mathcal{F}_1 = \{\{a, b\}; \{c\}\}. \\ u_2(\cdot, x_2) = \sqrt{x_2}; & e_2(a, b, c) = (20, 0, 20); & \mathcal{F}_2 = \{\{a, c\}; \{b\}\}. \end{array}$$

Consider the following risk sharing (Pareto optimal) redistribution of initial endowment:

$$\begin{array}{lll} x_1(a, b, c) & = & (20, 10, 10) \\ x_2(a, b, c) & = & (20, 10, 10). \end{array}$$

[24]The reader is also referred to Krasa and Yannelis (1994), Koutsougeras and Yannelis (1993) and Podczeck and Yannelis (2008) for an extensive discussion of the Bayesian incentive compatibility in asymmetric information economies.

Notice that the above allocation is not incentive compatible with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2)$. Indeed, suppose that the realized state of nature is a, agent 1 is in the event $\{a, b\}$ and he reports c, (observe that agent 2 cannot distinguish between a and c). If the realized state is b, then agent 2 knows that 1 lied, but we ignore the possible effects of this.[25] If the realized state is a, then 2 is not able to identify the lie and then gives to agent 1 ten units. Therefore, the utility of agent 1, when he misreports and the state is a is $u_1(a, e_1(a) + x_1(c) - e_1(c)) = u_1(a, 20 + 10 - 0) = \sqrt{30}$ which is greater than $u_1(a, x_1(a)) = \sqrt{20}$, the utility of agent 1 when he does not misreport. Hence, the allocation $x_1(a, b, c) = (20, 10, 10) \quad \text{and} \quad x_2(a, b, c) = (20, 10, 10)$ is not incentive compatible.

In order to make sure that the equilibrium contracts are stable, we must insist on a coalitional definition of incentive compatibility and not an individual one. As the following example shows, a contract which is individual incentive compatible may not be coalitional incentive compatible and therefore may not be viable.

Example 6.3 Consider an economy with three agents, two goods and three states of nature $S = \{a, b, c\}$. The primitives of the economy are given as follows: for all $i = 1, 2, 3$, $u_i(\cdot, x_i, y_i) = \sqrt{x_i y_i}$ and

$$\begin{array}{rl} \mathcal{F}_1 = \{\{a,b,c\}\}; & e_1(a,b,c) = ((15,0);(15,0);(15,0)). \\ \mathcal{F}_2 = \{\{a,b\},\{c\}\}; & e_2(a,b,c) = ((0,15);(0,15);(0,15)). \\ \mathcal{F}_3 = \{\{a\},\{b\},\{c\}\}; & e_3(a,b,c) = ((15,0);(15,0);(15,0)). \end{array}$$

Consider the following redistribution of the initial endowments:

$$\begin{array}{rcl} \mathbf{x}_1(a,b,c) & = & ((8,5),(8,5),(8,13)) \\ \mathbf{x}_2(a,b,c) & = & ((7,4),(7,4),(12,1)) \\ \mathbf{x}_3(a,b,c) & = & ((15,6),(15,6),(10,1)). \end{array} \tag{13}$$

Notice that the only agent who can misreport either state a or b to agents 1 and 2 is agent 3. Clearly, agent 3 cannot misreport state c since agent 2 would know it. Thus, agent 3 can only lie if either state a or state b occurs. However, agent 3 has no incentive to misreport since he gets the same consumption in both states a and b. Hence, the allocation (13) is individual incentive compatible with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3)$, but we will show that it is not coalitional incentive compatible with respect to $\mathcal{F}$. Indeed, if c is the realized state of nature, agents 2 and 3 have an incentive to cooperate against agent 1 and report b (notice that agent 1 cannot distinguish between b and c). The coalition $C = \{2, 3\}$ will

[25] We ignore the implication of being caught lying, thus allowing more lies to happen. This means that we are using a stronger incentive compatibility notion.

now be better off, i.e.,

$$\begin{aligned} u_2(c, e_2(c) + \mathbf{x}_2(b) - e_2(b)) &= u_2(c, (0,15) + (7,4) - (0,15)) \\ = u_2(c, (7,4)) &= \sqrt{28} > \sqrt{12} = u_2(c, \mathbf{x}_2(c)) \\ u_3(c, e_3(c) + \mathbf{x}_3(b) - e_3(b)) &= u_3(c, (15,0) + (15,6) - (15,0)) \\ = u_3(c, (15,6)) &= \sqrt{90} > \sqrt{10} = u_3(c, \mathbf{x}_3(c)). \end{aligned}$$

In Example 6.2 we have constructed an allocation which is Pareto optimal but it is not individual incentive compatible; while in Example 6.3 we have shown that an allocation, which is individual incentive compatible, need not be coalitional incentive compatible.

In view of Examples 6.2 and 6.3, it is easy to understand the meaning of Definition 6.1. An allocation is coalitional incentive compatible if no coalition of agents C can cheat the complementary coalition (i.e., $I \setminus C$) by misreporting the realized state of nature and make all its members better off. Notice that condition (i) indicates that coalition C can only cheat the agents not in C (i.e., $I \setminus C$) in the states that the agents in $I \setminus C$ cannot distinguish. If $C = \{i\}$ then the above definition reduces to individual incentive compatibility.

6.1 Maximin Incentive Compatibility

In this section, we will prove that the maximin rational expectations equilibrium is incentive compatible. To this end we need the following definition of maximin coalitional incentive compatibility, which is an extension of the Krasa and Yannelis (1994) definition to incorporate maximin preferences (see also de Castro and Yannelis (2011)).

Definition 6.4 *A feasible allocation x is said to be maximin coalitional incentive compatible (MCIC) with respect to information structure $\Pi = (\Pi)_{i \in I}$, if the following does not hold: there exists a coalition C and two states a and b such that*

$$\begin{array}{lll} (i) & \Pi_i(a) = \Pi_i(b) & for\ all\ i \notin C, \\ (ii) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^{\ell}_{+} & for\ all\ i \in C,\ and \\ (iii) & \underline{u}_i^{\Pi_i}(a, y_i) > \underline{u}_i^{\Pi_i}(a, x_i) & for\ all\ i \in C, \end{array}$$

where for all $i \in C$,

$$(*) \qquad y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a \\ x_i(s) & \text{otherwise}. \end{cases}$$

According to the above definition, an allocation is said to be maximin coalitional incentive compatible if it is not possible for a coalition to misreport the realized state of nature and have a distinct possibility of making its members better off in terms of maximin utility.

Remark 6.1 Example 6.2 shows that an efficient allocation may not be incentive compatible in the Krasa-Yannelis sense. We now show that it is not the case in our maximin sense[26]. Precisely, if agents take into account the worse possible state that can occur, then the allocation $x_i(a,b,c) = (20,10,10)$ for $i = 1,2$ in Example 6.2, is maximin incentive compatible. Indeed, if a is the realized state of nature, agent 1 does not have an incentive to report state c and benefit, because when he misreports he gets:

$$\underline{u}_1(a,y_1) = \min\{u_1(a, e_1(a)+x_1(c)-e_1(c)); u_1(b, x_1(b))\} = \min\{\sqrt{30}, \sqrt{10}\} = \sqrt{10}.$$

When agent 1 does not misreport, he gets:

$$\underline{u}_1(a,x_1) = \min\{u_1(a, x_1(a)); u_1(b, x_1(b))\} = \min\{\sqrt{20}, \sqrt{10}\} = \sqrt{10}.$$

Consequently, agent 1 does not gain by misreporting. Similarly, one can easily check that agent 2, when a is the realized state of nature, does not have an incentive to report state b and benefit. Indeed, if the realized state of nature is a, agent 2 is in the event $\{a,c\}$. If agent 2 reports the false event $\{b\}$ then his maximin utility does not increase since

$$\begin{aligned} \underline{u}_2(a,y_1) &= \min\{u_2(a, e_2(a)+x_2(b)-e_2(b)); u_2(c, x_2(c))\} \\ &= \min\{\sqrt{20+10-0}, \sqrt{10}\} = \sqrt{10} \\ &= \min\{\sqrt{20}, \sqrt{10}\} = \underline{u}_2(a,x_2). \end{aligned}$$

Proposition 6.5 *If x is CIC with respect to the information structure $\Pi = (\Pi_i)_{i\in I}$, then it is also maximin CIC with respect to Π. The converse may not be true.*

Proof: See Appendix.

Theorem 6.6 *Any maximin rational expectations equilibrium (x,p) is maximin coalitional incentive compatible with respect to the information structure $\mathcal{G} = (\mathcal{G}_i)_{i\in I}$, where $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$ for any $i \in I$.*

Proof: See Appendix.

6.2 Further remarks on the incentive compatibility of maximin REE

In this section we consider the incentive compatibility with respect to the initial private information $\mathcal{F} = (\mathcal{F}_i)_{i\in I}$ since the same considerations made in subsection 5.1 apply. In what follows, by the term "(private) incentive compatible", we mean incentive compatibility with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_i)_{i\in I}$.

[26]De Castro and Yannelis show that every efficient allocation is coalitional incentive compatible if and only if all individuals have maximin preferences (see Theorem 3.1 in de Castro and Yannelis (2011)).

Remark 6.2 Clearly, any non revealing maximin rational expectations equilibrium is (private) maximin CIC, simply because $\mathcal{G}_i = \mathcal{F}_i$ for all $i \in I$, and hence the result follows from Theorem 6.6. Example 6.8 below shows that a fully revealing maximin REE may not be (private) maximin CIC. This suggests that a weaker notion of maximin CIC is needed.

Definition 6.7 *A feasible allocation x is said to be weak maximin coalitional incentive compatible (weak MCIC) with respect to information structure Π, if the following does not hold: there exists a coalition C and two states a and b such that*

$$\begin{array}{lll} (I) & \Pi_i(a) = \Pi_i(b) & for\ all\ i \notin C, \\ (II) & u_i(a, x_i(a)) = u_i(a, x_i(b)) & for\ all\ i \notin C, \\ (III) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^\ell_+ & for\ all\ i \in C,\ and \\ (IV) & \underline{u}_i^{\Pi_i}(a, y_i) > \underline{u}_i^{\Pi_i}(a, x_i) & for\ all\ i \in C, \end{array}$$

where for all $i \in C$,

$$(*) \qquad y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a \\ x_i(s) & \text{otherwise.} \end{cases}$$

Condition (II) of Definition 6.7 does not necessarily mean that $x_i(\cdot)$ is Π_i-measurable for all $i \notin C$, neither that $x_i(a) = x_i(b)$. It just guarantees, together with (I), that individuals not in coalition C are not able to detect a misreport by coalition C.

Clearly, any maximin CIC allocation is also weak maximin CIC whatever is the information structure Π, but the converse may not be true as shown by the following example.

Example 6.8 We consider the Example 3.1 in Glycopantis, Muir, and Yannelis (2005) that we recall below.[27] There are two agents $I = \{1, 2\}$, two commodities and three states of nature $S = \{a, b, c\}$. The primitives of the economy are given as follows

$$\begin{array}{llll} e_1(a) = e_1(b) = (7,1) & e_1(c) = (4,1) & \mathcal{F}_1 = \{\{a,b\},\{c\}\} & u_1(\cdot, x_1, y_1) = \sqrt{x_1 y_1} \\ e_2(b) = e_2(c) = (1,7) & e_2(a) = (1,10) & \mathcal{F}_2 = \{\{a\},\{b,c\}\} & u_2(\cdot, x_2, y_2) = \sqrt{x_2 y_2}. \end{array}$$

In this economy the unique (Bayesian) REE is the following:

$$\begin{array}{lll} (p_1(a), p_2(a)) = \left(1, \frac{8}{11}\right) & (x_1(a), y_1(a)) = \left(\frac{85}{22}, \frac{85}{16}\right) & (x_2(a), y_2(a)) = \left(\frac{91}{22}, \frac{91}{16}\right) \\ (p_1(b), p_2(b)) = (1, 1) & (x_1(b), y_1(b)) = (4, 4) & (x_2(b), y_2(b)) = (4, 4) \\ (p_1(c), p_2(c)) = \left(1, \frac{5}{8}\right) & (x_1(c), y_1(c)) = \left(\frac{37}{16}, \frac{37}{10}\right) & (x_2(c), y_2(c)) = \left(\frac{43}{16}, \frac{43}{10}\right). \end{array}$$

Notice that (p, x) is a fully revealing (Bayesian) REE and hence it is also a maximin REE. Moreover, x is weak (private) maximin CIC, but it is not (private) maximin CIC.

[27] We thank Liu Zhiwei for having suggested this example us.

Indeed, take $C = \{2\}$ and the two states a and b, and observe that

$$\begin{array}{rcl} \mathcal{F}_1(a) & = & \mathcal{F}_1(b) \\ (e_2^1(a) + x_2(b) - e_2^1(b), e_2^2(a) + y_2(b) - e_2^2(b)) & = & (1+4-1, 10+4-7) = (4,7) \gg 0 \\ u_2(a, e_2^1(a) + x_2(b) - e_2^1(b), e_2^2(a) + y_2(b) - e_2^2(b)) & = & \sqrt{28} > \sqrt{\frac{91^2}{352}} = u_2(a, x_2(a), y_2(a)). \end{array}$$

Hence, x is not (private) maximin CIC, but there does not exist two states s_1 and s_2 and an agent i, such that

$$\begin{array}{rcl} \mathcal{F}_i(s_1) & = & \mathcal{F}_i(s_2) \\ \sqrt{x_i(s_1) y_i(s_1)} & = & \sqrt{x_i(s_2) y_i(s_2)}. \end{array}$$

Therefore, x is weak (private) maximin coalitional incentive compatible.

Proposition 6.9 *Assume that $e_i(\cdot)$ is $\mathcal{F}_i$-measurable for all $i \in I$ and let (p, x) be a maximin rational expectations equilibrium. If one of the following conditions holds true:*

1. *$u_i(\cdot, y)$ is $\mathcal{F}_i$-measurable*[28] *for any $i \in I$ and any $y \in \mathbb{R}_+^\ell$;*
2. *p is fully revealing, i.e., $\sigma(p) = \mathcal{F}$;*

then x is weak (private) maximin coalitional incentive compatible.

Proof: See Appendix.

Remark 6.3 Although in Kreps's example, the utility functions are not private information measurable, the unique maximin rational expectations equilibrium is (private) maximin coalitional incentive compatible, since the equilibrium price p is non revealing (see Remarks 6.4 and 6.2). On the other hand, in Example 6.8 both hypotheses of Proposition 6.9 are satisfied and the maximin REE is weak (private) maximin CIC. However, as it has been already observed, it is not (private) maximin CIC.

Remark 6.4 As a corollary of Theorem 6.6 we deduce that any maximin rational expectations equilibrium is maximin individual incentive compatible. Moreover, it should be noted that the maximin rational expectations equilibrium in Kreps' example (Example 3.3) is coalitional incentive compatible. Indeed if state s_1 occurs and agent 1 announces s_2, then

$$u_1(s_1, e_1^1(s_1) + x_1(s_2) - e_1^1(s_2), e_1^2(s_1) + y_1(s_2) - e_1^2(s_2)) = log2 + 1 < 2 = u_1(s_1, x_1(s_1), y_1(s_1)).$$

On the other hand, if state s_2 occurs and agent 1 announces s_1, then

$$u_1(s_2, e_1^1(s_2) + x_1(s_1) - e_1^1(s_1), e_1^2(s_2) + y_1(s_1) - e_1^2(s_1)) = 2 < 2log2 + 1 = u_1(s_2, x_1(s_2), y_1(s_2)).$$

Therefore, the unique maximin rational expectations equilibrium in Example 3.3 is maximin CIC.

[28] Notice that the measurability assumption of utility functions is not too strong when we deal with coalitional incentive compatibility notions (see for example Koutsougeras and Yannelis (1993), Krasa and Yannelis (1994), Angeloni and Martins-da Rocha (2009) where the utility functions are assumed to be state independent, and therefore $\mathcal{F}_i$-measurable.)

7 Discussion

We introduced a new rational expectations equilibrium notion which departs from the Bayesian (subjective expected utility) formulation. Our new rational expectations equilibrium notion is formulated in terms of (a particular version of) Gilboa-Schmeidler's maximin expected utility. Furthermore, the resulting equilibrium allocations need not to be measurable with respect to the private information and the information the equilibrium prices have generated as in the case of the Bayesian REE. Our new notion exists universally (and not generically). We also show that it is also Pareto efficient and incentive compatible. These results are false for the Bayesian REE (see Kreps (1977) and Glycopantis and Yannelis (2005)).

As the reader should have noticed, one important aspect of our theory is private information measurability. We discuss this in detail below.

7.1 Private information measurability and incentive compatibility

In view of the several examples in this paper, it seems that the private information measurability of allocations in the definition of the REE creates problems. Recall that in the ex ante expected utility case (e.g., Radner (1968) and Yannelis (1991)) the role of the private information measurability of allocations is two-fold.

First it highlights the relevance of asymmetric information. If this condition is relaxed, then agents behave as they have symmetric information and the information partition does not influence the payoff of each player. Hence the asymmetric information in the Radner (1968) model is modeled by the private information measurability of allocations. In contract, in our MEU modeling the asymmetry of information is captured by the definition of the MEU itself. Specifically priors are defined on the events of each partition of each agents and therefore the MEU itself models the information asymmetry. Consequently there is no need to assume that allocations are private information measurable as it is the case with the Bayesian modeling of Radner.

Second, in the one good case the private information measurability of allocations becomes a necessary and sufficient condition to ensure that trades are incentive compatible (e.g., Krasa and Yannelis (1994)), and in the multi good case it is a sufficient condition to ensure incentive compatibility. Thus, the private information measurability seems to be a desirable assumption in the ex ante case as it ensures that ex-ante private information Pareto optimal allocations are interim incentive compatible.

However, this is not the case with the Bayesian REE as it is not necessarily incentive compatible (Glycopantis, Muir, and Yannelis (2005)). Also, in the ex ante case as we mentioned above, the private information measurability amounts to asymmetric information but in the interim stage, (e.g., REE case), the interim expected utility is automatically private information measurable as it is conditioned on the event in the private information of each agent, thus constant on the individual's event. Hence, the asymmetric information in the interim case enters the model via the interim utility function of each agent. By also imposing the private information measurability on allocations we end up with an existence of equilibrium problem as the Kreps's example clearly indicates. To the best of our knowledge, we do not know what the private information measurability means or accomplishes in the interim framework, as the conditional expected utility is already private information measurable. We fail to see the importance or usefulness of the private information measurability of allocations because the counterexamples in Glycopantis and Yannelis (2005) show that the REE is not implementable, is not incentive compatible and it is not efficient.

In a general equilibrium model with asymmetric information, it is possible that the MEU choice does not reflect pessimistic behavior, but rather incentive compatible behavior. If an agent plays against the nature (e.g., Milnor game), since, nature is not strategic, it makes sense to view the MEU decision making as reflecting pessimistic behavior. However, when you negotiate the terms of a contract under asymmetric information and the other agents have an incentive to misreport the state of nature and benefit, then the MEU provides a mechanism to prevent others from cheating you. This in not pessimism, but incentive compatibility. It is exactly for this reason that the MEU solves the conflict between efficiency and incentive compatibility (see for example de Castro and Yannelis (2011)). This conflict seems to be inherent in the Bayesian analysis, where agents must assign probabilities to complete unknown states and those probabilities could be very far from the “true" ones.

7.2 Our maximin model

The reader may have noted that in some results[29], our MEU formulation is a particular form of the original Gilboa-Schmeiler model. Namely, we assume a particular set of probabilities, $\mathcal{C}_i^s$, which comprises all probabilities with support contained in the element of the partition $\Pi_i(s)$.

Some researchers have expressed the view that this model assumes too much pessimism and that it would be desirable to allow the set $\mathcal{C}_i$ of probabilities to be a strict subset of C_i^s.

There are at least two responses to this criticism. First, we can conceive the partition model as a description of all information that the individual has. If we take this principle seriously, this means that once individual i is informed of its element $\Pi_i(s)$, he knows nothing else. In particular, he has no information about the likelihood or probability of the states inside that partition. If the partition represents his knowledge, he is completely ignorant beyond it, that is, he has no relevant information to rule out any probability in $\mathcal{C}_i^s$. This is related to the literature of complete ignorance that flourished in 1950's. For example, Milnor (1954) discusses this hypothesis of complete ignorance in games against nature as follows:

> “Our basic assumption that the player has absolutely no information about Nature may seem too restrictive. However such no-information games may be used as normal form for a wider class of games in which certain types of partial information is allowed. For example if the information consists of bounds for the probabilities of the various states of Nature, then by considering only those mixed strategies for Nature which satisfy these bounds, we construct a new game having no information.” (Milnor, 1954, p.49)

[29] Incentive compatibility results only, since for the rest we may consider a more general framework by adopting suitable modifications.

Thus, according to this view, we can reduce the partial information that is outside the partition and is represented in some knowledge of the probabilities $\mathcal{C}_i$, in a new model with no information left; this would be the model that we are analyzing.

A second response to this criticism begins by recalling the standard practice in economic theory that an unrealistic assumption is used to capture in a simplistic form a phenomenon that is quite realistic. Even with unrealistic assumptions, economic theory was frequently able to provide good insights about the real world. In our case, the restrictive assumption about the preference is a simplistic way to capture a phenomenon that is universal: indifference among indistinguishable bundles. When people do not have a good reason to prefer an option over other, they are frequently indifferent. The main reason of why our result is true is the indifference between some specific bundles. Our formalization just captures this property in a way that allows us to obtain the result in a clear and straightforward way.

It is useful to put in perspective the relationship of this assumption with the problems of the REE mentioned in the introduction, namely, the unrealistic rationality required from agents by the REE paradigm. In our case, the indifference among bundles that are not clearly better restrain the implications of excessive rationality required by REE. Thus, this model may indicate, at least conceptually, a way out of the REE conundrum.

We refrain from taking a strong position in accordance to one or other response, but do submit that exploring variations of the REE concept, as we do here, may lead to a better understanding of the REE phenomenon. This is perhaps the best justification for our study of this model.

7.3 Open questions

We conclude this paper with some open questions.

Throughout we have used the assumption that there is a finite number of states. We conjecture that the main existence theorem can be extended to infinitely many states of nature of the world and even to an infinite dimensional commodity space.[30] Some progress in this direction has been made in Bhowmik, Cao, and Yannelis (2014).

In Glycopantis, Muir, and Yannelis (2005) it was shown that the Bayesian REE is not implementable as a perfect Bayesian equilibrium of an extensive form game. We conjecture that a new definition of perfect maximin equilibrium can be introduced, which will be compatible with the implementation of the maximin REE. What reinforces this conjecture is the fact that incentive compatible equilibrium notions, i.e., private core (Yannelis (1991)) and private value allocations (Krasa and Yannelis (1994)) are implementable as a perfect Bayesian equilibrium. Since, the maximin REE is also maximin incentive compatible, we believe that such a conjecture should be true. The recent papers de Castro, Yannelis, and Zhiwei (2015); Zhiwei (2015) analyze this issue and obtain interesting results. Indeed, in Zhiwei (2015) it is shown that the MREE is

[30] For the ex-ante case some existence and equivalence results are obtained in He and Yannelis (2015).

implementable as a "maximin equilibrium."[31]

It is also of interest to know if the results of this paper could be extended to a continuum of agents, or to a more general setup such as mixed markets.

Based on the Bayesian expected utility formulation, Sun, Wu, and Yannelis (2012) show that with a continuum of agents, whose private signals are independent conditioned on the macro states of nature, a REE universally exists, it is incentive compatible and efficient. These results have been obtained by means of the law of large numbers. It is of interest to know if the theorems of this paper can be extended in such a framework which makes the law of large numbers applicable.

Furthermore, it is of interest to know under what conditions the core-value-REE equivalence theorems hold for the maximin expected utility framework.

8 Appendix

8.1 Proofs of Section 3

Proof of Lemma 3.1: If $y_i \in L_i^{REE}$, that is, y_i is $\mathcal{G}_i$-measurable, then $p(s) \cdot y_i(s) \leq p(s) \cdot e_i(s)$ is equivalent to $p(s') \cdot y_i(s') \leq p(s') \cdot e_i(s')$ for all $s' \in \mathcal{G}_i(s)$, which establishes the equivalence of (i) and (iii).

$(ii) \Leftrightarrow (iii)$: Assume that y_i is $\mathcal{G}_i$-measurable (i.e., $y_i \in L_i^{REE}$). Since $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$, then $p(\cdot)$ is $\mathcal{G}_i$-measurable for all $i \in I$, as well as the initial endowment $e_i(\cdot)$ because it is $\mathcal{F}_i$-measurable and $\mathcal{F}_i \subseteq \mathcal{G}_i$. Therefore, for all $i \in I$ and all $s \in S$

$$\begin{aligned} \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot y_i(s') &= p(s) \cdot y_i(s) \, |\mathcal{G}_i(s)| \quad \text{and} \\ \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot e_i(s') &= p(s) \cdot e_i(s) \, |\mathcal{G}_i(s)| \, , \end{aligned}$$

where $|\mathcal{G}_i(s)|$ is the number of states in the event $\mathcal{G}_i(s)$. Hence, for all $i \in I$ and $y_i \in L_i^{REE}$,

$$p(s) \cdot y_i(s) \leq p(s) \cdot e_i(s) \quad \Leftrightarrow \quad \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot y_i(s') \leq \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot e_i(s').$$

□

Proof of Proposition 3.4: Assume on the contrary that there exists an agent $i \in I$ and two states $a, b \in S$ such that $a \in \mathcal{G}_i(b)$ and $x_i(a) \neq x_i(b)$. Consider $z_i(s) =$

[31] Maximin equilibrium is a non cooperative equilibrium notion that captures the idea that each player maximizes his interim payoff taking into account what is the worst possible state that can occur and also the worst possible announcement of all the other players against him, see de Castro, Yannelis, and Zhiwei (2015).

$\alpha x_i(a) + (1-\alpha)x_i(b)$ for all $s \in \mathcal{G}_i(b)$, where $\alpha \in (0,1)$, and notice that z_i is constant in the event $\mathcal{G}_i(b)$. Moreover,

$$\underline{u}_i^{REE}(b, z_i) = \min_{s\in\mathcal{G}_i(b)} u_i(s, z_i(s)) = \min_{s\in\mathcal{G}_i(b)} u_i(s, \alpha x_i(a) + (1-\alpha)x_i(b))$$

Since $u_i(\cdot, y)$ is $\mathcal{G}_i$-measurable for all $y \in \mathbb{R}^\ell_+$, from strict quasi concavity of u_i it follows that

$$\begin{aligned}\underline{u}_i^{REE}(b, z_i) = u_i(b, \alpha x_i(a) + (1-\alpha)x_i(b)) &> \min\{u_i(b, x_i(a)), u_i(b, x_i(b))\} \\ = \min\{u_i(a, x_i(a)); u_i(b, x_i(b))\} &\geq \min_{s\in\mathcal{G}_i(b)} u_i(s, x_i(s)) \\ &= \underline{u}_i^{REE}(b, x_i).\end{aligned}$$

Since (p, x) is a maximin rational expectations equilibrium it follows that $z_i \notin B_i(b, p)$, that is, there exists a state $s_i \in \mathcal{G}_i(b)$ such that

$$p(s_i)\cdot z_i(s_i) > p(s_i)\cdot e_i(s_i) \;\Rightarrow\; \alpha p(s_i)\cdot x_i(a) + (1-\alpha)p(s_i)\cdot x_i(b) > p(s_i)\cdot e_i(s_i).$$

Moreover, since $p(\cdot)$ and $e_i(\cdot)$ are $\mathcal{G}_i$-measurable and $p(s)\cdot x_i(s) \leq p(s)\cdot e_i(s)$ for all $s \in S$ (see condition (i) in Definition 3.2), it follows that $p(s_i)\cdot e_i(s_i) > p(s_i)\cdot e_i(s_i)$, which is a contradiction. □

Proof of Proposition 3.5: Because of Lemma 3.1, all we need to show is that the maximin utility and the (Bayesian) interim expected utility coincide. Since for all $i \in I$ and for all $y \in \mathbb{R}^\ell_+$, $u_i(\cdot, y)$ is $\mathcal{F}_i$-measurable and $\mathcal{F}_i \subseteq \mathcal{G}_i$, then $u_i(\cdot, y)$ is $\mathcal{G}_i$-measurable.

Moreover, since for each $i \in I$, $x_i(\cdot)$ is $\mathcal{G}_i$-measurable it follows that for all $i \in I$ and $s \in S$, both maximin and interim utility function are equal to the ex-post utility function. That is,

$$\underline{u}_i^{REE}(s, x_i) = \min_{s'\in\mathcal{G}_i(s)} u_i(s', x_i(s')) = u_i(s, x_i(s)) \tag{14}$$

and

$$v_i(x_i|\mathcal{G}_i)(s) = \sum_{s'\in\mathcal{G}_i(s)} u_i(s', x_i(s'))\frac{\pi_i(s')}{\pi_i\left(\mathcal{G}_i(s)\right)} = u_i(s, x_i(s)). \tag{15}$$

From (14) and (15) it follows that for all i and s, $\underline{u}_i^{REE}(s, x_i) = v_i(x_i|\mathcal{G}_i)(s)$. Therefore, we can conclude that if (p, x) is a Bayesian REE, then (p, x) is a MREE; the converse is also true if $x_i(\cdot)$ is $\mathcal{G}_i$-measurable for all $i \in I$. □

Proof of Proposition 3.6: For each $s \in S$, let

$$H(s) = \{h \in \{1, \dots, \ell\} \;:\; p^h(s) = 0\},$$

and let

$$\bar{S} = \{s \in S \ : \ H(s) \neq \emptyset\}.$$

Since (p, x) is a maximin REE, we consider the information generated by the equilibrium price, that is the algebra $\sigma(p)$. Clearly, $H(\cdot)$ is $\sigma(p)$-measurable[32], because $p(s_1) = p(s_2)$ whenever $\sigma(p)(s_1) = \sigma(p)(s_2)$. Moreover, since for any $i \in I$, $\sigma(p)$ is coarser than $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$, it follows that for all $i \in I$

$$H(\cdot) \ \text{ is } \mathcal{G}_i - \text{measurable}. \tag{16}$$

Now, assume on the contrary that $\bar{S}$ is non empty and let $\bar{s} \in \bar{S}$. Hence, $H(\bar{s}) \neq \emptyset$, i.e., there exists at least a "free" good h such that $p^h(\bar{s}) = 0$. Let $i \in I$ be the agent such that $u_i(s, \cdot)$ is strictly monotone for any $s \in S$; and define the following allocation:

$$z_i^h(s) = \begin{cases} x_i^h(s) + K & \text{if } s \in \mathcal{G}_i(\bar{s}) \text{ and } h \in H(s) \\ x_i^h(s) & \text{otherwise,} \end{cases}$$

where $K > 0$.

Notice that for any $s \in \mathcal{G}_i(\bar{s})$, since $H(s) = H(\bar{s}) \neq \emptyset$ (see (16)), from the strict monotonicity it follows that $u_i(s, z_i(s)) > u_i(s, x_i(s))$ for all $s \in \mathcal{G}_i(\bar{s})$, and hence

$$\underline{u}_i^{REE}(\bar{s}, z_i) > \underline{u}_i^{REE}(\bar{s}, x_i).$$

Since (p, x) is a maximin REE, $z_i \notin B_i(\bar{s}, p)$, that is there exists a state $s_i \in \mathcal{G}_i(\bar{s})$ such that

$$p(s_i) \cdot [z_i(s_i) - e_i(s_i)] > 0.$$

From (16), it follows that $H(s_i) = H(\bar{s}) \neq \emptyset$, and therefore

$$\begin{aligned} 0 &< p(s_i) \cdot [z_i(s_i) - e_i(s_i)] = \\ \sum_{h \in H(s_i)} p^h(s_i)[x_i^h(s_i) + K - e_i^h(s_i)] &+ \sum_{h \notin H(s_i)} p^h(s_i)[x_i^h(s_i) - e_i^h(s_i)] = \\ 0 &+ \sum_{h \notin H(s_i)} p^h(s_i)[x_i^h(s_i) - e_i^h(s_i)] = \\ \sum_{h \in H(s_i)} p^h(s_i)[x_i^h(s_i) - e_i^h(s_i)] &+ \sum_{h \notin H(s_i)} p^h(s_i)[x_i^h(s_i) - e_i^h(s_i)] = \\ p(s_i) \cdot [x_i(s_i) - e_i(s_i)] &\leq 0. \end{aligned}$$

This is a contradiction, hence $p(s) \gg 0$ for each $s \in S$. □

Proof of Proposition 3.7: Let (p, x) be a maximin rational expectations equilibrium and define for each agent $i \in I$ and state $s \in S$ the following set:

$$M_i(s) = \left\{ s' \in \mathcal{G}_i(s) : \ \underline{u}_i^{REE}(s, x_i) = u_i(s', x_i(s')) \right\}.$$

[32] We mean that $H(s_1) = H(s_2)$ if $\sigma(p)(s_1) = \sigma(p)(s_2)$.

Clearly, since S is finite, for all $i \in I$ and $s \in S$, the set $M_i(s)$ is nonempty, i.e., $M_i(s) \neq \emptyset$. Moreover, if $s' \in \mathcal{G}_i(s) \setminus M_i(s)$ it means that $\underline{u}_i^{REE}(s, x_i) < u_i(s', x_i(s'))$. Thus, we want to show that for all $i \in I$ and $s \in S$, $M_i(s) = \mathcal{G}_i(s)$.

Assume on the contrary that there exists an agent $j \in I$ and a state $\bar{s} \in S$ such that $\mathcal{G}_j(\bar{s}) \setminus M_j(\bar{s}) \neq \emptyset$. Notice that

$$\underline{u}_j^{REE}(\bar{s}, x_j) < u_j(s, x_j(s)) \quad \text{for any } s \in \mathcal{G}_j(\bar{s}) \setminus M_j(\bar{s}).$$

Fix $s' \in \mathcal{G}_j(\bar{s}) \setminus M_j(\bar{s})$ and define the following allocation

$$y_j(s) = \begin{cases} x_j(s) & \text{if } s \in G_j(\bar{s}) \setminus M_j(\bar{s}) \\ x_j(s') & \text{if } s \in M_j(\bar{s}). \end{cases}$$

Since the utility functions are assumed to be private information measurable, it follows that $u_j(s, y_j(s)) > \underline{u}_j^{REE}(\bar{s}, x_j)$ for any $s \in \mathcal{G}_j(\bar{s})$, and hence $\underline{u}_j^{REE}(\bar{s}, y_j) > \underline{u}_j^{REE}(\bar{s}, x_j)$. Recall that (p, x) is a maximin REE, therefore there exists $s \in \mathcal{G}_j(\bar{s})$ such that $p(s) \cdot y_j(s) > p(s) \cdot e_j(s)$. If $s \in M_j(\bar{s})$, then $p(s) \cdot x_j(s') > p(s) \cdot e_j(s)$. Since $p(\cdot)$ and $e_j(\cdot)$ are both $\mathcal{G}_j$-measurable, it follows that $p(s') = p(s)$ and $e_j(s') = e_j(s)$. This implies that $p(s') \cdot x_j(s') > p(s') \cdot e_j(s')$, which is clearly a contradiction. On the other hand, if $s \in G_j(\bar{s}) \setminus M_j(\bar{s})$, thus we have that $p(s) \cdot x_j(s) > p(s) \cdot e_j(s)$ which is a contradiction as well. Therefore, for each $i \in I$ and $s \in S$, $M_i(s) = \mathcal{G}_i(s)$. □

8.2 Proofs of Section 4

The following section has an its own meaning as it presents some comparisons between maximin REE and other solutions concepts. It is also useful to prove the existence of a maximin REE.

8.2.1 Some comparisons

Given a differential information economy $\mathcal{E}$ described in Section 2, since S is finite, there is a finite number of complete information economies $\{\mathcal{E}(s)\}_{s \in S}$. For each fixed s in S, the complete information economy $\mathcal{E}(s)$ is given as follows:

$$\mathcal{E}(s) = \left\{ I, \mathbb{R}_+^\ell, (u_i(s), e_i(s))_{i \in I} \right\},$$

where $I = \{1, \ldots, n\}$ is still the set of n agents, and for each $i \in I$, $u_i(s) = u_i(s, \cdot) : \mathbb{R}_+^\ell \to \mathbb{R}$ and $e_i(s) \in \mathbb{R}_+^\ell$ represent respectively the utility function and the initial endowment of agent i. A feasible allocation x is said to be an ex-post Walrasian equilibrium allocation if there exists a price $p : S \to \mathbb{R}_+^\ell$ such that for any state of nature $s \in S$, the pair $(x(s), p(s))$ is a Walrasian equilibrium for the complete information economy $\mathcal{E}(s)$. It is well now that any ex-post Walrasian equilibrium is ex-post Pareto optimal (see Definitions 5.1 and 5.6).

We now investigate on some relationships between maximin REE and ex-post Walrasian equilibria.

We first prove that the set of V-REE allocations and a fortiori of maximin REE allocations (see Remark 4.5) contains all the selections from the Walrasian equilibrium correspondence of the associated family of complete information economies.

Lemma 8.1 *If (x, p) is an ex-post Walrasian equilibrium, then (x, p) is a V-REE, and in particular it is a maximin REE.*

Proof: Let (x, p) an ex-post Walrasian equilibrium, we want to show that (x, p) is a V-REE. First, notice that x is feasible in the economy $\mathcal{E}$ since so is $x(s)$ in the economy $\mathcal{E}(s)$ for each s, and p is a price function since for any $s \in S$, $p(s) > 0$. Consider the algebra generated by p denoted by $\sigma(p)$, and for each agent i let $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. We show that (p, x) is a V-rational expectations equilibrium for $\mathcal{E}$. Clearly, $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$ for all i and s, hence $x_i \in B_i(s, p)$ for all i and s. It remains to prove that x_i maximizes $V_i(\cdot, \mathcal{G}_i, s)$ on $B_i(s, p)$. Assume, on the contrary, that there exists an alternative allocation $y \in L$ such that for some agent i and some state s,

$$V_i(y_i, \mathcal{G}_i, s) > V_i(x_i, \mathcal{G}_i, s), \text{ and} \tag{17}$$

$y_i \in B_i(s, p)$, that is

$$p(s') \cdot y_i(s') \leq p(s') \cdot e_i(s') \quad \text{for all } s' \in \mathcal{G}_i(s). \tag{18}$$

From $(*)$ it follows that there exists a state $\bar{s} \in \mathcal{G}_i(s)$ such that

$$u_i(\bar{s}, y_i(\bar{s})) > u_i(\bar{s}, x_i(\bar{s})).$$

Since $(p(\bar{s}), x(\bar{s}))$ is a Walrasian equilibrium for $\mathcal{E}(\bar{s})$, it follows that, $p(\bar{s}) \cdot y_i(\bar{s}) > p(\bar{s}) \cdot e_i(\bar{s})$, which clearly contradicts (18). □

In particular, any ex-post Walrasian equilibrium is a maximin REE. The converse is not true (see Example 4.2) unless agents' utility functions and initial endowments are private information measurable. The next lemma holds true for the general MEU formulation (8) provided that for any agent i and state s, the set $\mathcal{M}_i^s$ contains only positive priors (see section 8.5).

Lemma 8.2 *If $(u_i, e_i) \subseteq \mathcal{F}_i$ for all $i \in I$, then any ex-post Walrasian equilibrium is a maximin REE and viceversa.*

Proof: One inclusion is shown in Lemma 8.1 for which no measurability assumption is needed. In order to prove the converse, let (x, p) a maximin REE and consider for any agent $i \in I$ the algebra $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. The monotonicity assumption on agents' utility function ensures that for any $s \in S$ the equilibrium price $p(s)$ is positive in $\mathcal{E}(s)$, i.e., $p(s) \neq 0$ for any $s \in S$. Clearly, feasibility and budget constrains hold. Assume on the contrary that for some state s the pair $(x(s), p(s))$ is not a Walrasian equilibrium for the complete information economy $\mathcal{E}(s)$. This means that there exist an agent j and an alternative allocation $y \in \mathbb{R}^{\ell}_{+}$ such that

$$\begin{array}{ll} (i) & u_j(s,y) > u_j(s,x_j(s)) \\ (ii) & p(s)\cdot y \leq p(s)\cdot e_i(s). \end{array}$$

Let $z_j(s') = y$ for any $s' \in \mathcal{G}_j(s)$, and notice that since $u_j(\cdot, z)$ is $\mathcal{F}_j$-measurable and a fortiori $\mathcal{G}_j$-measurable, from (i) it follows that

$$\underline{u}_j^{REE}(s,z_j) = u_j(s,y) > u_j(s,x_j(s)) \geq \underline{u}_i^{REE}(s,x_i).$$

Recall that (x,p) is a maximin rational expectations equilibrium, thus there exists $\bar{s} \in \mathcal{G}_j(s)$ such that

$$p(\bar{s})\cdot z_j(\bar{s}) > p(\bar{s})\cdot e_j(\bar{s}).$$

Since $e_j(\cdot)$ and $p(\cdot)$ are $\mathcal{G}_j$-measurable, it follows that

$$p(s)\cdot y > p(s)\cdot e_i(s),$$

which contradicts (ii). □

For the general case of $V-REE$ we need a stronger version of Axiom 4 in de Castro, Pesce, and Yannelis (2011), as Proposition 4.6 states. Here there is the proof.

Proof of Proposition 4.6: One inclusion is shown in Lemma 8.1. In order to prove the converse, let (x,p) a V-REE and consider for any agent $i \in I$ the algebra $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Condition $(***)$ implies that for any $s \in S$ the equilibrium price $p(s)$ is positive in $\mathcal{E}(s)$, i.e., $p(s) \neq 0$ for any $s \in S$. Clearly, feasibility and budget constrains hold. Assume on the contrary that for some state s the pair $(x(s),p(s))$ is not a Walrasian equilibrium for the complete information economy $\mathcal{E}(s)$. This means that there exist an agent j and an alternative allocation $y \in \mathbb{R}^{\ell}_{+}$ such that

$$\begin{array}{ll} (i) & u_j(s,y) > u_j(s,x_j(s)) \\ (ii) & p(s)\cdot y \leq p(s)\cdot e_i(s). \end{array}$$

Let $z_j(s) = y$ and $z_j(s') = y$ for any $s' \in \mathcal{G}_j(s) \setminus \{s\}$. Condition $(***)$ implies that $V_j(z_j,\mathcal{G}_j,s) > V_j(x_j,\mathcal{G}_j,s)$, and hence there must exist $\bar{s} \in \mathcal{G}_j(s)$ such that

$$p(\bar{s})\cdot z_j(\bar{s}) > p(\bar{s})\cdot e_j(\bar{s}).$$

This is impossible by the definition of z_j, because it contradicts (ii). □

Remark 8.3 Theorem 4.3 states that if in addition $u_i(s,\cdot)$ is strict quasi-concave for all $i \in I$ and $s \in S$, the ex-post Walrasian equilibria coincide also with the (Bayesian) rational expectations equilibrium (see also Einy, Moreno, and Shitovitz (2000) and De Simone and Tarantino (2010)). Moreover, Lemma 8.2 also holds if the budget set of MREE allocations is $B_i^*(s,p)$ defined as (11).

8.2.2 The existence proof

Proof of Theorem 4.5: Since S is finite, there is a finite number of complete information economies $\mathcal{E}(s) = \{I, \mathbb{R}^\ell_+, (u_i(s), e_i(s))_{i\in I}\}$, where for any $i \in I$ and any $s \in S$, $u_i(s) := u_i(s, \cdot) : \mathbb{R}^\ell_+ \to \mathbb{R}$ is continuous, quasi-concave; and $e_i(s) \gg 0$. For any $s \in S$, let $W(\mathcal{E}(s))$ the set of Walrasian equilibrium allocations for the economy $\mathcal{E}(s)$. The above assumptions ensure that for any $s \in S$, $W(\mathcal{E}(s)) \neq \emptyset$. We prove it for sake of completeness. Fix $s \in S$ and let $X(s)$ be a non empty, compact and convex subset of $\mathbb{R}^\ell_+$ defined as follows

$$X(s) = \left\{ y \in \mathbb{R}^\ell_+ \ : \ 0 \le y^k \le \sum_{h=1}^{\ell} \sum_{i\in I} e^h_i(s) \text{ for all } k \in \{1, \dots, \ell\} \right\},$$

and let Δ be the $(\ell - 1)$-dimensional unit simplex in $\mathbb{R}^\ell_+$. For any $i \in I$ define the following correspondences

$$\begin{aligned} B_i(s,\cdot) : \Delta &\to 2^{X(s)} \text{ such that} \\ B_i(s,q) &:= \{y \in X(s) \ : \ q \cdot y \le q \cdot e_i(s)\} \\ \phi_i(s,\cdot) : \Delta &\to 2^{X(s)} \text{ such that} \\ \phi_i(s,q) &:= \{y \in B_i(s,q) \ : \ u_i(s,y) \ge u_i(s,x) \ \forall\, x \in B_i(s,q)\}. \end{aligned}$$

It is easy to verify that for any i, $B_i(s,\cdot)$ is a continuous, nonempty compact convex valued correspondence. By the Berge Maximum Theorem, the correspondence $\phi_i(s,\cdot)$ is nonempty-valued, compact-valued and upper hemicontinuous. Furthermore, the quasi concavity of the utility function $u_i(s,\cdot)$ and the convex-valuedness of $B_i(s,\cdot)$ implies that $\phi_i(s,\cdot)$ is convex-valued.

Define the excess demand correspondence $Z(s,\cdot) : \Delta \to 2^{\mathbb{R}^\ell}$ by $Z(s,q) = \sum_{i\in I}[\phi_i(s,q) - e_i(s)]$. Then, $Z(s,\cdot)$ is nonempty compact convex valued and upper hemicontinuous. Moreover, for any $q \in \Delta$ and any $z \in Z(s,q)$, there exists $x_i \in \phi_i(s,q)$ for any $i \in I$, so that, by definition, $q \cdot x_i \le q \cdot e_i(s)$ for all $i \in I$. Therefore, by summing up, for any $q \in \Delta$, there exists $z \in Z(s,q)$ such that $q \cdot z \le 0$. Thus, by the Debreu-Gale-Nikaido Theorem there exists $p^*(s) \in \Delta$ such that $Z(s,p^*(s)) \cap \mathbb{R}^\ell_- \neq \emptyset$. Take $z^*(s) \in Z(s,p^*(s)) \cap \mathbb{R}^\ell_-$. Then for any i, there exists $x^*_i(s) \in \phi_i(s,p^*(s))$ such that $\sum_{i\in I}[x^*_i(s) - e_i(s)] = z^*(s) \le 0$. Since $u_i(s,\cdot)$ is monotone for all $i \in I$, with standard arguments it follows that

$$p^*(s) \cdot x^*_i(s) = p^*(s) \cdot e_i(s) \quad \text{for all } \ i \in I. \tag{19}$$

Thus, for any $i \in I$,

$$\begin{aligned} x^*_i(s) &\in argmax_{y\in B_i(s,p^*(s))} u_i(s,y), \text{ where} \\ B_i(s,p^*(s)) &= \{y \in X(s) \ : \ p^*(s) \cdot y \le p^*(s) \cdot e_i(s)\}. \end{aligned} \tag{20}$$

We now show that for any $i \in I$,

$$\begin{aligned} x^*_i(s) &\in argmax_{y\in B^*_i(s,p^*(s))} u_i(s,y), \text{ where} \\ \tilde{B}_i(s,p^*(s)) &= \{y \in \mathbb{R}^\ell_+ \ : \ p^*(s) \cdot y \le p^*(s) \cdot e_i(s)\} \end{aligned}$$

Assume on the contrary that there exists $y \in \mathbb{R}^{\ell}_{+}$ such that for some $i \in I$, $u_i(s, y) > u_i(s, x_i^*(s))$ and $y \in \tilde{B}_i(s, p^*(s))$. Since $u_i(s, \cdot)$ is continuous and $e_i(s) \gg 0$, we may assume, without loss of generality that $p^*(s) \cdot y < p^*(s) \cdot e_i(s)$. Clearly from (20) it follows that $y \notin X(s)$, that is $K = \left\{ k \in \{1, \ldots, \ell\} \, : \, y^k > \sum_{h=1}^{\ell} \sum_{i \in I} e_i^h(s) \right\}$ is non empty. Since for any h, $x_i^{h*}(s) < \sum_{h=1}^{\ell} \sum_{i \in I} e_i^h(s)$, then for any $k \in K$ there exists $\epsilon^k \in (0,1)$ such that $\epsilon^k y^k + (1-\epsilon^k) x_i^{*k}(s) < \sum_{h=1}^{\ell} \sum_{i \in I} e_i^h(s)$. Define $\epsilon = \min_{k \in K} \epsilon^k$ and $z_i(s) = \epsilon y + (1-\epsilon) x_i^*(s)$. Notice that $z_i^h(s) < \sum_{h=1}^{\ell} \sum_{i \in I} e_i^h(s)$ for all h; $u(s, z_i(s)) \geq \min\{u_i(s, y), u_i(s, x_i^*(s))\} = u_i(s, x_i^*(s))$ and

$$p^*(s) \cdot z_i(s) = \epsilon p^*(s) \cdot y + (1-\epsilon) p^*(s) \cdot x_i^*(s) < \epsilon p^*(s) \cdot e_i(s) + (1-\epsilon) p^*(s) \cdot e_i(s) = p^*(s) \cdot e_i(s).$$

This implies that $z_i(s) \in B_i(s, p^*(s))$ and $u_i(s, z_i(s)) = u_i(s, x_i^*(s))$. Notice that $1[p^*(s) \cdot e_i(s) - p^*(s) \cdot z_i(s)] \in intX(s)$, where $1 = (1, \ldots, 1) \in \mathbb{R}^{\ell}_{+}$. Let $\delta \in (0,1)$ be such that

$$\tilde{z}_i(s) = z_i(s) + \delta 1[p^*(s) \cdot e_i(s) - p^*(s) \cdot z_i(s)] \in intX(s).$$

Then, $\tilde{z}_i(s) \gg z_i(s)$ and hence $u_i(s, \tilde{z}_i(s)) > u_i(s, z_i(s)) = u_i(s, x_i^*(s))$. Moreover,

$$\begin{aligned} p^*(s) \cdot \tilde{z}_i(s) &= p^*(s) \cdot z_i(s) + \delta p^*(s) \cdot e_i(s) - \delta p^*(s) \cdot z_i(s) \\ &= (1-\delta) p^*(s) \cdot z_i(s) + \delta p^*(s) \cdot e_i(s) \\ &< (1-\delta) p^*(s) \cdot e_i(s) + \delta p^*(s) \cdot e_i(s) = p^*(s) \cdot e_i(s). \end{aligned}$$

Therefore, $\tilde{z}_i(s) \in B_i(s, p^*(s))$ and $u_i(s, \tilde{z}_i(s)) > u_i(s, x_i^*(s))$, which contradicts (20). Hence $(p^*(s), x^*(s))$ constitutes a free disposal Walrasian equilibrium for the economy $\mathcal{E}(s)$.

We now show that there exists a Walrasian equilibrium in the economy $\mathcal{E}(s)$ satisfying the exact feasibility. Assume[33] that $\epsilon(s) = \sum_{i \in I} [e_i(s) - x_i^*(s)] > 0$ and notice that $p^*(s) \cdot \epsilon(s) = 0$ because of (19). Define for any $i \in I$ the allocation $y_i(s) = x_i^*(s) + \frac{\epsilon(s)}{n} > x_i^*(s)$ which satisfies the exact feasibility in $\mathcal{E}(s)$ (i.e., $\sum_{i \in I} [y_i(s) - e_i(s)] = 0$). Thanks to monotonicity of $u_i(s, \cdot)$, $u_i(s, y_i(s)) \geq u_i(s, x_i^*(s))$ for any $i \in I$. Moreover from (19) it follows that for any $i \in I$

$$p^*(s) \cdot y_i(s) = p^*(s) \cdot x_i^*(s) + p^*(s) \cdot \frac{\epsilon(s)}{n} = p^*(s) \cdot x_i^*(s) + 0 = p^*(s) \cdot e_i(s).$$

Hence, $u_i(s, y_i(s)) = u_i(s, x_i^*(s))$ for any $i \in I$ and thus $(p^*(s), y(s))$ is a Walrasian equilibrium of the economy $\mathcal{E}(s)$ satisfying the exact feasibility, i.e., $y(s) \in W(\mathcal{E}(s))$. Therefore for any $s \in S$, $W(\mathcal{E}(s)) \neq \emptyset$.

Let W be the following set:

$$W = \{x \in L \,|\, x(s) \in W(\mathcal{E}(s)) \text{ for all } s \in S\},$$

and notice that, as observed above, W is non empty. An element of W is an ex post Walrasian equilibrium allocation and from Lemma 8.1 it is a V-REE. □

[33] If $\epsilon(s) = 0$, then x^* satisfies the exact feasibility.

Proof of Theorem 4.1: It trivially follows from Theorem 4.5 and the observation that a MREE is a particular case of V-REE (see Remark 4.5). □

Proof of Theorem 4.3: The equivalence between (1) and (2) is obtained by combing Propositions 3.4 and 3.5 (see Remark 3.4). The equivalence between (1) and (3) is instead stated in Lemma 8.2. □

8.3 Proofs of Section 5

Proof of Proposition 5.3: Let x be a maximin Pareto optimal allocation with respect to the information structure Π and assume on the contrary that there exists a feasible allocation y such that

$$u_i(s, y_i(s)) \geq u_i(s, x_i(s)) \quad \text{for all } i \in I \text{ and for all } s \in S, \text{ with at least a strict inequality.}$$

Let $j \in I$ and $\bar{s} \in S$ such that $u_j(\bar{s}, y_j(\bar{s})) > u_j(\bar{s}, x_j(\bar{s}))$. Thanks to continuity of $u_j(\bar{s}, \cdot)$ there exists $\epsilon \in (0,1)$ for which $u_j(\bar{s}, \epsilon y_j(\bar{s})) > u_j(\bar{s}, x_j(\bar{s}))$. Consider the feasible allocation z given by $z_i(s) = y_i(s)$ for any $i \in I$ and $s \in S \setminus \{\bar{s}\}$; while in $\bar{s}$

$$z_i(\bar{s}) = \begin{cases} \epsilon y_j(\bar{s}) & \text{if } i = j \\ y_i(\bar{s}) + \frac{1-\epsilon}{n-1} y_j(\bar{s}) & \text{otherwise.} \end{cases}$$

From the strict monotonicity it follows that the feasible allocation z is such that

$$\begin{aligned} u_i(s, z_i(s)) &\geq u_i(s, x_i(s)) \quad \text{for any } i \in I \text{ and } s \in S, \\ u_i(\bar{s}, z_i(\bar{s})) &> u_i(\bar{s}, x_i(\bar{s})) \quad \text{for any } i \in I. \end{aligned}$$

Let $k \in I$ be such that $\Pi_k(\bar{s}) = \{\bar{s}\}$, thus

$$\begin{aligned} \underline{u}_i^{\Pi_i}(s, z_i) &\geq \underline{u}_i^{\Pi_i}(s, x_i) \quad \text{for any } i \in I \text{ and } s \in S, \\ \underline{u}_k^{\Pi_k}(\bar{s}, z_k) = u_k(\bar{s}, z_k(\bar{s})) &> u_k(\bar{s}, x_k(\bar{s})) = \underline{u}_k^{\Pi_k}(\bar{s}, x_k). \end{aligned}$$

This means that x is not maximin efficient with respect to the information structure Π, which is a contradiction. We now show that the converse may not be true[34]. To this end consider a differential information economy with two agents $I = \{1, 2\}$, two goods and two states $S = \{a, b\}$. The primitives are as follows:

$$\begin{array}{cc} \Pi_1 = \{\{a\}, \{b\}\} & \Pi_2 = \{\{a, b\}\} \\ e_1(a) = (1, 2) & e_1(b) = (2, 1) \\ e_2(a) = (1, 1) & e_2(b) = (1, 1) \\ u_i(a, x, y) = log\sqrt{xy} & u_i(b, x, y) = log(xy) \quad \text{for any } i \in I. \end{array}$$

Notice that since the first agent is fully informed, the information structure Π satisfies the assumption that for any state s there exists an agent i such that $\Pi_i(s) = \{s\}$. The following feasible allocation is ex-post efficient

$$(x_i(a), y_i(a)) = \left(1, \frac{3}{2}\right) \quad \text{for any } i \in I$$

[34] Kreps's example can also be used to show that an ex-post efficient allocation may not be maximin Pareto optimal (see Remark 5.3).

$$(x_i(b), y_i(b)) = \left(\frac{3}{2}, 1\right) \quad \text{for any } i \in I.$$

Indeed assume on the contrary the existence of an alternative feasible allocation (t, z) such that $t_i(s)z_i(s) \geq \frac{3}{2}$ for all $i \in I$ and $s \in S$, with at least one strict inequality.

Without loss of generality let $t_1(a)z_1(a) > \frac{3}{2}$, which means that[35] $z_1(a) > \frac{3}{2t_1(a)}$. This implies that

$$(2 - t_1(a))\left(3 - \frac{3}{2t_1(a)}\right) > (2 - t_1(a))(3 - z_1(a)) = t_2(a)z_2(a) \geq \frac{3}{2},$$

which implies the contradiction $(t_1(a) - 1)^2 < 0$. Hence (x, y) is ex-post Pareto optimal. We now show that it is not maximin efficient with respect to the information structure Π. To this end consider the following feasible allocation

$$\begin{aligned}
(t_i(a), z_i(a)) &= (x_i(a), y_i(a)) \quad \text{for any } i \in I, \\
(t_1(b), z_1(b)) &= \left(\frac{7}{4}, 1\right) \\
(t_2(b), z_2(b)) &= \left(\frac{5}{4}, 1\right),
\end{aligned}$$

and notice that

$$\begin{aligned}
\underline{u}_1^{\Pi_1}(a, t_1, z_1) &= \underline{u}_1^{\Pi_1}(a, x_1, y_1) \\
\underline{u}_1^{\Pi_1}(b, t_1, z_1) &= log\frac{7}{4} > log\frac{3}{2} = \underline{u}_1^{\Pi_1}(b, x_1, y_1) \\
\underline{u}_2^{\Pi_2}(a, t_2, z_2) &= \underline{u}_2^{\Pi_2}(b, t_2, z_2) = \min\left\{log\sqrt{\frac{3}{2}}, log\frac{5}{4}\right\} = log\sqrt{\frac{3}{2}} \\
&= \min\left\{log\sqrt{\frac{3}{2}}, log\frac{3}{2}\right\} = \underline{u}_2^{\Pi_2}(b, x_2, y_2) = \underline{u}_2^{\Pi_2}(a, x_2, y_2).
\end{aligned}$$

Thus the allocation (x, y) is ex-post efficient but not maximin Pareto optimal with respet to the information structure Π. □

The next example shows that the assumption that for any state s there exists an agent $i \in I$ such that $\Pi_i(s) = \{s\}$ is crucial in the proof of Proposition 5.3.

Example 8.1 Consider a differential information economy with two agents $I = \{1, 2\}$, two goods and three states $S = \{a, b, c\}$, whose primitives are given as follows:

$\Pi_1 = \{\{a\}, \{b, c\}\}$	$\Pi_2 = \{\{a, b\}, \{c\}\}$
$e_1(a) = (1, 1)$	$e_2(a) = (2, 2)$
$e_1(b) = (2, 2)$	$e_2(b) = (2, 2)$
$e_1(c) = (2, 2)$	$e_2(c) = (1, 1)$

[35]Notice that $(t_i(s), z_i(s)) \gg 0$ for any $i \in I$ and any $s \in S$.

$$u_i(a,x,y) = log(xy) \; u_i(b,x,y) = \sqrt{xy} \; u_i(c,x,y) = xy \quad \text{for any } i \in I.$$

Notice that $\Pi_i(b) \setminus \{b\} \neq \emptyset$ for any $i \in I$. The following feasible allocation

$$\begin{array}{lll} (x_1(a), y_1(a)) = \left(\frac{3}{2}, \frac{2}{3}\right) & (x_1(b), y_1(b)) = (2,2) & (x_1(c), y_1(c))) = \left(\frac{3}{2}, \frac{7}{3}\right) \\ (x_2(a), y_2(a)) = \left(\frac{3}{2}, \frac{7}{3}\right) & (x_2(b), y_2(b)) = (2,2) & (x_2(c), y_2(c)) = \left(\frac{3}{2}, \frac{2}{3}\right) \end{array}$$

is not ex-post efficient since it is blocked by the initial endowment, but it is maximin Pareto optimal with respect to the information structure (Π_1, Π_2). Indeed, assume by the way of contradiction the existence of an alternative feasible allocation (t, z) such that $\underline{u}_i^{\Pi_i}(s, t_i, z_i) \geq \underline{u}_i^{\Pi_i}(s, x_i, y_i)$ for any i and any s with at least a strict inequality. This means in particular that

$$\begin{aligned} t_1(c)z_1(c) \geq \min\{\sqrt{t_1(b)z_1(b)}, t_1(c)z_1(c)\} &\geq \min\left\{2, \frac{7}{2}\right\} = 2 \\ (3 - t_1(c))(3 - z_1(c)) &\geq 1, \end{aligned}$$

which implies the inequality $3z_1^2(c) - 10z_1(c) + 6 \leq 0$ with no solution.

Proof of Theorem 5.4: Let (p, x) be a maximin rational expectations equilibrium.

I CASE: If $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for each $i \in I$, Lemma 8.2 ensures that x is an ex-post Walrasian equilibrium allocation and therefore it is ex-post efficient. We now want to show that it is also maximin Pareto optimal. To this end, assume on the contrary that there exists an alternative allocation y such that

$$\begin{array}{ll} (i) & \underline{u}_i^{REE}(s, y_i) \geq \underline{u}_i^{REE}(s, x_i) \quad for\ all\ i \in I\ and\ for\ all\ s \in S, \\ & with\ at\ least\ a\ strict\ inequality. \\ (ii) & \sum_{i \in I} y_i(s) = \sum_{i \in I} e_i(s) \quad for\ all\ s \in S. \end{array}$$

Proposition 3.7 implies that for any agent $i \in I$ and any state $s \in S$

$$u_i(s, y_i(s)) \geq \underline{u}_i^{REE}(s, y_i) \geq \underline{u}_i^{REE}(s, x_i) = u_i(s, x_i(s)),$$

with at least a strict inequality. This means that x is not ex-post efficient and from Lemma 8.2 we get a contradiction.

II CASE: Assume that p is fully revealing. Clearly since $\mathcal{G}_i(s) = \{s\}$ for all i and s, maximin Pareto optimality with respect to the information structure $\mathcal{G}$ coincides with the ex-post efficiency. We have already observed that in this case a maximin REE is an ex-post Walrasian equilibrium and hence it is both ex-post and maximin efficient.

Example 8.2 and Remark 5.3 show that if none of the above conditions is satisfied, a maximin REE may not be maximin efficient. □

Proof of Proposition 5.7: Let x be a weak maximin efficient allocation and assume, on the contrary, that there exists an alternative allocation y such that

$$
\begin{array}{ll}
(i) & u_i(s, y_i(s)) > u_i(s, x_i(s)) \quad \text{for all } i \in I \text{ and for all } s \in S \\
(ii) & \sum_{i \in I} y_i(s) = \sum_{i \in I} e_i(s) \quad \text{for all } s \in S.
\end{array}
$$

Thus, for each agent $i \in I$ whatever his information partition is Π_i, it follows from (i) above that $\underline{u}_i^{\Pi_i}(s, y_i) > \underline{u}_i^{\Pi_i}(s, x_i)$ for each state s. Hence, a contradiction since x is weak maximin Pareto optimal. In order to show that the converse may not be true, consider an economy with two agents, three states of nature, $S = \{a, b, c\}$, and two goods, such that

$$
\begin{aligned}
u_i(a, x_i, y_i) &= \sqrt{x_i y_i} \quad u_i(b, x_i, y_i) = log(x_i y_i) \quad u_i(c, x_i, y_i) = x_i^2 y_i \quad \text{for all } i = 1, 2. \\
e_1(a) &= (2, 1) \quad e_2(a) = e_1(b) = e_2(b) = e_1(c) = e_2(c) = (1, 2) \\
\Pi_1 &= \{\{a, c\}, \{b\}\} \quad \Pi_2 = \{\{a\}, \{b, c\}\}.
\end{aligned}
$$

Consider the following feasible allocation:

$$
\begin{aligned}
(x_1(a), y_1(a)) &= \left(3, \frac{1}{3}\right) \quad (x_2(a), y_2(a)) = \left(0, \frac{8}{3}\right) \\
(x_1(b), y_1(b)) &= (1, 2) \quad (x_2(b), y_2(b)) = (1, 2)\,, \\
(x_1(c), y_1(c)) &= (2, 1) \quad (x_2(c), y_2(c)) = (0, 3)\,.
\end{aligned}
$$

Notice that it is weak ex post efficient, since if on the contrary there exists (t, z) such that

$$
u_i(s, t_i(s), z_i(s)) > u_i(s, x_i(s), y_i(s)) \quad \text{for all } i \in I \text{ and all } s \in S,
$$

in particular,

$$
\begin{cases}
log(t_1(b) z_1(b) > log 2 \\
log(t_2(b) z_2(b)) > log 2 \\
t_1(b) + t_2(b) = 2 \\
z_1(b) + z_2(b) = 4,
\end{cases}
$$

then[36]

$$
\begin{cases}
z_1(b) > \frac{2}{t_1(b)} \\
(2 - t_1(b))(2t_1(b) - 1) > t_1(b).
\end{cases}
$$

This implies that $(t_1(b) - 1)^2 < 0$, which is impossible. Thus, the above allocation is weak ex post Pareto optimal, but it is not weak maximin efficient with respect to the information structure Π, since it is (maximin) blocked by the following feasible allocation:

[36] Clearly, $(t_i(b), z_i(b)) \gg (0, 0)$ for each $i = 1, 2$.

$$
\begin{aligned}
(t_1(a), z_1(a)) &= \left(\frac{5}{4}, \frac{5}{2}\right) \quad (t_2(a), z_2(a)) = \left(\frac{7}{4}, \frac{1}{2}\right) \\
(t_1(b), z_1(b)) &= \left(1, \frac{8}{3}\right) \quad (t_2(b), z_2(b)) = \left(1, \frac{4}{3}\right) \\
(t_1(c), z_1(c)) &= \left(\frac{3}{4}, 2\right) \quad (t_2(c), z_2(c)) = \left(\frac{5}{4}, 2\right).
\end{aligned}
$$

Indeed,

$$
\begin{aligned}
\underline{u}_1^{\Pi_1}(a, t_1, z_1) &= \underline{u}_1^{\Pi_1}(c, t_1, z_1) = \min\left\{\sqrt{\frac{25}{8}}, \frac{9}{8}\right\} = \frac{9}{8} \\
&> 1 = \min\{1, 4\} = \underline{u}_1^{\Pi_1}(c, x_1, y_1) = \underline{u}_1^{\Pi_1}(a, x_1, y_1) \\
\underline{u}_1^{\Pi_1}(b, t_1, z_1) &= u_1(b, t_1(b), z_1(b)) = log\frac{8}{3} \\
&> log2 = u_1(b, x_1(b), y_1(b)) = \underline{u}_1^{\Pi_1}(b, x_1, y_1) \\
\underline{u}_2^{\Pi_2}(a, t_2, z_2) &= u_2(a, t_2(a), z_2(a)) = \sqrt{\frac{7}{8}} \\
&> 0 = u_2(a, x_2(a), y_2(a)) = \underline{u}_2^{\Pi_2}(a, x_2, y_2) \\
\underline{u}_2^{\Pi_2}(b, t_2, z_2) &= \underline{u}_2^{\Pi_2}(c, t_2, z_2) = \min\left\{log\frac{4}{3}, \frac{25}{8}\right\} = log\frac{4}{3} \\
&> 0 = \min\{log2, 0\} = \underline{u}_2^{\Pi_2}(c, x_2, y_2) = \underline{u}_2^{\Pi_2}(b, x_2, y_2).
\end{aligned}
$$

□

Proof of Theorem 5.8: Clearly in the first two cases the result easily follows from Theorem 5.4 and from the observation that any allocation maximin efficient with respect to Π is weak maximin Pareto optimal with respect to Π.

Let (p, x) be a maximin rational expectations equilibrium, and assume on the contrary that there exists an alternative allocation $y \in L$ such that

$$
\begin{aligned}
&(i) \qquad \underline{u}_i^{REE}(s, y_i) > \underline{u}_i^{REE}(s, x_i) \quad \text{for all } i \in I \text{ and for all } s \in S, \\
&(ii) \qquad \sum_{i \in I} y_i(s) = \sum_{i \in I} e_i(s) \quad \text{for all } s \in S.
\end{aligned}
$$

(iii) CASE: there exists a state of nature $\bar{s} \in S$, such that $\{\bar{s}\} = \mathcal{G}_i(\bar{s})$ for all $i \in I$.

Since for each $i \in I$, $\{\bar{s}\} = \mathcal{G}_i(\bar{s})$; from (i) it follows that $\underline{u}_i^{REE}(\bar{s}, y_i) = u_i(\bar{s}, y_i(\bar{s})) > u_i(\bar{s}, x_i(\bar{s})) = \underline{u}_i^{REE}(\bar{s}, x_i)$ for all $i \in I$. Hence, since (p, x) is a MREE, for each agent i there exists at least one state $s_i \in \mathcal{G}_i(\bar{s}) = \{\bar{s}\}$ (that is $s_i = \bar{s}$ for all $i \in I$) such that $p(\bar{s}) \cdot y_i(\bar{s}) > p(\bar{s}) \cdot e_i(\bar{s})$. Therefore,

$$
\sum_{i \in I} p(\bar{s})[y_i(\bar{s}) - e_i(\bar{s})] > 0,
$$

which contradicts (ii).

(iv) CASE: $n-1$ agents are fully informed.

Since (p,x) is a MREE, from (i) it follows that for any state $s \in S$ and any agent $i \in I$ there exists at least one state $s_i \in \mathcal{G}_i(s)$ such that $p(s_i) \cdot y_i(s_i) > p(s_i) \cdot e_i(s_i)$. Let j be the unique not fully informed agent, and consider the state s_j for which $p(s_j) \cdot y_j(s_j) > p(s_j) \cdot e_j(s_j)$. Since each agent $i \neq j$ is fully informed, it follows that $\mathcal{G}_i(s_j) = \{s_j\}$ for all $i \neq j$. Thus,

$$p(s_j) \cdot y_i(s_j) > p(s_j) \cdot e_i(s_j) \quad \text{for all } i \in I.$$

Hence,

$$\sum_{i \in I} p(s_j) \cdot y_i(s_j) > \sum_{i \in I} p(s_j) \cdot e_i(s_j),$$

which is a contradiction.

Example 8.2 below shows that if no condition of Theorem 5.8 is satisfied, then a maximin REE may not be weak maximin efficient (and a fortiori it may not be maximin Pareto optimal). □

Example 8.2 Consider a differential information economy with three states of nature, $S = \{a, b, c\}$, two goods, $\ell = 2$ (the first good is considered as numerarie) and three agents, $I = \{1, 2, 3\}$ whose characteristics are given as follows:

$$\begin{array}{ccc}
e_1(a) = e_1(b) = (2,1) & e_1(c) = (3,1) & \mathcal{F}_1 = \{\{a,b\};\{c\}\} \\
e_2(a) = e_2(c) = (1,2) & e_2(b) = (2,2) & \mathcal{F}_2 = \{\{a,c\};\{b\}\} \\
e_3(b) = e_3(c) = (2,1) & e_3(a) = (3,1) & \mathcal{F}_3 = \{\{a\};\{b,c\}\}. \\
u_1(a,x,y) = \sqrt{xy} & u_1(b,x,y) = log(xy) & u_1(c,x,y) = \sqrt{xy}, \\
u_2(a,x,y) = log(xy) & u_2(b,x,y) = \sqrt{xy} & u_2(c,x,y) = \sqrt{xy}, \\
u_3(a,x,y) = \sqrt{xy} & u_3(b,x,y) = \sqrt{xy} & u_3(c,x,y) = log(xy).
\end{array}$$

Consider the following maximin rational expectations equilibrium

$$\begin{array}{cccc}
(p(a),q(a)) = \left(1,\frac{3}{2}\right) & (x_1(a),y_1(a)) = \left(\frac{7}{4},\frac{7}{6}\right) & (x_2(a),y_2(a)) = \left(2,\frac{4}{3}\right) & (x_3(a),y_3(a)) = \left(\frac{9}{4},\frac{3}{2}\right) \\
(p(b),q(b)) = \left(1,\frac{3}{2}\right) & (x_1(b),y_1(b)) = \left(\frac{7}{4},\frac{7}{6}\right) & (x_2(b),y_2(b)) = \left(\frac{5}{2},\frac{5}{3}\right) & (x_3(b),y_3(b)) = \left(\frac{7}{4},\frac{7}{6}\right) \\
(p(c),q(c)) = \left(1,\frac{3}{2}\right) & (x_1(c),y_1(c)) = \left(\frac{9}{4},\frac{3}{2}\right) & (x_2(c),y_2(c)) = \left(2,\frac{4}{3}\right) & (x_3(c),y_3(c)) = \left(\frac{7}{4},\frac{7}{6}\right),
\end{array}$$

and notice that it is a non revealing equilibrium, since $(p(a),q(a)) = (p(b),q(b)) = (p(c),q(c))$ and hence $\sigma(p,q) = \{\{a,b,c\}\}$, that is $\mathcal{G}_i = \mathcal{F}_i$ for any $i \in I$. Moreover, notice that no condition of Theorems 5.4 and 5.8 is satisfied. We now show that the equilibrium allocation is not weak maximin Pareto optimal with respect to the information structure $\mathcal{G} = (\mathcal{G}_i)_{i \in I}$ and a fortiori it is neither maximin efficient. Indeed, consider the following feasible allocation

$$
\begin{aligned}
(t_1(a), z_1(a)) &= \left(\frac{20}{12}, \frac{13}{12}\right) \quad (t_2(a), z_2(a)) = \left(\frac{25}{12}, \frac{16}{12}\right) \quad (t_3(a), z_3(a)) = \left(\frac{27}{12}, \frac{19}{12}\right) \\
(t_1(b), z_1(b)) &= \left(\frac{22}{12}, \frac{14}{12}\right) \quad (t_2(b), z_2(b)) = \left(\frac{30}{12}, \frac{21}{12}\right) \quad (t_3(b), z_3(b)) = \left(\frac{20}{12}, \frac{13}{12}\right) \\
(t_1(c), z_1(c)) &= \left(\frac{28}{12}, \frac{18}{12}\right) \quad (t_2(c), z_2(c)) = \left(\frac{23}{12}, \frac{15}{12}\right) \quad (t_3(c), z_3(c)) = \left(\frac{21}{12}, \frac{15}{12}\right),
\end{aligned}
$$

and notice that,

$$
\begin{aligned}
\underline{u}_1^{REE}(a, t_1, z_1) &= \underline{u}_1^{REE}(b, t_1, z_1) = \min\left\{\sqrt{\frac{260}{144}}, log\frac{308}{144}\right\} = log\frac{308}{144} > log\frac{49}{24} = \\
\min\left\{\sqrt{\frac{49}{24}}, log\frac{49}{24}\right\} &= \underline{u}_1^{REE}(a, x_1, y_1) = \underline{u}_1^{REE}(b, x_1, y_1),
\end{aligned}
$$

$$
\underline{u}_1^{REE}(c, t_1, z_1) = u_1(c, t_1(c), z_1(c)) = \sqrt{\frac{504}{144}} > \sqrt{\frac{27}{8}} = u_1(c, x_1(c), y_1(c)) = \underline{u}_1^{REE}(c, x_1, y_1),
$$

$$
\begin{aligned}
\underline{u}_2^{REE}(a, t_2, z_2) &= \underline{u}_2^{REE}(c, t_2, z_2) = \min\left\{log\frac{400}{144}, \sqrt{\frac{345}{144}}\right\} = log\frac{400}{144} > log\frac{8}{3} = \\
\min\left\{log\frac{8}{3}, \sqrt{\frac{8}{3}}\right\} &= \underline{u}_2^{REE}(a, x_2, y_2) = \underline{u}_2^{REE}(c, x_2, y_2),
\end{aligned}
$$

$$
\underline{u}_2^{REE}(b, t_2, z_2) = u_2(b, t_2(b), z_2(b)) = \sqrt{\frac{630}{144}} > \sqrt{\frac{25}{6}} = u_2(b, x_2(b), y_2(b)) = \underline{u}_2^{REE}(b, x_2, y_2),
$$

$$
\underline{u}_3^{REE}(a, t_3, z_3) = u_3(a, t_3(a), z_3(a)) = \sqrt{\frac{513}{144}} > \sqrt{\frac{27}{8}} = u_3(a, x_3(a), y_3(a)) = \underline{u}_3^{REE}(a, x_3, y_3),
$$

$$
\begin{aligned}
\underline{u}_3^{REE}(b, t_3, z_3) &= \underline{u}_3^{REE}(c, t_3, z_3) = \min\left\{\sqrt{\frac{260}{144}}, log\frac{315}{144}\right\} = log\frac{315}{144} > log\frac{49}{24} = \\
\min\left\{\sqrt{\frac{49}{24}}, log\frac{49}{24}\right\} &= \underline{u}_3^{REE}(b, x_3, y_3) = \underline{u}_3^{REE}(c, x_3, y_3).
\end{aligned}
$$

Hence, the equilibrium allocation (x, y) is not weak maximin Pareto optimal with respect to the information structure $\mathcal{G} = (\mathcal{G}_i)_{i \in I}$.

The following example shows that if there exists a state that everybody may distinguish (see condition (iii) of Theorem 5.8) then according to Theorem 5.8, a maximin REE allocation is weak maximin efficient with respect to the information structure $(\mathcal{G}_i)_{i\in I}$, but it is not maximin Pareto optimal.

Example 8.3 Consider a differential information economy with five states of nature, $S = \{a, b, c, d, f\}$, two goods and two agents, $I = \{1, 2\}$ whose characteristics are given as follows:

$$\begin{array}{ccc} e_1(a) = e_1(b) = (1,2) & e_1(c) = e_1(d) = e_1(f) = (2,1) & \mathcal{F}_1 = \{\{a,b\};\{c,d\};\{f\}\} \\ e_2(a) = e_2(c) = e_2(d) = e_2(f) = (2,1) & e_2(b) = (1,2) & \mathcal{F}_2 = \{\{a,c\};\{b\};\{d,f\}\}. \\ u_i(a,x,y) = u_i(c,x,y) = \sqrt{xy} & u_i(b,x,y) = u_i(d,x,y) = log(xy) & u_i(f,x,y) = xy. \end{array}$$

Consider the following maximin rational expectations equilibrium

$$\begin{array}{ccc} (p(a), q(a)) = (1,1) & (x_1(a), y_1(a)) = \left(\frac{3}{2}, \frac{3}{2}\right) & (x_2(a), y_2(a)) = \left(\frac{3}{2}, \frac{3}{2}\right) \\ (p(b), q(b)) = \left(1, \frac{1}{2}\right) & (x_1(b), y_1(b)) = (1,2) & (x_2(b), y_2(b)) = (1,2) \\ (p(c), q(c)) = (1,2) & (x_1(c), y_1(c)) = (2,1) & (x_2(c), y_2(c)) = (2,1) \\ (p(d), q(d)) = (1,2) & (x_1(d), y_1(d)) = (2,1) & (x_2(d), y_2(d)) = (2,1) \\ (p(f), q(f)) = (1,2) & (x_1(f), y_1(f)) = (2,1) & (x_2(f), y_2(f)) = (2,1)\,, \end{array}$$

and notice that $\sigma(p,q) = \{\{a\}, \{b\}, \{c, d, f\}\}$ and hence, $\mathcal{G}_1 = \{\{a\}, \{b\}, \{c, d\}, \{f\}\}$ and $\mathcal{G}_2 = \{\{a\}, \{b\}, \{c\}, \{d, f\}\}$.

For any $i \in I$ the equilibrium allocation (x_i, y_i) is $\mathcal{G}_i$-measurable but not $\mathcal{F}_i$-measurable. Moreover notice that the utility functions are not $\mathcal{F}_i$-measurable neither $\mathcal{G}_i$-measurable, the equilibrium price is not fully revealing, and no agent is fully informed. On the other hand, there exists a state s such that $\mathcal{G}_i(s) = \{s\}$ for any agent i, for example states a and b, but such a condition does not hold for the initial information structure $(\mathcal{F}_i)_{i\in I}$. Thus only condition (iii) of Theorem 5.8 is satisfied. From this it follows that the equilibrium allocation (x, y) is weak efficient with respect to the information structure $(\mathcal{G}_i)_{i\in I}$. We now show that x is not maximin efficient with respect to the information structure $(\mathcal{G}_i)_{i\in I}$. To this end, consider the following feasible allocation

$$\begin{aligned} (t_i(s), z_i(s)) &= (x_i(s), y_i(s)) \quad \text{for any } i = \{1,2\} \text{ and any } s \in \{a, b, d\} \\ (t_1(c), z_1(c)) &= \left(\frac{3}{2}, 1\right) \quad (t_2(c), z_2(c)) = \left(\frac{5}{2}, 1\right) \\ (t_1(f), z_1(f)) &= \left(\frac{5}{2}, 1\right) \quad (t_2(f), z_2(f)) = \left(\frac{3}{2}, 1\right), \end{aligned}$$

and notice that,

$$\begin{aligned}
\underline{u}_i^{REE}(s,t_i,z_i) &= \underline{u}_i^{REE}(s,x_i,y_i) \quad \text{for any } i\in\{1,2\} \text{ and any } s\in\{a,b\} \\
\underline{u}_1^{REE}(c,t_1,z_1) &= \underline{u}_1^{REE}(d,t_1,z_1) = \min\left\{\sqrt{\frac{3}{2}}, log2\right\} = log2 \\
&= \min\{\sqrt{2}, log2\} = \underline{u}_1^{REE}(d,x_1,y_1) = \underline{u}_1^{REE}(c,x_1,y_1) \\
\underline{u}_2^{REE}(c,t_2,z_2) &= u_2(c,t_2(c),z_2(c)) = \sqrt{\frac{5}{2}} \\
&> \sqrt{2} = u_2(c,x_2(c),y_2(c)) = \underline{u}_2^{REE}(c,x_2,y_2) \\
\underline{u}_1^{REE}(f,t_1,z_1) &= u_1(f,t_1(f),z_1(f)) = \frac{5}{2} \\
&> 2 = u_1(f,x_1(f),y_1(f)) = \underline{u}_1^{REE}(f,x_1,y_1) \\
\underline{u}_2^{REE}(d,t_2,z_2) &= \underline{u}_2^{REE}(f,t_2,z_2) = \min\left\{log2, \frac{3}{2}\right\} = log2 \\
&= \min\{log2, 2\} = \underline{u}_2^{REE}(f,x_2,y_2) = \underline{u}_2^{REE}(d,x_2,y_2).
\end{aligned}$$

Hence, the equilibrium allocation is not maximin Pareto optimal with respect to the information structure $\mathcal{G} = (\mathcal{G}_i)_{i\in I}$.

The next example shows that if all agents except one are fully informed (i.e., condition (iv) of Theorem 5.8 holds), then a maximin REE allocation is weak maximin efficient with respect to the information structure $(\mathcal{G}_i)_{i\in I}$ but it may not be maximin Pareto optimal.

Example 8.4 Consider a differential information economy with two states of nature, $S = \{a,b\}$, two goods and three agents, $I = \{1,2,3\}$ whose characteristics are given as follows:

$$\begin{array}{cc}
e_1(a) = e_1(b) = \left(\frac{1}{3}, \frac{1}{3}\right) & \mathcal{F}_1 = \{\{a\};\{b\}\} \\
e_2(a) = e_2(b) = \left(\frac{1}{3}, \frac{1}{3}\right) & \mathcal{F}_2 = \{\{a\};\{b\}\}. \\
e_3(a) = e_3(b) = \left(\frac{1}{3}, \frac{1}{3}\right) & \mathcal{F}_3 = \{\{a,b\}\}. \\
u_i(a,x,y) = \sqrt{xy} & u_i(b,x,y) = log(xy) \text{ for all } i\in I.
\end{array}$$

Notice that for any $i \in I$ $e_i(\cdot)$ is $\mathcal{F}_i$-measurable, while u_i is not. Two agents are fully informed. The initial endowment is a non-revealing maximin rational expectations equilibrium and there does not exist a state s such that $\mathcal{G}_i(s) = \{s\}$ for any i, neither $\mathcal{F}_i(s) = \{s\}$ for any i. Thus, only condition (iv) of Theorem 5.8 is satisfied. From this it follows that the equilibrium allocation e is weak efficient with respect to the information structure $(\mathcal{G}_i)_{i\in I}$, and since it is a non-revealing maximin REE it is also weak efficient with respect to the information structure $(\mathcal{F}_i)_{i\in I}$ (because $\mathcal{G}_i = \mathcal{F}_i$ for any $i \in I$). We now show that e is not maximin efficient with respect to the information structure $(\mathcal{G}_i)_{i\in I}$ and hence neither with respect to $(\mathcal{F}_i)_{i\in I}$. To this end, consider the following feasible allocation

$$
\begin{aligned}
(t_i(a), z_i(a)) &= \left(\frac{5}{12}, \frac{5}{12}\right) \quad \text{for any } i \in \{1,2\}, \\
(t_3(a), z_3(a)) &= \left(\frac{1}{6}, \frac{1}{6}\right), \\
(t_i(b), z_i(b)) &= \left(\frac{1}{3}, \frac{1}{3}\right) \quad \text{for any } i \in \{1,2,3\}.
\end{aligned}
$$

Notice that,

$$
\begin{aligned}
\underline{u}_i^{REE}(a, t_i, z_i) &= \frac{5}{12} > \frac{1}{3} = \underline{u}_i^{REE}(a, x_i, y_i) \quad \text{for any } i \in \{1,2\} \\
\underline{u}_i^{REE}(b, t_i, z_i) &= \underline{u}_i^{REE}(b, x_i, y_i) \quad \text{for any } i \in \{1,2\} \\
\underline{u}_3^{REE}(a, t_3, z_3) &= \underline{u}_3^{REE}(b, t_3, z_3) = \min\left\{\frac{1}{6}, log\frac{1}{9}\right\} = log\frac{1}{9} \\
&= \min\left\{\frac{1}{3}, log\frac{1}{9}\right\} = \underline{u}_3^{REE}(a, x_3, y_3) = \underline{u}_3^{REE}(b, x_3, y_3).
\end{aligned}
$$

Hence, the equilibrium allocation e is not maximin Pareto optimal with respect to the information structure $\mathcal{G} = (\mathcal{G}_i)_{i \in I}$ neither with respect to $\mathcal{F} = (\mathcal{F}_i)_{i \in I}$.

Remark 8.4 The above Examples 8.3 and 8.4 also show that a weak maximin efficient allocation may not be maximin Pareto optimal.

Proof of Theorem 5.10: Let (p, x) be a maximin rational expectations equilibrium and assume on the contrary that there exists an alternative feasible allocation y such that

$$
\underline{u}_i(s, y_i) > \underline{u}_i(s, x_i) \quad \text{for all } i \in I \text{ and } s \in S. \tag{21}
$$

(a) CASE: If there exists a state of nature $\bar{s} \in S$, such that $\{\bar{s}\} = \mathcal{F}_i(\bar{s})$ for all $i \in I$, then in particular from (21) it follows that for all $i \in I$

$$
\underline{u}_i^{REE}(\bar{s}, y_i) = u_i(\bar{s}, y_i(\bar{s})) = \underline{u}_i(\bar{s}, y_i) > \underline{u}_i(\bar{s}, x_i) = u_i(\bar{s}, x_i(\bar{s})) = \underline{u}_i^{REE}(\bar{s}, x_i).
$$

Thus, since (p, x) is a maximin rational expectations equilibrium for all $i \in I$ there exists a state $s_i \in \mathcal{G}_i(\bar{s}) = \{\bar{s}\}$ (i.e., $s_i = \bar{s}$ for all $i \in I$) such that $p(s_i) \cdot y_i(s_i) > p(s_i) \cdot e_i(s_i)$, that is

$$
p(\bar{s}) \cdot y_i(\bar{s}) > p(\bar{s}) \cdot e_i(\bar{s}) \quad \text{for all } i \in I.
$$

Hence

$$
p(\bar{s}) \cdot \sum_{i \in I} [y_i(\bar{s}) - e_i(\bar{s})] > 0,
$$

which contradicts the feasibility of the allocation y. Thus x is weak maximin efficient with respect to the information structure $\mathcal{F}$. Moreover, notice that if there is a state of nature $\bar{s}$ such that $\mathcal{F}_i(\bar{s}) = \{\bar{s}\}$ for all $i \in I$, then a fortiori $\mathcal{G}_i(\bar{s}) = \{\bar{s}\}$ for all $i \in I$.

This means that condition (iii) of Theorem 5.8 is satisfied and hence x is maximin Pareto optimal also with respect to the information structure $\mathcal{G}$.

(b) CASE: If the $n-1$ agents are fully informed, condition (iv) of Theorem 5.8 holds and hence x is weak maximin efficient with respect to the information structure $\mathcal{G}$. We want to show that x is maximin Pareto optimal also with respect to the information structure $\mathcal{F}$. To this end, assume without loss of generality that 1 is the unique non fully informed agent and let s be a state of nature. From (21) it follows in particular that there exists $\bar{s} \in \mathcal{F}_1(s)$ such that

$$\underline{u}_1^{REE}(\bar{s}, y_1) \geq \underline{u}_1(\bar{s}, y_1) > \underline{u}_1(\bar{s}, x_1) = \underline{u}_1^{REE}(\bar{s}, x_1).$$

Since (p, x) is a maximin REE, there exists a state $s' \in \mathcal{G}_1(\bar{s})$ such that

$$p(s') \cdot y_1(s') > p(s') \cdot e_1(s'). \tag{22}$$

Any agent $i \neq 1$ is fully informed, then (21) implies that

$$\underline{u}_i^{REE}(s', y_i) = u_i(s', y_i(s')) = \underline{u}_i(s', y_i) > \underline{u}_i(s', x_i) = u_i(s', x_i(s')) = \underline{u}_i^{REE}(s', x_i).$$

Thus, for all $i \neq 1$ there exists a state $s_i \in \mathcal{G}_i(s') = \{s'\}$ (i.e., $s_i = s'$ for all $i \neq 1$, because they are all fully informed) such that $p(s_i) \cdot y_i(s_i) > p(s_i) \cdot e_i(s_i)$, that is

$$p(s') \cdot y_i(s') > p(s') \cdot e_i(s') \quad \text{for all } i \neq 1. \tag{23}$$

Hence from (22) and (23) it follows that

$$p(s') \cdot \sum_{i \in I} [y_i(s') - e_i(s')] > 0,$$

which contradicts the feasibility of the allocation y.

We now show that if none of the above conditions is satisfied, then a maximin REE may not be weak maximin efficient with respect to the information structure $\mathcal{F} = (\mathcal{F}_i)_{i \in I}$ and a fortiori maximin Pareto optimal. To this end, consider a differential information economy with two states of nature, $S = \{a, b\}$, two goods, $\ell = 2$ (the first good is considered as numerarie) and three agents, $I = \{1, 2, 3\}$ whose characteristics are given as follows:

$$\begin{array}{llll}
e_1(a) = (2,1) & e_1(b) = (1,2) & \mathcal{F}_1 = \{\{a\}; \{b\}\} & \\
e_2(a) = (1,2) & e_2(b) = (1,2) & \mathcal{F}_2 = \{\{a, b\}\} & \\
e_3(a) = (2,1) & e_3(b) = (2,1) & \mathcal{F}_3 = \{\{a, b\}\}. & \\
u_1(s, x, y) = x^2 y & u_2(s, x, y) = \sqrt{xy} & u_3(s, x, y) = log(xy) & \text{for any } s \in S.
\end{array}$$

Notice that agents' initial endowments and utility functions are private information measurable. Consider the following fully revealing maximin rational expectations equilibrium

$$(p(a), q(a)) = (1, 1) \quad (x_1(a), y_1(a)) = (2, 1) \quad (x_2(a), y_2(a)) = \left(\tfrac{3}{2}, \tfrac{3}{2}\right) \quad (x_3(a), y_3(a)) = \left(\tfrac{3}{2}, \tfrac{3}{2}\right)$$

$$(p(b), q(b)) = \left(1, \tfrac{11}{17}\right) \quad (x_1(b), y_1(b)) = \left(\tfrac{26}{17}, \tfrac{13}{11}\right) \quad (x_2(b), y_2(b)) = \left(\tfrac{39}{34}, \tfrac{39}{22}\right) \quad (x_3(b), y_3(b)) = \left(\tfrac{45}{34}, \tfrac{45}{22}\right).$$

The above fully revaling maximin REE is maximin efficient (and a fortiori weak maximin Pareto otpimal) with respect to the information structure $\mathcal{G} = (\mathcal{G}_i)_{i\in I}$ (see Theorem 5.4). Of course it is also ex post efficient since it coincides with an ex post Walrasian equilibrium. On the other hand, we now show that it is not weak maximin efficient (and a fortiori it is not maximin Pareto optimal) with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_i)_{i\in I}$. To this end, consider the following feasible allocation (t, z)

$$(t_1(a), z_1(a)) = \left(\tfrac{33}{16}, 1\right) \quad (t_1(b), z_1(b)) = \left(\tfrac{105}{68}, \tfrac{13}{11}\right)$$

$$(t_2(a), z_2(a)) = \left(\tfrac{22}{16}, \tfrac{3}{2}\right) \quad (t_2(b), z_2(b)) = \left(\tfrac{79}{68}, \tfrac{39}{22}\right)$$

$$(t_3(a), z_3(a)) = \left(\tfrac{25}{16}, \tfrac{3}{2}\right) \quad (t_3(b), z_3(b)) = \left(\tfrac{88}{68}, \tfrac{45}{22}\right),$$

and notice that,

$$\begin{aligned}
\underline{u}_1(a, t_1, z_1) &= u_1(a, t_1(a), z_1(a)) = \left(\frac{33}{16}\right)^2 > 4 = u_1(a, x_1(a), y_1(a)) = \underline{u}_1(a, x_1, y_1) \\
\underline{u}_1(b, t_1, z_1) &= u_1(b, t_1(b), z_1(b)) = \left(\frac{105}{68}\right)^2 \frac{13}{11} > \left(\frac{26}{17}\right)^2 \frac{13}{11} = u_1(b, x_1(b), y_1(b)) = \underline{u}_1(b, x_1, y_1) \\
\underline{u}_2(a, t_2, z_2) &= \underline{u}_2(b, t_2, z_2) = \min\left\{\sqrt{\frac{22}{16}\frac{3}{2}}, \sqrt{\frac{79}{68}\frac{39}{22}}\right\} = \sqrt{\frac{79}{68}\frac{39}{22}} > \sqrt{\frac{39}{34}\frac{39}{22}} \\
&= \min\left\{\frac{3}{2}, \sqrt{\frac{39}{34}\frac{39}{22}}\right\} = \underline{u}_2(b, x_2, y_2) = \underline{u}_2(a, x_2, y_2) \\
\underline{u}_3(a, t_3, z_3) &= \underline{u}_3(b, t_3, z_3) = \min\left\{log\left(\frac{25}{16}\frac{3}{2}\right), log\left(\frac{88}{68}\frac{45}{22}\right)\right\} = log\left(\frac{25}{16}\frac{3}{2}\right) > log\frac{9}{4} \\
&= \min\left\{log\frac{9}{4}, log\left(\frac{45}{34}\frac{45}{22}\right)\right\} = \underline{u}_3(b, x_3, y_3) = \underline{u}_3(a, x_3, y_3).
\end{aligned}$$

Hence, the equilibrium allocation (x, y) is not weak maximin Pareto optimal with respect to the information structure $\mathcal{F} = (\mathcal{F}_i)_{i\in I}$.

□

8.4 Proofs of Section 6

Before proving Proposition 6.5 the following lemma is needed.

Lemma 8.5 *Condition (iii) and $(*)$ in the Definition 6.4, imply that for all $i \in C$,*

$$u_i(a, x_i(a)) = \min_{s\in\Pi_i(a)} u_i(s, x_i(s)) = \underline{u}_i^{\Pi_i}(a, x_i),$$

and

$$u_i(a, x_i(a)) < u_i(s, x_i(s)) \quad \text{for all } s \in \Pi_i(a) \setminus \{a\}.$$

Proof: Assume, on the contrary, there exists an agent $i \in C$ and a state $s_1 \in \Pi_i(a) \setminus \{a\}$ such that $\underline{u}_i^{\Pi_i}(a, x_i) = \min_{s \in \Pi_i(a)} u_i(s, x_i(s)) = u_i(s_1, x_i(s_1))$.

Notice that

$$\underline{u}_i^{\Pi_i}(a, y_i) = \min\{u_i(a, e_i(a) + x_i(b) - e_i(b)); \min_{s \in \Pi_i(a) \setminus \{a\}} u_i(s, x_i(s))\}.$$

If, $u_i(a, e_i(a) + x_i(b) - e_i(b)) = u_i(a, y_i(a)) = \underline{u}_i^{\Pi_i}(a, y_i)$, then in particular $u_i(a, y_i(a)) \leq u_i(s_1, x_i(s_1)) = \underline{u}_i^{\Pi_i}(a, x_i)$. This contradicts (iii). On the other hand, if there exists $s_2 \in \Pi_i(a) \setminus \{a\}$ such that $u_i(s_2, x_i(s_2)) = \underline{u}_i^{\Pi_i}(a, y_i)$, then in particular $\underline{u}_i^{\Pi_i}(a, y_i) = u_i(s_2, x_i(s_2)) \leq u_i(s_1, x_i(s_1)) = \underline{u}_i^{\Pi_i}(a, x_i)$. This again contradicts (iii). Thus, for each member i of C, there does not exist a state $s \in \Pi_i(a) \setminus \{a\}$ such that $\underline{u}_i^{\Pi_i}(a, x_i) = u_i(s, x_i(s))$. This means that

$$u_i(a, x_i(a)) = \min_{s \in \Pi_i(a)} u_i(s, x_i(s)) = \underline{u}_i^{\Pi_i}(a, x_i),$$

and

$$u_i(a, x_i(a)) < u_i(s, x_i(s)) \quad \text{for all } s \in \Pi_i(a) \setminus \{a\}. \qquad \square$$

Proof of Proposition 6.5: Let x be a CIC with respect to the information structure Π and assume on the contrary that there exist a coalition C and two states a and b such that

$$\begin{array}{lll} (i) & \Pi_i(a) = \Pi_i(b) & \text{for all } i \notin C, \\ (ii) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}_+^\ell & \text{for all } i \in C, \text{ and} \\ (iii) & \underline{u}_i^{\Pi_i}(a, y_i) > \underline{u}_i^{\Pi_i}(a, x_i) & \text{for all } i \in C, \end{array}$$

where for all $i \in C$,

$$y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a \\ x_i(s) & \text{otherwise.} \end{cases}$$

Notice that from (iii) and Lemma 8.5 it follows that for all $i \in C$,

$$u_i(a, e_i(a) + x_i(b) - e_i(b)) = u_i(a, y_i(a)) \geq \underline{u}_i^{\Pi_i}(a, y_i) > \underline{u}_i^{\Pi_i}(a, x_i) = u_i(a, x_i(a)).$$

Hence x is not CIC with respect to the information structure Π, which is a contradiction. For the converse, we construct the following counterexample. Consider the economy, described in Example 6.2, with two agents, three states of nature, denoted by a, b and c, and one good per state denoted by x. Assume that

$$\begin{array}{lll} u_1(\cdot, x_1) = \sqrt{x_1}; & e_1(a, b, c) = (20, 20, 0); & \mathcal{F}_1 = \{\{a, b\}; \{c\}\}. \\ u_2(\cdot, x_2) = \sqrt{x_2}; & e_2(a, b, c) = (20, 0, 20); & \mathcal{F}_2 = \{\{a, c\}; \{b\}\}. \end{array}$$

Consider the allocation

$$\begin{aligned} x_1(a, b, c) &= (20, 10, 10) \\ x_2(a, b, c) &= (20, 10, 10). \end{aligned}$$

We have already noticed that such an allocation is not Krasa-Yannelis incentive compatible with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2)$ (see Example 6.2), but it is maximin CIC with respect to $\mathcal{F}$ (see Remark 6.1). $\square$

Proof of Theorem 6.6: Let (p, x) be a maximin rational expectations equilibrium. Since agents take into account the information generated by the equilibrium price p, the private information of each individual i is given by $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Thus, for each agent $i \in I$, $\Pi_i = \mathcal{G}_i$ and $\underline{u}_i^{\Pi_i} = \underline{u}_i^{REE}$. Assume on the contrary that (p, x) is not maximin CIC. This means that there exists a coalition C and two states $a, b \in S$ such that

$$\begin{array}{lll} (i) & \mathcal{G}_i(a) = \mathcal{G}_i(b) & \text{for all } i \notin C, \\ (ii) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}_+^\ell & \text{for all } i \in C, \text{ and} \\ (iii) & \underline{u}_i^{REE}(a, y_i) > \underline{u}_i^{REE}(a, x_i) & \text{for all } i \in C, \end{array}$$

where for all $i \in C$,

$$y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a \\ x_i(s) & \text{otherwise.} \end{cases}$$

Notice that condition (i) implies that $p(a) = p(b)$, meaning that the equilibrium price is partially revealing.[37] Clearly, if p is fully revealing, since for any $i \in I$, $\mathcal{G}_i = \mathcal{F}$, then there does not exist a coalition C and two states a and b such that $\mathcal{G}_i(a) = \mathcal{G}_i(b)$ for all $i \notin C$. Therefore, any fully revealing MREE is maximin coalitional incentive compatible. On the other hand, since (p, x) is a maximin rational expectations equilibrium, it follows from (iii) that for all $i \in C$ there exists a state $s_i \in \mathcal{G}_i(a)$ such that

$$p(s_i) \cdot y_i(s_i) > p(s_i) \cdot e_i(s_i) \geq p(s_i) \cdot x_i(s_i).$$

By the definition of y_i, it follows that for all $i \in C$, $s_i = a$, that is $p(a) \cdot y_i(a) > p(a) \cdot e_i(a)$, and hence $p(a) \cdot [x_i(b) - e_i(b)] > 0$. Furthermore, since $p(a) = p(b)$ it follows that $p(b) \cdot x_i(b) > p(b) \cdot e_i(b)$. This contradicts the fact that (p, x) is a maximin rational expectations equilibrium. $\square$

Proof of Proposition 6.9: Let (p, x) be a maximin REE and assume on the contrary that there exist a coalition C and two states $a, b \in S$ such that

$$\begin{array}{lll} (I) & \mathcal{F}_i(a) = \mathcal{F}_i(b) & \text{for all } i \notin C, \\ (II) & u_i(a, x_i(a)) = u_i(a, x_i(b)) & \text{for all } i \notin C, \\ (III) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}_+^\ell & \text{for all } i \in C, \text{ and} \\ (IV) & \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) & \text{for all } i \in C, \end{array}$$

where for all $i \in C$,

[37] Notice that for all i, $\sigma(p) \subseteq \mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Thus, for all i, $p(\cdot)$ is $\mathcal{G}_i$-measurable. Therefore, condition (i) implies that $p(a) = p(b)$.

$$y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a \\ x_i(s) & \text{otherwise.} \end{cases}$$

If (p, x) is a non revealing MREE, then the proposition holds true with no additional assumptions on utility functions (see Remark 6.2).

I CASE: Assume that for any $i \in I$ $u_i(\cdot, t)$ is $\mathcal{F}_i$-measurable for each $t \in \mathbb{R}^\ell_+$. Observe that if p is partially revealing and $\mathcal{G}_i(a) \setminus \{a\} \neq \emptyset$ for some agent i in C, then the allocation x is (private) maximin coalitional incentive compatible and hence weak (private) maximin CIC. Indeed, from Lemma 8.5 and condition (IV), it follows that

$$\underline{u}_i^{REE}(a, x_i) = \underline{u}_i(a, x_i) = u_i(a, x_i(a)) < u_i(s, x_i(s)) \quad \text{for all } s \in \mathcal{F}_i(a) \setminus \{a\}.$$

In particular the above inequality holds for all $s \in \mathcal{G}_i(a) \setminus \{a\}$, and this contradicts Proposition 3.7. Moreover, if for some agent $i \notin C$, $\mathcal{G}_i(a) = \mathcal{G}_i(b)$, then it follows that $p(a) = p(b)$, and hence p is partially revealing. However, even if utility functions are not private information measurable, we can conclude that x is (private) maximin coalitional incentive compatible and hence weak (private) maximin CIC. In fact, from (IV) and Lemma 8.5, it follows that for all $i \in C$,

$$\underline{u}_i^{REE}(a, y_i) \geq \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) = u_i(a, x_i(a)) = \underline{u}_i^{REE}(a, x_i).$$

Therefore, since (p, x) is a maximin REE, from the definition of the allocation y, it follows that for each $i \in C$, $p(a) \cdot y_i(a) > p(a) \cdot e_i(a)$, and hence $p(a) \cdot x_i(b) > p(a) \cdot e_i(b)$, which is a contradiction because $p(a) = p(b)$.

Thus, let us assume that $\mathcal{G}_i(a) = \{a\}$ for all $i \in C$ and $\mathcal{G}_i(a) \neq \mathcal{G}_i(b)$ for any $i \notin C$. Again from (IV) and Lemma 8.5, it follows that for all $i \in C$,

$$\underline{u}_i^{REE}(a, y_i) \geq \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) = u_i(a, x_i(a)) = \underline{u}_i^{REE}(a, x_i),$$

while from (II) it follows that for all $i \notin C$,

$$\begin{aligned} \underline{u}_i^{REE}(a, y_i) &= \min\{\min_{s \in \mathcal{G}_i(a) \setminus \{a\}} u_i(s, x_i(s)), u_i(a, y_i(a))\} \\ &= \min\{\min_{s \in \mathcal{G}_i(a) \setminus \{a\}} u_i(s, x_i(s)), u_i(a, x_i(b))\} \\ &= \min\{\min_{s \in \mathcal{G}_i(a) \setminus \{a\}} u_i(s, x_i(s)), u_i(a, x_i(a))\} \\ &= \underline{u}_i^{REE}(a, x_i). \end{aligned}$$

Moreover, y is feasible. Indeed, for each state $s \neq a$, y is feasible because so is x. On the other hand, if $s = a$, then

$$\sum_{i \in I} y_i(a) = \sum_{i \in I} e_i(a) + \sum_{i \in I} x_i(b) - \sum_{i \in I} e_i(b) = \sum_{i \in I} e_i(a).$$

Hence, there exists a feasible allocation y such that

$$\underline{u}_i^{REE}(s, y_i) \geq \underline{u}_i^{REE}(s, x_i) \quad \text{for all } i \in I \text{ and all } s \in S,$$

with a strict inequality for each $i \in C$ in state a. Since x is a maximin REE and $\mathcal{G}_i(a) = \{a\}$ for all $i \in C$, it follows that

$$p(a) \cdot y_i(a) > p(a) \cdot e_i(a) \quad \text{for any } i \in C.$$

Moreover, since y is feasible, there exists at least one agent $j \notin C$ such that

$$p(a) \cdot y_j(a) < p(a) \cdot e_j(a).$$

Notice that

$$p(s) \cdot y_j(a) < p(s) \cdot e_j(s) \qquad \text{for all } s \in \mathcal{G}_j(a), \tag{24}$$

because $p(\cdot)$ and $e_j(\cdot)$ are $\mathcal{G}_j$-measurable. Define the allocation[38] z_j as follows:

$$z_j(s) = y_j(a) + \frac{\mathbf{1}p(s) \cdot [e_j(s) - y_j(a)]}{\sum_{h=1}^{\ell} p^h(s)} \qquad \text{for any } s \in \mathcal{G}_j(a),$$

where $\mathbf{1}$ is the vector with ℓ components each of them equal to one, i.e., $\mathbf{1} = (1, \dots, 1)$. Notice that $z_j(\cdot)$ is constant in the event $\mathcal{G}_j(a)$; for any $s \in \mathcal{G}_j(a)$ $z_j(s) \gg y_j(a)$ and $p(s) \cdot z_j(s) = p(s) \cdot e_j(s)$. Therefore, since (p, x) is a maximin REE and $u_j(\cdot, x)$ is $\mathcal{F}_j$-measurable, from the monotonicity of $u_j(a, \cdot)$, it follows that

$$\underline{u}_j^{REE}(a, x_j) \geq \underline{u}_j^{REE}(a, z_j) = u_j(a, z_j(a)) > u_j(a, y_j(a)) \geq \underline{u}_j^{REE}(a, y_j) = \underline{u}_j^{REE}(a, x_j),$$

a contradiction.

II CASE: Assume now that the equilibrium price p is fully revealing; hence $\mathcal{G}_i(a) = \{a\}$ for any $i \in I$. From (IV) and Lemma 8.5 it follows that for all $i \in C$,

$$\underline{u}_i^{REE}(a, y_i) \geq \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) = u_i(a, x_i(a)) = \underline{u}_i^{REE}(a, x_i),$$

and hence

$$p(a) \cdot y_i(a) > p(a) \cdot e_i(a) \quad \text{for any } i \in C.$$

while from (II) it follows that for all $i \notin C$,

$$\underline{u}_i^{REE}(a, y_i) = u_i(a, x_i(b)) = u_i(a, x_i(a)) = \underline{u}_i^{REE}(a, x_i).$$

Since, we have already observed that y is feasible, we conclude that for some agent $j \notin C$,

$$p(a) \cdot y_j(a) < p(a) \cdot e_j(a).$$

Define the following bundle[39]

$$z_j(a) = y_j(a) + \frac{\mathbf{1}p(a) \cdot [e_j(a) - y_j(a)]}{\sum_{h=1}^{\ell} p^h(a)} \gg y_j(a),$$

[38] Notice that for any $s \in \mathcal{G}_j(a)$, $\sum_{h=1}^{\ell} p^h(s) > 0$, because $p(s) \in \mathbb{R}_+^{\ell} \setminus \{0\}$ for any $s \in S$.

[39] Notice that $\sum_{h=1}^{\ell} p^h(a) > 0$, because agents' utility functions are monotone and consequently $p(s) \in \mathbb{R}_+^{\ell} \setminus \{0\}$ for any $s \in S$.

where $\mathbf{1}$ is the vector with ℓ components each of them equal to one, i.e., $\mathbf{1} = (1, \ldots, 1)$. Notice that $p(a) \cdot z_j(a) = p(a) \cdot e_j(a)$ and

$$\underline{u}_j^{REE}(a, z_j) = u_j(a, z_j(a)) > u_j(a, y_j(a)) = \underline{u}_j^{REE}(a, y_j) = \underline{u}_j^{REE}(a, x_j),$$

contradicts the fact that x is a maximin REE allocation. □

8.5 Counterexamples for general set of priors

As we commented above, Propositions 3.4, 3.7, Theorems 4.3, 5.4, 5.8, and Lemma 8.2 are valid for the general MEU models, provided that all priors are strictly positive. In this section, we give counterexamples for these results if some priors are not strictly positive.

Consider the following differential information economy:

$$\begin{array}{ccc}
I = \{1,2,3\} & S = \{a,b,c,d\} & \ell = 2 \\
\mathcal{F}_1 = \{\{a,b,c\},\{d\}\} & \mathcal{F}_2 = \{\{a,b,c,d\}\} & \mathcal{F}_3 = \{\{a\},\{b\},\{c\},\{d\}\} \\
e_1(s) = (1,3) \text{ for all } s \in \{a,b,c\} & e_1(d) = (2,2) & e_2(s) = (2,1) \text{ for all } s \in S \\
e_3(a) = (1,4) & e_3(b) = (2,6) & e_3(c) = (0,2) \\
e_3(d) = (1,7) & u_i(s,x,y) = \sqrt{xy} & \forall i \text{ and } \forall s \in S
\end{array}$$

Notice that $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ and $u_i(s, \cdot)$ is strict quasi concave. Let

$$\begin{array}{rcl}
\mathcal{M}_1^s & = & \{\alpha : S \to [0,1] \, : \, \alpha(a) + \alpha(b) = 1\} \quad \text{for all } s \in \{a,b,c\} \\
\mathcal{M}_2^s & = & \{\alpha : S \to [0,1] \, : \, \alpha(a) + \alpha(b) + \alpha(d) = 1\} \quad \text{for all } s \in S.
\end{array}$$

$\mathcal{M}_1$ and $\mathcal{M}_2$ are proper subsets of $\mathcal{C}_1$ and $\mathcal{C}_2$ and they do not contain only positive priors.

Consider the following allocation $\{(x_i^*(s), y_i^*(s))\}_{i \in I, s \in S}$

$$(x_1(a), y_1(a)) = \left(\tfrac{5}{4}, \tfrac{5}{2}\right) \quad (x_2(a), y_2(a)) = \left(\tfrac{5}{4}, \tfrac{5}{2}\right) \quad (x_3(a), y_3(a)) = \left(\tfrac{3}{2}, 3\right)$$

$$(x_1(b), y_1(b)) = \left(\tfrac{5}{4}, \tfrac{5}{2}\right) \quad (x_2(b), y_2(b)) = \left(\tfrac{5}{4}, \tfrac{5}{2}\right) \quad (x_3(b), y_3(b)) = \left(\tfrac{5}{2}, 5\right)$$

$$(x_1(c), y_1(c)) = \left(\tfrac{9}{4}, \tfrac{1}{2}\right) \quad (x_2(c), y_2(c)) = \left(\tfrac{1}{4}, \tfrac{9}{2}\right) \quad (x_3(c), y_3(c)) = \left(\tfrac{1}{2}, 1\right)$$

$$(x_1(d), y_1(d)) = \left(\tfrac{3}{2}, 3\right) \quad (x_2(d), y_2(d)) = \left(\tfrac{5}{4}, \tfrac{5}{2}\right) \quad (x_3(d), y_3(d)) = \left(\tfrac{9}{4}, \tfrac{9}{2}\right)$$

and the following price $(p(s), q(s)) = \left(1, \frac{1}{2}\right)$ for all $s \in S$. Thus, (p, q) is non revealing and hence $\mathcal{G}_i = \mathcal{F}_i$ for all i.

We now show that the allocation above is a MREE where agents' preferences are represented by (the general) G-S- maximin expected utility. Indeed, $\{(x_i^*(s), y_i^*(s))\}_{i \in I, s \in S}$

is feasible and it satisfies the budget constraints:

$\frac{5}{4}+\frac{5}{4}=1+\frac{3}{2}$	$\frac{5}{4}+\frac{5}{4}=2+\frac{1}{2}$	$\frac{3}{2}+\frac{3}{2}=1+2$
$\frac{5}{4}+\frac{5}{4}=1+\frac{3}{2}$	$\frac{5}{4}+\frac{5}{4}=2+\frac{1}{2}$	$\frac{5}{2}+\frac{5}{2}=2+3$
$\frac{9}{4}+\frac{1}{4}=1+\frac{3}{2}$	$\frac{1}{4}+\frac{9}{4}=2+\frac{1}{2}$	$\frac{1}{2}+\frac{1}{2}=0+1$
$\frac{3}{2}+\frac{3}{2}=2+1$	$\frac{5}{4}+\frac{5}{4}=2+\frac{1}{2}$	$\frac{9}{4}+\frac{9}{4}=1+\frac{7}{2}$

Moreover it maximizes the MEU subjet to the budget constraint. Indeed, assume on the contrary that there exists

(I case: $i=1$ and $s\in\{a,b,c\}$) a random bundle $(x_1(s),y_1(s))$ such that

$$\inf_{\alpha\in\mathcal{M}_1^s}\sum_{s'\in\{a,b,c\}}\sqrt{x_1(s')y_1(s')}\alpha(s')>\inf_{\alpha\in\mathcal{M}_1^s}\sum_{s'\in\{a,b,c\}}\sqrt{x_1^*(s')y_1^*(s')}\alpha(s'). \tag{25}$$

and

$$x_1(s)+\frac{1}{2}y_1(s)\leq 1+\frac{3}{2}\quad \text{for all } s\in\{a,b,c\}.$$

Since for all $\alpha\in\mathcal{M}_1^s$, $\alpha(c)=0$ and there exists $\beta\in\mathcal{M}_1^s$ such that $\beta(a)=1$ and $\beta(b)=\beta(c)=0$, from (25) it follows in particular that

$$\begin{aligned}\sqrt{x_1(a)y_1(a)} &> \sqrt{\frac{25}{8}}\\ x_1(a)+\frac{1}{2}y_1(a) &\leq \frac{5}{2}.\end{aligned}$$

Thus, $\frac{1}{2}(5-y_1(a))y_1(a)>\frac{25}{8}$, i.e., $\left(y_1(a)-\frac{5}{2}\right)^2<0$ a contradiction.

(II case: $i=1$ and $s=d$) a random bundle $(x_1(d),y_1(d))$ such that

$$\sqrt{x_1(d)y_1(d)}>\sqrt{\frac{9}{2}}$$

and

$$x_1(d)+\frac{1}{2}y_1(d)\leq 3.$$

This implies that $(3-\frac{1}{2}y_1(d))y_1(d)>\frac{9}{2}$, i.e., $(y_1(a)-3)^2<0$ a contradiction.

(III case: $i=2$ and $s\in S$) a random bundle $(x_2(s),y_2(s))$ such that

$$\inf_{\alpha\in\mathcal{M}_2^s}\sum_{s'\in S}\sqrt{x_2(s')y_2(s')}\alpha(s')>\inf_{\alpha\in\mathcal{M}_2^s}\sum_{s'\in S}\sqrt{x_2^*(s')y_2^*(s')}\alpha(s'). \tag{26}$$

and

$$x_2(s) + \frac{1}{2}y_2(s) \leq 2 + \frac{1}{2} \quad \text{for all } s \in S.$$

Since for all $\alpha \in \mathcal{M}_2^s$, $\alpha(c) = 0$ and there exists $\beta \in \mathcal{M}_2^s$ such that $\beta(a) = 1$ and $\beta(b) = \beta(c) = 0$, from (26) it follows in particular that

$$\begin{aligned} \sqrt{x_2(a)y_2(a)} &> \sqrt{\frac{25}{8}} \\ x_2(a) + \frac{1}{2}y_2(a) &\leq \frac{5}{2}. \end{aligned}$$

As in the first case, this implies a contradiction.

(IV case: $i = 3$ and $s = a$) a random bundle $(x_3(a), y_3(a))$ such that

$$\sqrt{x_3(a)y_3(a)} > \sqrt{\frac{9}{2}}$$

and

$$x_3(a) + \frac{1}{2}y_3(a) \leq 3.$$

As in the second case, this implies a contradiction.

(V case: $i = 3$ and $s = b$) a random bundle $(x_3(b), y_3(b))$ such that

$$\sqrt{x_3(b)y_3(b)} > \sqrt{\frac{25}{2}}$$

and

$$x_3(b) + \frac{1}{2}y_3(b) \leq 5.$$

This implies that $(5 - \frac{1}{2}y_3(b))y_3(b) > \frac{25}{2}$, i.e., $(y_3(b) - 5)^2 < 0$ a contradiction.

(VI case: $i = 3$ and $s = c$) a random bundle $(x_3(c), y_3(c))$ such that

$$\sqrt{x_3(c)y_3(c)} > \sqrt{\frac{1}{2}}$$

and

$$x_3(c) + \frac{1}{2}y_3(c) \leq 1.$$

This implies that $\left(1 - \frac{1}{2}y_3(c)\right) y_3(c) > \frac{1}{2}$, i.e., $(y_3(c) - 1)^2 < 0$ a contradiction.

(VII case: $i = 3$ and $s = d$) a random bundle $(x_3(d), y_3(d))$ such that

$$\sqrt{x_3(d)y_3(d)} > \sqrt{\frac{81}{8}}$$

and

$$x_3(d) + \frac{1}{2}y_3(d) \leq \frac{9}{2}.$$

This implies that $\left(\frac{9}{2} - \frac{1}{2}y_3(d)\right) y_3(d) > \frac{81}{8}$, i.e., $\left(y_3(d) - \frac{9}{2}\right)^2 < 0$ a contradiction.

Notice that

- the allocation $(x_i^*(\cdot), y_i^*(\cdot))$ is not $\mathcal{G}_i$-measurable. Thus, this is a counterexample for Proposition 3.7 for the general MEU case if the set of priors contains priors that are not strictly positive.
- agents' utilities are not constant in the event $\mathcal{G}_i(s)$. Thus, this is a counterexample for Proposition 3.11 for the general MEU case if the set of priors contains priors that are not strictly positive.
- the allocation $(x_i^*(\cdot), y_i^*(\cdot))$ is not ex-post efficient, since it is blocked by

$$\begin{aligned}
(t_i(s), z_i(s)) &= (x_i^*(s), y_i^*(s)) \ \forall i \in I \text{ if } s \neq c, \ \textit{and} \\
(t_i(c), z_i(c)) &= \left(\frac{5}{4}, \frac{5}{2}\right) \ \forall i \in \{1, 2\} \\
(t_3(c), z_3(c)) &= (x_3^*(c), y_3^*(c)) = \left(\frac{1}{2}, 1\right).
\end{aligned}$$

  Indeed (t, z) is feasible, and $u_i(s, t, z) = u_i(s, x^*, y^*)$ for all $i \in I$ if $s \neq c$, and

$$\begin{aligned}
u_1(t_1(c), z_1(c)) &= \sqrt{\frac{25}{8}} > \sqrt{\frac{9}{8}} = u_1(x_1^*(c), y_1^*(c)) \\
u_2(t_2(c), z_2(c)) &= \sqrt{\frac{25}{8}} > \sqrt{\frac{9}{8}} = u_2(x_2^*(c), y_2^*(c)) \\
u_3(t_3(c), z_3(c)) &= u_3(x_3^*(c), y_3^*(c))
\end{aligned}$$

  Thus, this is a counterexample for Theorem 5.4 for the general MEU case if the set of priors contains priors that are not strictly positive.
- the allocation $(x_i^*(\cdot), y_i^*(\cdot)$ is not maximin efficient, since it is blocked by

$$\begin{aligned}
(t_i(s), z_i(s)) &= (x_i^*(s), y_i^*(s)) \ \forall i \in I \text{ if } s \neq c, \ \textit{and} \\
(t_i(c), z_i(c)) &= (0, 0) \ \forall i \in \{1, 2\} \\
(t_3(c), z_3(c)) &= (3, 6).
\end{aligned}$$

Indeed (t, z) is feasible, and $u_i(s,t,z) = u_i(s,x^*,y^*)$ for all $i \in I$ if $s \neq c$, and

$$\begin{aligned}
\inf_{\alpha \in \mathcal{M}_1^s} \sum_{s' \in S} \sqrt{t_1(s')z_1(s')}\alpha(s') &= \sqrt{\frac{25}{8}} = \inf_{\alpha \in \mathcal{M}_1^s} \sum_{s' \in S} \sqrt{x_1^*(s')y_1^*(s')}\alpha(s') \\
\inf_{\alpha \in \mathcal{M}_2^s} \sum_{s' \in S} \sqrt{t_2(s')z_2(s')}\alpha(s') &= \sqrt{\frac{25}{8}} = \inf_{\alpha \in \mathcal{M}_2^s} \sum_{s' \in S} \sqrt{x_2^*(s')y_2^*(s')}\alpha(s') \\
u_3(t_3(c), z_3(c)) &= \sqrt{18} > \sqrt{\frac{1}{2}} = u_3(x_3^*(c), y_3^*(c))
\end{aligned}$$

Thus, this is a counterexample for Theorem 5.9 for the general MEU case if the set of priors contains priors that are not strictly positive.

- the allocation $(x_i^*(\cdot), y_i^*(\cdot))$ is not an ex-post Walrasian equilibrium allocation. Indeed consider for example agent 2 in state c and the bundle $\left(\frac{5}{4}, \frac{5}{2}\right)$ which is such that
$$\sqrt{\frac{25}{8}} > \sqrt{\frac{9}{8}} \quad \text{and} \quad \frac{5}{4} + \frac{5}{4} = 2 + \frac{1}{2}.$$
Thus, this is a counterexample for Theorem 4.6 and Lemma 8.2 for the general MEU case if the set of priors contains priors that are not strictly positive.

References

ALLAIS, M. (1953): "Le Comportement de l'Homme Rationnel devant le Risque: Critique des Postulats et Axiomes de l'Ecole Americaine," *Econometrica*, 21(4), 503–546.

ALLEN, B. (1981): "Generic Existence of Completely Revealing Equilibria for Economies with Uncertainty when Prices Convey Information," *Econometrica*, 49(5), 1173–1199.

ALLEN, B., AND J. JORDAN (1998): "The existence of rational expectations equilibrium: a retrospective," *Staff Report*.

ANGELONI, L., AND V. MARTINS-DA ROCHA (2009): "Large economies with differential information and without free disposal," *Economic Theory*, 38(2), 263–286.

ANGELOPOULOS, A., AND L. C. KOUTSOUGERAS (2015): "Value allocation under ambiguity," *Economic Theory*, 59(1), 147–167.

ARYAL, G., AND R. STAUBER (2014): "Trembles in extensive games with ambiguity averse players," *Economic Theory*, 57(1), 1–40.

BHOWMIK, A., J. CAO, AND N. C. YANNELIS (2014): "Aggregate preferred correspondence and the existence of a maximin REE," *Journal of Mathematical Analysis and Applications*, 414(1), 29–45.

BODOH-CREED, A. L. (2012): "Ambiguous beliefs and mechanism design," *Games and Economic Behavior*, 75(2), 518 – 537.

CABALLERO, R., AND A. KRISHNAMURTHY (2008): "Collective risk management in a flight to quality episode," *The Journal of Finance*, 63(5), 2195–2230.

CONDIE, S., AND J. GANGULI (2011a): "Ambiguity and rational expectations equilibria," *The Review of Economic Studies*, 78(3), 821.

——— (2011b): "Informational efficiency with ambiguous information," *Economic Theory*, pp. 1–14.

CORREIA-DA SILVA, J., AND C. HERVÉS-BELOSO (2008): "Subjective expectations equilibrium in economies with uncertain delivery," *Journal of Mathematical Economics*, 44(7), 641–650.

——— (2009): "Prudent expectations equilibrium in economies with uncertain delivery," *Economic Theory*, 39(1), 67–92.

——— (2012): "General equilibrium in economies with uncertain delivery," *Economic Theory*, 51(3), 729–755.

——— (2014): "Irrelevance of private information in two-period economies with more goods than states of nature," *Economic Theory*, 55(2), 439–455.

DE CASTRO, L., M. PESCE, AND N. YANNELIS (2011): "Core and equilibria under ambiguity," *Economic Theory*, 48, 519–548, 10.1007/s00199-011-0637-3.

DE CASTRO, L. I., AND N. C. YANNELIS (2011): "Uncertainty, Efficiency and Incentive Compatibility," Discussion paper, Northwestern University.

DE CASTRO, L. I., N. C. YANNELIS, AND L. ZHIWEI (2015): "Impementation under ambiguity," Discussion paper, University of Iowa.

DE SIMONE, A., AND C. TARANTINO (2010): "Some new characterization of rational expectation equilibria in economies with asymmetric information," *Decisions in Economics and Finance*, 33, 7–21, 10.1007/s10203-009-0094-7.

DOW, J., AND S. WERLANG (1992): "Uncertainty Aversion, Risk Aversion, and the Optimal Choice of Portfolio," *Econometrica*, 60(1), 197–204.

EINY, E., D. MORENO, AND B. SHITOVITZ (2000): “Rational expectations equilibria and the ex-post core of an economy with asymmetric information,” *Journal of Mathematical Economics*, 34(4), 527 – 535.

ELLSBERG, D. (1961): “Risk, Ambiguity, and the Savage Axioms,” *The Quarterly Journal of Economics*, 75(4), 643–669.

EPSTEIN, L., AND M. SCHNEIDER (2007): “Learning Under Ambiguity,” *Review of Economic Studies*, 74, 1275–1303, University of Rochester.

EPSTEIN, L., AND M. SCHNEIDER (2008): “Ambiguity, Information Quality, and Asset Pricing,” *The Journal of Finance*, 63(1), 197–228.

EPSTEIN, L. G., AND M. SCHNEIDER (2010): “Ambiguity and Asset Markets,” *Annual Review of Financial Economics*, 2(1), 315–346.

GILBOA, I., AND D. SCHMEIDLER (1989): “Maxmin expected utility with non-unique prior,” *Journal of Mathematical Economics*, 18(2), 141 – 153.

GLYCOPANTIS, D., A. MUIR, AND N. YANNELIS (2005): “Non-implementation of rational expectations as a perfect Bayesian equilibrium,” *Economic Theory*, 26(4), 765–791.

GLYCOPANTIS, D., AND N. YANNELIS (2005): *Differential Information Economies*, vol. 19 of *Studies in Economic Theory*. Springer.

GREEN, J. (1977): “The Non-Existence of Informational Equilibria,” *The Review of Economic Studies*, 44(3), pp. 451–463.

GROSSMAN, S. J. (1981): “An Introduction to the Theory of Rational Expectations Under Asymmetric Information,” *The Review of Economic Studies*, 48(4), pp. 541–559.

HANSEN, L., AND T. SARGENT (2001): “Acknowledging Misspecification in Macroeconomic Theory,” *Review of Economic Dynamics*, 4(3), 519–535.

HANSEN, L. P., AND T. J. SARGENT (2012): “Three types of ambiguity,” *Journal of Monetary Economics*, 59(5), 422–445.

HAYEK, F. A. (1945): “The Use of Knowledge in Society,” *The American Economic Review*, 35(4), 519–530.

HE, W., AND N. C. YANNELIS (2015): “Equilibrium Theory under ambiguity,” Discussion paper, University of Iowa.

ILUT, C., AND M. SCHNEIDER (2012): “Ambiguous Business Cycles,” *NBER Working Paper No. 17900*.

Ju, N., and J. Miao (2012): "Ambiguity, Learning, and Asset Returns," *Econometrica*, 80(2), 559–591.

Koutsougeras, L., and N. Yannelis (1993): "Incentive compatibility and information superiority of the core of an economy with differential information," *Economic Theory*, 3(2), 195–216.

Krasa, S., and N. C. Yannelis (1994): "The value allocation of an economy with differential information," *Econometrica*, 62, 881–900.

Kreps, D. M. (1977): "A note on 'fulfilled expectations' equilibria," *Journal of Economic Theory*, 14(1), 32 – 43.

Mas-Colell, A., M. Whinston, and J. Green (1995): *Microeconomic theory*. Oxford University Press New York.

Milnor, J. (1954): "Games against nature," *Decision Processes*, pp. 49–59.

Podczeck, K., and N. C. Yannelis (2008): "Equilibrium theory with asymmetric information and with infinitely many commodities," *Journal of Economic Theory*, 141(1), 152 – 183.

Radner, R. (1968): "Competitive Equilibrium Under Uncertainty," *Econometrica*, 36(1), 31–58.

——— (1979): "Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices," *Econometrica*, 47(3), 655–678.

——— (1982): "Equilibrium under uncertainty," in *Handbook of Mathematical Economics*, ed. by K. J. Arrow, and M. D. Intriligator, vol. 2, Chapter 20, pp. 923 – 1006. Elsevier.

Savage, L. J. (1954): *The foundations of statistics*. John Wiley & Sons Inc., New York.

Sun, Y., L. Wu, and N. C. Yannelis (2012): "Existence, incentive compatibility and efficiency of the rational expectations equilibrium," *Games and Economic Behavior*, 76(1), 329 – 339.

Yannelis, N. C. (1991): "The core of an economy with differential information," *Economic Theory*, 1(2), 183–197.

Zhiwei, L. (2014): "A note on the welfare of the maximin rational expectations," *Economic Theory Bulletin*, 2(213-218).

——— (2015): "The implementation of maximin rational expectations equilibrium," Discussion paper, University of Iowa.