Investment brings change: Implications for news-driven business cycles

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Abstract

Purchasing investment goods does not directly increase the productive capacity of the firm. Changes in the firm through installation of capital, worker training, and workplace reorganization are often required . These changes themselves are not easily automated. Change requires workers. We build a model where investment requires a complementary labor input. This mechanism is embedded in a real business cycle model with capacity utilization, adjustment costs, and separable preferences. We show that this environment can yield positive comovement between consumption, investment and labor hours when the economy experiences a news shock regarding future productivity. As such we provide an additional channel through which news shocks can generate key business cycle features.

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1 Introduction

Capital investment at the firm level is often a component of broader change. A new computer system means that workers will have new tasks added to their workload while others are eliminated. New heavy equipment will not simply replace the old, but will change the production process in important ways. Investment installation itself is a sort of change as this activity is a departure from routine. Even additional capital which simply scales capacity will cause change. Adding more trucks to a shipping fleet, for example, will change how the firm is optimally managed.

The notion that investment brings change has received considerable attention in the literature. So to has the complementary notion that change requires labor. Much of this literature focuses in particular on the change brought about by the revolution in information technology. A prominent example is the work of Bresnahan, Brynjolfsson, and Hitt (2002). They find a close relationship between improved information technology and workplace reorganization. They also show that this change itself cannot easily be automated. Labor is required to implement the reorganization.

In this paper we take a broad view of the investment/labor demand relationship, assuming that investment of any sort is more productive when 'implementation labor' is employed to accommodate the firm-level changes. We consider the implications of implementation labor for news driven business cycles. We build a model where news of future productivity improvements changes firms' investment demand. Changes in investment demand cause changes in the demand for implementation labor. The productivity increase may be specific to investment. Investment-specific technology change is often associated with information technology. In this way, we consider improvements in information technology as highlighted by Bresnahan et al. (2002). The productivity increase may instead affect all production symmetrically. It this case, we consider the effects of adding implementation labor in a more standard setting.

A common relationship resulting from optimal agent behavior in real business cycle models, and many other models, is that the marginal rate of substitution between consumption and labor is equal to the marginal productivity of labor. Beaudry and Portier (2004, 2007), among others, point out that this relationship presents a challenge for modeling business cycles as resulting from news about future productivity. If news of future productivity results in increased current consumption, the marginal rate of substitution will increase. To preserve the relationship, the marginal productivity of labor must increase; i.e. the labor input must fall. This violates a key feature of business cycles: consumption and labor hours are positively correlated. Beaudry and Portier (2007) refer to this as the 'static problem' and we adopt this terminology. This challenge is not present with contemporaneous productivity shocks since such shocks increase the marginal product of labor even after accounting for the general equilibrium increase in employment.

The literature related to news-driven business cycles explores several modifications of the baseline real business cycle model that overcome the static problem. We show that implementation labor provides an additional useful modification of this sort. In essence, a news shock influences the supply of labor used to implement new investment capital as well as labor used in the production of a final good. Labor used for implementation enhances capital accumulation but has no impact on current production. Importantly, then, this labor has no direct effect on the marginal product of labor. This allows more freedom of movement between the marginal rate of substitution and total labor employed.

Our model is most closely related to Jaimovich and Rebelo (2009). Their model has three key features which together allow consumption, investment, and labor hours to increase in response to a positive news shock. First they set the depreciation rate for capital equal to the endogenously determined rate of capacity utilization. An increase in capacity utilization has a similar effect to an increase is capital. With more capital employed, the marginal product of labor increases. This helps to overcome the static problem. Second they include adjustment costs which helps in assuring that current consumption is positively correlated with news of future productivity changes.

Jaimovich and Rebelo demonstrate that these two features of the law of motion for the capital stock fall short in generating the desired comovements in response to a news shock. A third feature, non-separable preferences, is essential. This weakens the relationship between the marginal rate of substitution and the marginal product of labor. Our model does not include this third feature. We instead include an additional term in the law of motion for capital meant to capture the salient features the investment/labor demand relationship.

We first consider a special case of our model with no adjustment costs which allows us to analytically examine conditions allowing positive comovement between consumption, investment, and total labor hours. Importantly, we show that our new feature gives a boost to the effects of the capacity utilization rate. With this boost, capacity utilization can respond sufficiently to a news shock to allow a general equilibrium increase in the marginal product of labor at the same time that labor employed increases. Absent implementation labor, this cannot occur.

Moreover, we show that whether this occurs is closely related to returns to scale in the production of investment goods. In particular, we show that increasing returns to scale is a sufficient condition for comovement across these key variables. Because of this, our results are related to those of Guo et al. (2015) in two ways. They show that a production externality can overcome the static problem. They focus on a production spillover that results in increasing returns to scale at the social level despite constant returns to scale at the firm level. There is no similar externality in our model. However, investment is produced using a final good and implementation labor. The final good is a standard Cobb-Douglas combination of capital and labor with constant returns to scale. Investment combines this final good with a second sort of labor input. This allows the possibility of constant returns to scale in the production of the final good with increasing returns to scale for investment. Another commonality of our model with Guo et al. (2015) is that with increasing returns to scale, the model may be indeterminate. We characterize conditions which give rise to indeterminacy in an unpublished appendix. Here, we restrict our parameter choices to cases where the model is determinate.

In our special case, the model can overcome the static problem. Beaudry and Portier (2007) also articulate the 'dynamic problem' of news driven business cycles. A future productivity increase is a positive lifetime income shock. The resulting consumption smoothing is a positive force affecting current consumption. At the same time, investment may increase as firms gear up for the anticipated productivity increase. This weighs in weighs in favor of decreased current consumption through the resources constraint. A news driven business cycle model must find a way for output to respond sufficiently, and its allocation to respond properly, such that consumption and investment both increase.

Our special case does not overcome the dynamic problem. As in Rebelo and Jaimovich (2009),we need adjustment costs for positive productivity shocks to yield both an increase is consumption and positive comovement between consumption, investment, labor hours, and output. The remainder of the paper, then, includes adjustment costs and considers the impact of implementation labor in our full model. We show that our model with implementation labor, variable capacity utilization, and investment adjustment cost can generate qualitatively realistic aggregate fluctuations driven by news shock to total factor productivity and investment specific technological change. In the existing literature of expectation driven business cycles, the results rely on non-separable preferences. Our analysis does not rely on preference rather a new law of motion of capital that considers labor for implementing the investment. When the economy is subject to anticipated technological shock, the relative importance of implementation labor in capital production is needed to be enhanced for strong labor demand in the economy to exhibit positive comovements among macroeconomic variables.

2 Literature Review

Our work is based on two strands of literature. First, an impressive body of empirical studies shows that the technical change in the USA is associated with shifts in demand toward more specific workers.[1] We motivate the agent's decision on investment-specific labor input choice based on empirical facts that technological change increases the demand for investment specific labor inputs (Bresnahan et al. 2002).[2] Bresnahan et al. (2002) investigate the relationships between firms' labor demand behavior and complementary relationship of three innovations, namely, i) increased use of Information Technology (IT), ii) changes in organization practices, and iii) changes products and services. As a result, technological innovation affects labor demand raises the labor demand not only directly as shown in the previous studies, but indirectly through firm level changes.[3] In other words, technological innovation is also associated with other changes in the firm through the installation of capital. Several studies suggest that IT enabled organizational change results in better measurement and communication with in the organization. As a result, internal organization of the firm is determined by the economics of information and communication (Milgrom and Roberts [1990], Brynjolfsson and Mendelson [1993], and Radner [1993]) and workplace reorganization.[4]

[1] See the works by Autor, Katz, and Krueger (1998) and Berman, Bound, and Griliches (1994) and Black and Lynch (2001) among others.

[2] Bresnahan et al. 2002 show that the increased demand for skilled labor is related to a particular cluster of technological change. This change not only increased use of that particular technology but also changes in workplace organization and changes in product and service quality. Using the firm level data on IT and labor demand and IT and organization, they showed that the most common combinations of IT-enabled organizational and strategic change increase the relative demand for skilled workers. IT use is correlated with increases in the demand for various indicators of human capital and workforce skills.

[3] Computers have influenced the principles of job design and changed the way the way that human work is measured, controlled, or reported, mediated by organizational change. This creates a number of additional indirect flows from computers to labor demand. For instance, supervisors will need more skills in dealing with customers and suppliers, influencing teammates and colleagues, and inspiring and coaching subordinates [Brynjolfsson, Renshaw, and Van Alstyne 1997]. More generally, the changes involve providing the "people skills" that computers lack.

[4] In other words, as computers have become faster, smaller, cheaper, more flexible, and easier to network together, the quality-adjusted real price of computers has been declining at a compound rate of about 20 percent per year. The rapid decline in IT prices lead to very rapidly growing demand for IT. As a result, optimal structure of the organization is determined by the change in IT.For example: the sources of complementarity between investments in IT and reorganization of the firm changes could be understood by the authority relationships, decentralization of

Moreover, a common idea is that technological change could be a substitution out of certain kinds of human effort.[5] However, a technological change could also increase the demand for labor. For example, highly computerized organizational processes within the firm level leading to generate a large volume of data every day. This raw data is being used to analyze customer needs to target new product development, increasing the value of workers, managers, and professionals. This will directly lead to a greater demand for skilled labor at the firm.[6] Additionally, innovations of new products and workplace organization associated with technological change require greater levels of cognitive skill, flexibility, and autonomy than in traditional employee roles where the production process is fixed and includes limited discretion. Bartel and Lichtenberg [1987] suggest that cognitive skills may be important in adapting to change generally, notably in the adoption of new technologies. Adaptation to IT-based change may be a standing requirement of the modern firm, resulting in a lasting shift in labor demands. This cluster of inventions, playing out over multiple years and across many firms, constitute the technical change that is associated with an increased demand for specific labor.

Secondly, we embedded our idea of the connection between firms' labor demand and technological innovations in an expectation-driven business cycle model. Following the developments of expectation-driven business cycles by Beaudry and Portier (2004, 2007), economists have built models, identifying few channels through which agent's expectations about the future fundamentals could lead to qualitatively realistic expectations-driven business cycles. Guo et al. (2015) show that production externalities are a channel through which news about fundamentals cause the positive co-movement among macroeconomic aggregates. More specifically, they focus on production spillover that is external to the production process of an individual agent, and the individual agent is more productive the higher the production level of other agents. It tends to be the case if there are sufficiently high production externalities leading to aggregate increasing returns to scale, which

decision authority, shifts in the task content of clerks', operatives', professionals', and managers' work, and changes in reward schemes, among others (Bresnahan, Brynjolfsson, and Hitt 1999). Similarly, the combination of organizational and technological innovation is required to deliver consistently high levels of customer service [Davenport 1994]. All this suggests a three-way cluster of complementarity among product quality improvements (broadly understood), reorganization, and IT investment.

[5] Especially in record keeping, remembering, simple calculating, and similar tasks, IT use has led firms to systematically substitute computer decision-making for human decision-making in clerical (and similar routine) work.

[6] In general, more complex and cognitively demanding work, such as that of managers and professionals, has proved to be remarkably difficult to automate. Computer automation of such work has been correspondingly limited in its scope. Computer automation of clerical and blue-collar work typically does not directly substitute for all of a worker's tasks, but instead for a subset of ancillary tasks, and in particular, those that do not require exception processing, visual or spatial skills, or nonalgorithmic reasoning [Autor, Levy, and Murnane 2000; Levy, Beamish, Murnane, and Autor 1999].

is imperative to comply with the necessary conditions for macroeconomic comovements. Jaimovich and Rebelo (2009) introduced a unified utility function through which they able to control the wealth effect and showed that for weaker wealth effects, news shocks generate positive comovements. In their case, the labor supply will increase due to weaker wealth effects. In our analysis, we show that because of firms' strategic choice on labor input that complements with investment demand, and due to the anticipation of technological innovations firms' demand for labor would increase. Others focus on the different structure of labor markets, like Denhaan and Kaltenbrunner (2005) study the effects of news in a matching friction model that is similar to labor adjustment cost. Wang (2012) introduce different labor market structures to generate business cycles co-movements through labor market diagrams. Guido (2005) studies a model where productivity has one permanent and one temporary component and agent has imperfect information about the relative importance of the components. Gilchirst and Leahy (2002) discuss effects of news in a sticky price model. Blanchard (2007) emphasize the importance of news in an open market set up.

Based on these ideas above, we forge a theoretical framework to show the connection between the technological innovations and agents' demand for specific labor required for workplace organization change. We build a model where investment requires a complementary labor input. The effect of innovations that cause a change in labor input decisions and dorganizational changes are embedded in our investment function. When we incorporate this idea in neo-classical setting, we find that the condition for expectation driven business cycles can also be achieved with an extra boost in labor demand through the channel of change in workplace organization due to news shock.

3 The model

Our model is closely related to Jaimovich and Rebelo (2009). However, their results rely on preferences with some degree of non-separability in consumption and hours worked. We highlight that this feature is not required in our model by considering a version of their special case with separable preferences. We generalize their model through including a term in law of motion meant to capture salient features of the discussion in the previous section.

The economy is populated by a mass of identical agents who derive utility from consuming a final good in each period and disutility from providing a labor input. With c_t and n_t defined as consumption and the labor input in period time period t, the expected lifetime utility of a representative agent is

$$U = E_o \sum_{t=o}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \phi \frac{n_t^{1+\gamma}}{1+\gamma} \right]. \tag{1}$$

Here E_o is the expectations operator, $\beta < 1$ is the discount rate, $\phi > 0$ gauges the disutility of hours worked and $\sigma, \gamma \geq 0$ govern elasticities.

A representative firm combines capital and labor to produce the final good subject to a Cobb-Douglas production function with share parameter $\alpha \in [0, 1]$ and general productivity parameter $a_t > 0$. In general, capital employed by the firm will be some share, u_t, of the total capital stock available to the economy, k_t. Moreover, the labor input in the final good production, $n_{f,t}$, will be only part of the total labor input. Output of the final good, y_t, then, is

$$y_t = a_t \left(u_t k_t \right)^{\alpha} n_{f,t}^{1-\alpha}. \tag{2}$$

The final good can be utilized as a consumption good or an investment good, i_t, with the resource constraint given by

$$c_t + \frac{i_t}{v_t} = y_t. \tag{3}$$

The rate at which a unit of the final good can be converted to a unit of the investment good depends on the technology parameter v_t. An increase v_t reflects investment-specific technological progress and a decrease is technological regress. General and investment-specific technological progress are stochastic and governed by

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} + \varepsilon_{a,t-j} \tag{4}$$

$$v_t = \rho_v v_{t-1} + \varepsilon_{v,t} + \varepsilon_{v,t-j} \tag{5}$$

where $\rho_a, \rho_v \in (0, 1)$ and $j > 0$.

The law of motion for capital contains two of the key features driving business cycles in Jaimovich and Rebelo (2009). First, the rate of depreciation is positively related to the endogenous rate of capacity utilization. Specifically, we set depreciation equal to $\frac{\varphi_2 u_t^{\varphi_1}}{\varphi_1}$ with $\varphi_1 > 1$ and $\varphi_2 > 0$. Second, we include adjustment costs so that investment, i_t, is scaled by $1 - \frac{\psi_2}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2$ where $\psi_2 > 0$.

Jaimovich and Rebelo demonstrate that these features of the law of motion fall short in generating the desired comovements in response to a news shock. Their third feature, non-separable preferences, is essential. Since preferences are separable in our model, an alternative feature is required. We assume that purchasing investment goods can more effectively add to the capital stock when combined with separate labor input, $n_{i,t}$. We refer to this separate labor input as implementation labor. Our law of motion is given by

$$k_{t+1} = \left(1 - \frac{\varphi_2 u_t^{\varphi_1}}{\varphi_1}\right) k_t + i_t \left(\psi_1 - \frac{\psi_2}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) + \psi_3 \left(\theta_i i_t^\kappa + \theta_n n_{i,t}^\kappa\right)^{\frac{1}{\kappa}}. \tag{6}$$

The term $\psi_2 \left(\theta_i i_t^\kappa + \theta_n n_{i,t}^\kappa\right)^{\frac{1}{\kappa}}$ represents our generalization of the law of motion consistent with the discussion in the previous section. Here $\theta_i, \theta_n \geq 0$ gauge the relative importance of investment and implementation services in producing physical capital and $\kappa \geq 0$ governs their substitutability.

The value of output in this economy is the sum of final goods production (our numeraire good) and the value of the services provided in putting investment goods into production. Looking ahead to an equilibrium, labor will have the same wage, w_t, whether employed in final goods production or investment implementation. The value of the implementation services, then will be equal to $w_t n_{i,t}$ and total output is given by

$$Y_t = y_t + w_t n_{i,t}. \tag{7}$$

To solve the model, the social planner chooses c_t, $n_{f,t}$, $n_{i,t}$, i_t, k_{t+1} and u_t to maximize (1) subject to equations (3), (6) and (2) and $n_{f,t} + n_{i,t} = n_t$. Substituting in for the last two constraints, and defining λ_t and $\lambda_{k,t}$ as the Lagrangian multipliers on the first two constraints, first order conditions are given by

$$c_t^{-\sigma} = \lambda_t \tag{8}$$

$$\phi \left(n_{f,t} + n_{i,t}\right)^\gamma = \lambda_t (1-\alpha) \frac{y_t}{n_{f,t}} \tag{9}$$

$$\phi \left(n_{f,t} + n_{i,t}\right)^\gamma = \lambda_{k,t} \theta_n \psi_3 \left(\theta_i i_t^\kappa + \theta_n n_{i,t}^\kappa\right)^{\frac{1}{\kappa}-1} n_{i,t}^{\kappa-1} \tag{10}$$

$$\lambda_{k,t} z\left(i_{t-1}, i_t, n_{i,t}\right) + \lambda_{k,t+1} \beta \psi_2 \frac{i_{t+1}^2}{i_t^2}\left(\frac{i_{t+1}}{i_t} - 1\right) = \frac{\lambda_t}{v_t} \tag{11}$$

$$\beta \lambda_{t+1} \frac{y_{t+1}}{k_{t+1}} \alpha + \beta \lambda_{k,t+1} \left(1 - \frac{u_{t+1}^{\varphi_1}}{\varphi_1}\right) = \lambda_{k,t} \tag{12}$$

$$\lambda_t \alpha y_t = \lambda_{k,t} \varphi_2 u_t^{\varphi_1} k_t. \tag{13}$$

where

$$z\left(i_{t-1}, i_t, n_{i,t}\right) \equiv \psi_1 - \frac{\psi_2}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 + \frac{\theta_i \psi_3 \left(\theta_i i_t^\kappa + \theta_n n_{i,t}^\kappa\right)^{\frac{1}{\kappa}-1}}{i_t^{1-\kappa}} - \frac{\psi_2 i_t}{i_{t-1}}\left(\frac{i_t}{i_{t-1}} - 1\right)$$

Combining equations (**??**) and (9) gives

$$\phi n_t^\gamma c_t^\sigma = (1-\alpha)\,\frac{y_t}{n_{f,t}} \tag{14}$$

which is the usual relationship equating the marginal rate of substitution between n_t and c_t to the marginal product of labor. As noted by Beaudry and Portier (2014), this fundamental relationship exposes two challenges to modeling new-driven business cycles. The first challenge is to have positive news cause an increase in c_t and the second is to preserve the equality given this increase.

To replicate key business cycle facts, consumption, investment and labor hours should all have positive comovement. Through the resource constraint, this will assure positive comovement with output. News of a positive future productivity shock may lead to an increase in current consumption. A future productivity increase is a positive lifetime income shock. The resulting consumption smoothing is a positive force affecting current consumption. At the same time, investment may increase as firms gear up for the anticipated productivity increase. This weighs in weighs in favor of decreased current consumption through the resources constraint. Absent an increase in output, increased current investment requires decreased current consumption. A news driven business cycle model must find a way for output to respond sufficiently, and its allocation to respond properly, such that consumption and investment both increase. This output increase cannot rely on technology changes or increased capital as these are fixed in the current period. This is what Beaudry and Portier refer to as the 'dynamic challenge' news driven business cycles.

A model that overcomes this dynamic challenge assures comovement between consumption, investment, and output. It still faces a 'static challenge' to assure positive comovement with hours.[7] With c_t increased, other general equilibrium adjustments must preserve equality in equation (14). This might be a decrease in n_t on the left-hand-side. However, this violates the required positive correlation between c_t and n_t. The other possibility is to increase the right-hand-side of the equality. That is, the model could generate an increase in the marginal product of labor when consumption increases. This, too, is problematic. In general, the marginal product of labor is decreasing in labor. To increase this would require less labor employed in producing the final good. We show later with numerical exercises that n_t and $n_{f,t}$ are positively correlated over a wide range or parameters and make this assumption for now. In this case, increasing the marginal product of labor through a decrease in $n_{f,t}$ means that n_t falls. This again violates positive correlation between c_t and n_t.

[7] We are again using the terminology of Beaudry and Portier (2014).

One feature of Jaimovich and Rebelo (2009) helps toward overcoming both the dynamic and the static problem. Variable capacity utilization in their model allows the economy to respond to future productivity by utilizing, and hence depreciating, capital at a higher rate. This allows output to increase with fixed capital and technology, creating the possibility that investment and consumption could both increase. A second feature of their model, properly calibrated adjustment costs, assures a proper allocation of this increased output and generates the appropriate relationship between c_t and i_t.

Aside from its part in solving the dynamic problem, variable capacity utilization mitigates, but does not eliminate, the static problem. To see this, substitute the production function in for y_t in (14) and simplify to get

$$c_t^{\sigma} = \frac{(1-\alpha)\, a_t}{\phi\alpha} \frac{(u_t k_t)^{\alpha}}{n_{f,t}^{\alpha} n_t^{\gamma}}. \tag{15}$$

With a_t and k_t fixed at time t, an increase in the left-hand side through an increase in c_t, and an increase in the numerator through comovement between c_t and $n_{f,t}$, might be accommodated through an increase in the capacity utilization rate. Essentially, an increase in u_t increases the marginal product of labor at any level of $n_{f,t}$. This allows for the possibility that both $n_{f,t}$ and the marginal product of labor could increase. The increase in $n_{f,t}$, through its impact on y_t, amplifies the effect of increased capacity utilization in overcoming the dynamic problem.

3.1 A special case

In this subsection we show that in a special case, capacity utilization falls short of overcoming the static problem. We consider a simpler law of motion where $\psi_1 = \psi_2 = \kappa = 0$ and $\theta_i = 1$ so that

$$k_{t+1} = \left(1 - \frac{\varphi_2 u_t^{\varphi_1}}{\varphi_1}\right) k_t + i_t n_{i,t}^{\theta_n} \tag{16}$$

In this case there are no adjustment costs, implementation labor scales investment is creating physical capital. Then equations (**??**), (13) and (2) yield

$$u_t = \left(\frac{v_t \alpha a_t}{k_t^{1-\alpha}} n_{i,t}^{\theta_n} n_{f,t}^{1-\alpha}\right)^{\frac{1}{\varphi_1 - \alpha}}. \tag{17}$$

Substituting in for you u_t in equation (15) gives

$$c_t^{\sigma} = \frac{x_1 n_{i,t}^{\frac{\theta_n \alpha}{\varphi_1 - \alpha}}}{n_t^{\gamma} n_{f,t}^{\frac{\alpha(\varphi_1 - 1)}{\varphi_1 - \alpha}}} \tag{18}$$

where x_1 is a scalar. All exponents in equation (18) are positive. With $\theta_n = 0$, hours show up only in the denominator so it not possible for c_t and hours worked to both increase while preserving the equality. With θ_n positive, labor hours are also in the numerator. This makes comovement possible as the denominator may increase more than the numerator when hours increase. It is not clear whether the denominator will increase sufficiently since $n_{i,t}$ and $n_{f,t}$ are related and $n_t = n_{i,t} + n_{f,t}$. However, when preferences are logarithmic in consumption and linear in the labor input $(\sigma = 1, \gamma = 0)$, $n_{f,t}$ and $n_{i,t}$ are linearly related to n_t such that

$$n_{i,t} = \frac{\theta_n n_t}{\theta_n + 1 - \alpha} - \frac{\theta_n (1-\alpha)}{\phi(\theta_n + 1 - \alpha)} \tag{19}$$

$$n_{f,t} = \frac{n_t (1-\alpha)}{\theta_n + 1 - \alpha} + \frac{\theta_n (1-\alpha)}{\phi(\theta_n + 1 - \alpha)}. \tag{20}$$

We show in Appendix that in this case $\theta > (\varphi_1 - 1)$ is a sufficient condition for $\frac{\partial c_t}{\partial n_t} > 0$ in equation (18).

Returning to equation (14), the static problem is that (i) the marginal rate of substitution is increasing in consumption and (ii) the marginal product of labor is decreasing in labor. Either (i) or (ii) must be overcome in some way to allow consumption and labor to simultaneously increase. Capacity utilization in Jaimovich and Rebelo (2009) is not enough to overcome (ii). They instead overcome (i) by introducing a more generalized set of preferences that allows the MRS to decrease in c_t. We instead allow a more general setting in the law of motion for capital to overcome (ii). This works through increasing the response of capacity utilization to a news shock.

It may appear that our model is highly susceptibility to a particular concern with using capacity utilization as the channel through which news drives business cycles. With variable capacity utilization, a news shock increases capital through increased investment. However, it also decreases capital through increased depreciation. This can associate news shocks with decreased capital in the subsequent period. As our model gives increased scope to capacity utilization, the second effect is amplified. In our model, however, there is another feature favoring an increase in capital. As seen in equation (17) capacity utilization is associated with an increase in $n_{i,t}$. This, in turn, is associated with more productive implementation of investment through equation (6). This makes it easier for our model to generate an increase in k_{t+1} resulting from a news shock. We return to this point in our numerical exercises where we show capital accumulation in our baseline model.

A nice feature of the mechanism we introduce is that it makes it easier for news shocks to be associated with current increases in labor productivity. This is easiest to see in the final goods market. Using equations (2) and (17) we arrive at the following expression for labor productivity:

$$\frac{y_t}{n_{f,t}} = \left(\frac{a_t^{\varphi_1} v_t^{\alpha} \alpha^{\alpha} k_t^{\varphi_1 - 1} n_{i,t}^{\alpha \theta_n}}{n_{f,t}^{\alpha(\varphi_1 - 1)}} \right)^{\frac{1}{\varphi_1 - \alpha}}$$

Each exponent here is positive. We see that with $\theta = 0$, hours appear only in the denominator so productivity and hours move in opposite directions. With $\theta > 0$, both can increase. In the special case expressed in equations (19) and (20), we can show that $\theta > (\varphi - 1)$ is a sufficient condition for positive comovement between hours and labor productivity.

3.2 The static problem with no adjustment costs.

The argument above suggests that implementation effort can overcome the static challenge of news driven business cycles and shows this analytically for a special case. It does not suggest that this mechanism is helpful in overcoming the dynamic problem, and indeed we find that is not. As in Rebelo and Jaimovich (2009) we need adjustment costs in order for positive productivity shocks to yield both an increase is consumption and positive comovement between consumption, investment, labor hours, and output. In this subsection we omit adjustment costs in order to focus on overcoming the static problem. Without adjustment costs, consumption decreases when the model generates the proper comovements. When we later add adjustment costs, the model generates these comovements along with an increase in consumption.

We first consider our special case above and then relax several of the assumption in turn to show their impact on the comovement between consumption, hours, and investment. We show in an unpublished appendix that in our special case the model can be indeterminate and find sufficient conditions where this can hold. For current purposes, it is sufficient to have some intuition for why this can occur. Let s_t be the endogenously determined share of the final good that is invested in period t so that from equation (3) $i_t = s_t v_t y_t$. From Then with no adjustment costs we can write equation (6) as

$$k_{t+1} - k_t + \frac{\varphi_2 u_t^{\varphi_1}}{\varphi_1} = \psi_3 \left(\theta_i \left(s_t v_t y_t \right)^{\kappa} + \theta_n n_{i,t}^{\kappa} \right)^{\frac{1}{\kappa}}.$$

The left-hand side of this is gross capital formation. Gross capital formation is a function of y_t and $n_{i,t}$. The final good, y_t, is constant returns to scale in k_t and $n_{f,t}$. Given this, gross capital formation is constant returns to scale in all inputs if $\theta_i + \theta_n = 1$. However, if $\theta_i + \theta_n > 1$, we have increasing returns to scale in this aggregate. Prior literature shows that indeterminacy can arise in models with increasing returns to scale in the production of the final good. Increasing returns to scale in gross capital formation gives rise to similar concerns in our model. While this is most clear in our special case, the issue of indeterminacy is robust. In particular we show in numerical exercises that the model can generally be indeterminate when θ_n+ θ_n exceeds 1 and β or φ_1 is sufficiently large.

Our goal is to generate news driven business cycles in a deterministic setting rather than consider sunspot equilibria. For this reason, the possibility of indeterminacy restricts our parameter choices. The model in determinate in our baseline calibration of the full model below. This holds also for the sensitivity analysis conducted around the baseline. However, the parameter restrictions in this special case are more severe. In particular, we show that sufficient conditions for indeterminacy restrict us to relatively small values of β. With small values of β the current response to future productivity changes is small. Nonetheless, the special case is useful for showing how implementation effort can yield the proper directional changes in the aggregates of interest.

Figure 1 below shows the current period response to news of a one standard deviation total factor productivity news shock to arrive in period 2. It common in the literature to consider news that pre-dates productivity shocks by multiple periods. We choose a one-period-ahead shock for this discussion only to make the response larger for expositional purposes. In this simple setting we find that β must be set at a relatively low level to avoid indeterminacy. We set $\beta = .82$ which means that shocks further in the future have a small impact in the current period. Our qualitative findings are not sensitive to this and we later consider a more standard time frame. We set parameters in the baseline for this exercise consistent with our special case: $\psi_1 = \psi_2 = \kappa = \gamma = 0$ and $\sigma = 1$. We further $\alpha = .33$, $\varphi_1 = 1.1$, set $v_t = \psi_2 = 1$ and $\rho_a = .9$. The parameter φ_2 influences the capacity utilization rate. With this set to 1, we have a capacity utilization rate in excess of 1. While this does not cause mathematical problems in the model, it creates as challenge for interpreting the results. For this reason we set $\varphi_2 = 5$. While the dynamics are qualitatively similar between $\varphi_2 = 5$ and $\varphi_2 = 1$, this larger value allows for reasonable capacity utilitization values.

The main point of figure 1 that the model can overcome the static problem with while θ large but cannot with θ small. While we show this only for the special case and for the total factor productivity shock, we find that it holds also with the an investment specific shock and over a wide range of parameter values.

4 Result

This subsection quantitatively examines a calibrated version of our model in response to agent's optimistic expectation about an upcoming change in productivity in the economy while maintaining saddle path stability and unique equilibrium. We assume, at time zero the economy is in steady state. At period one an anticipated shock arrive. Agent learns that there will be a one percentage increase in TFP or Investment Specific Technological (IST) change three periods later, in period 4. Following one standard deviation of news shock, the level of a_t and v_t does not increase immediately, by construction, but rises sharply at period 4 and go down to zero over the horizon.

This subsection quantitatively examines a calibrated version of our model in response to agent's optimistic expectation of an upcoming change in productivity in the economy while maintaining saddle path stability and unique equilibrium. We begin by looking at the model in the special case in section 2.1. For the particular example, we assume the news shock arrives one period before the actual shock realizes. We focus on the response of major macroeconomic variables namely consumption, investment and labor hour in response to TFP news shock for different values θ_i, keeping θ_n fixed at 0.1. We assume the news shock arrive one period before the shock materialized. We focus only on news shock for broad range values of θ_i, since this is where we contribute to the literature by showing how our feature help to produce comovement among the variables. Figure 1 shows percentage deviation of each variable from its steady state value due to the TFP shock for a range of θ_i. In the first panel, the solid line represents the percentage deviation of consumption from it's the steady state in period 1 when the news shock arrives. Consumption goes down when θ_i is very low, then gradually increases and drop again when θ_i is high. In the second panel, the solid line and the dashed line belongs to the response of labor hour and investment for news shock in period 1. Both of the variables response positively with low θ_i and falls gradually with higher θ_i.Comparing both of the panels, it is clear that without our feature, when θ_i is low, consumption, investment and labor hour moves in opposite direction. They all move in the same direction when our feature become prominent, i.e., θ_i is high. The results are unchanged when we consider variation in θ_n holding θ_i constant. The analysis indicates our model can achieve comovement among the variables with substitutability of investment and implementation labor hour and without adjustment cost.

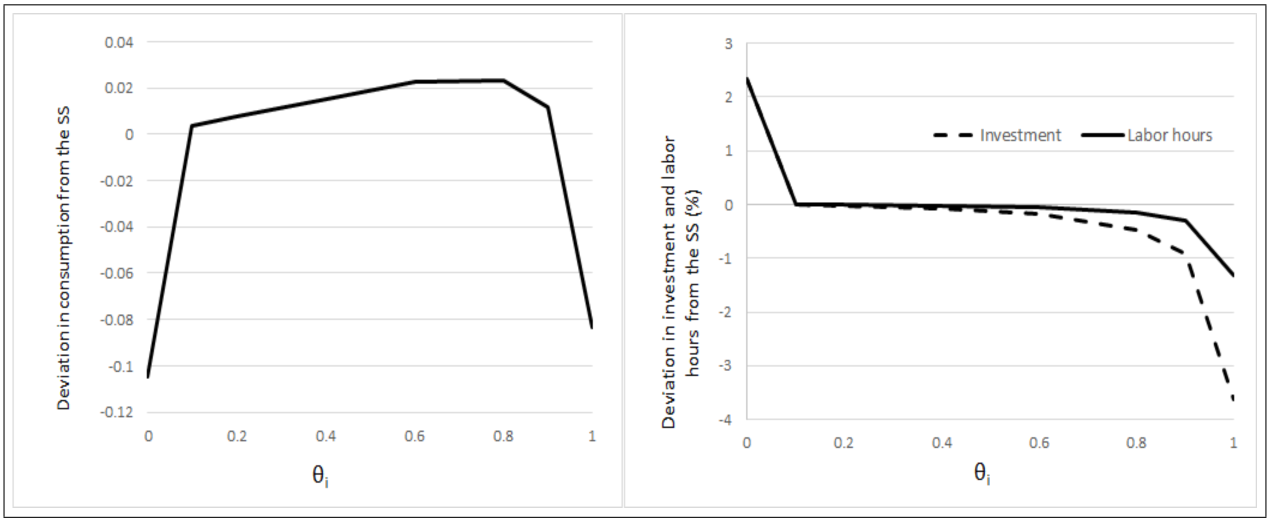

Figure 1:Figure 1: Comovements for different values of θ_i

We interpret the result as following economic process. From the Equation (16), $\theta_i + \theta_n < 1$ and $\theta_i + \theta_n > 1$ make the investment function decreasing returns to scale(DRTS) and increasing returns to scale (IRTS) respectively. With DRTS, it is optimal for the agent to increase labor hour and reduce the consumption when the news arrives as the marginal productivity of labor may fall in future. The shock will realize in next period, the return on hourly labor will increase, and the agent intends to supply more labor. If they postponed labor supply today and tend to work more in next periods, their future productivity will go down rapidly for the decreasing return to scale in investment production. The opposite is true for IRTS. When the marginal productivity of labor increases with more labor hour, agent will hold supply of labor till the shock materialize, because, the return of labor would be much higher with rising in labor supply. In summary, IRTS in investment is very important in our special case to generate comovements.

Figure 2 shows the impulse response results of our baseline model that use CES investment function as a measure of complementarity between investment and implementation labor. In each figure, eight panels are plotted for ten periods. In every panel, the solid line represents the impulse response of the respective variable obtained from the TFP news shock. We assume, at time zero the economy is in steady state. At period one an anticipated shock arrive. Agent learns that there will be a one percentage increase in TFP or Investment Specific Technological (IST) change three periods later, in period 4. Following one standard deviation of news shock, the level of a_t and v_t does not increase immediately, by construction, but rises sharply at period 4 and go down to zero over the horizon.

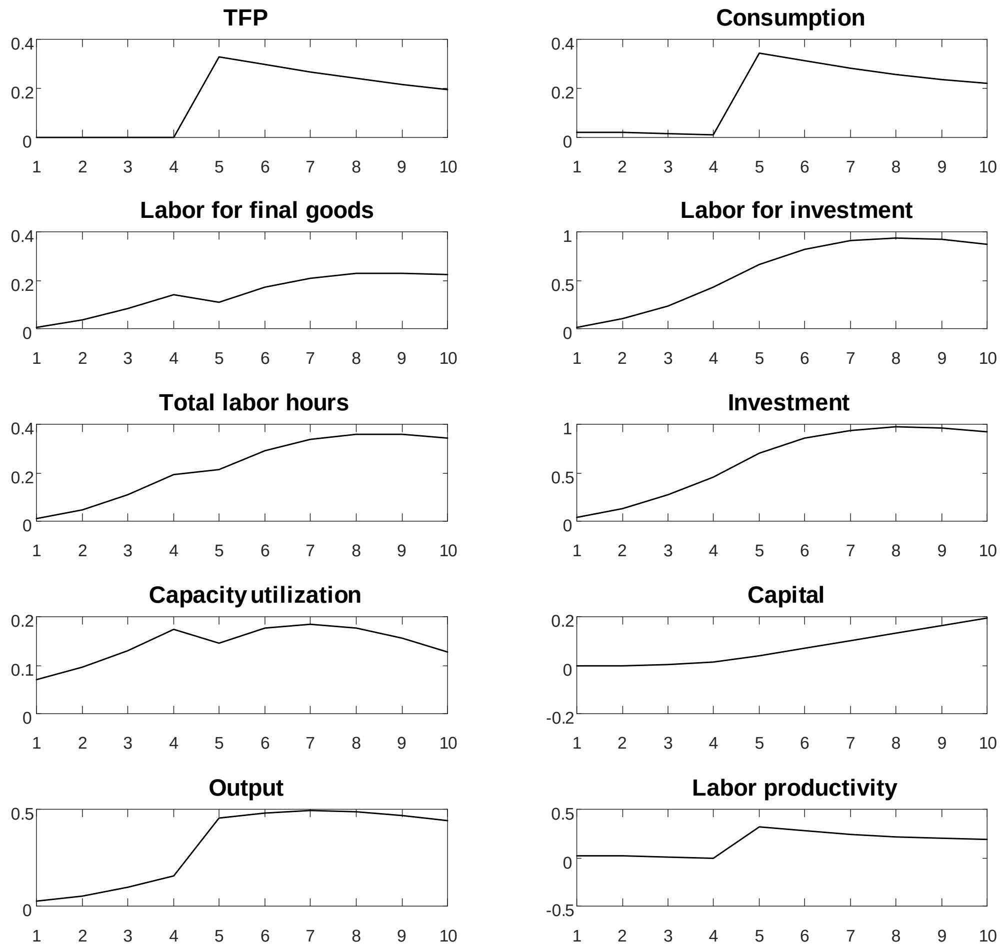

Figure 2: Impulse responses from TFP shocks in the baseline model

We adopt the following parameterization that is commonly used in the real business cycle literature the income share of capital $\alpha = 0.33$, the discount factor $\beta = 0.985$, labor supply elasticity $\gamma = 0$ (i.e., perfectly elastic or indivisible labor supply) and $\sigma = 1$ (logarithmic utility in consumption). The preference parameter $\phi = 1$, the capital utilization parameter $\varphi = 1.3$, the adjustment cost parameters $\Psi_1 = 1, \Psi_2 = 2$, and $\Psi_3 = 3$. We consider the elasticity of substitution parameter $\kappa = -8$, negatively infinity so we get a Leontief, or perfect complements investment function. The relative importance of investment θ_i and implementation labor θ_n in capital production are 1 and 0.1 respectively.

All the panels show similarities between the impulse response patterns in response to news shock of total productivity. In all cases, there is an expansion in periods one to three even the positive shock occurs only in period four. Once a_t shocks materialize it directly impact on the output. On the other hand, news about the future increase in a_t affect output through changes in labor demand and the capital utilization. An expectation of future technological change increases demands for investment today for investment adjustment cost. As investment requires 'implementation labor', labor demand increases, that raise the marginal productivity of labor. However, the capital stock falls initially as the capital utilization dominates the investment. Firms increase the capital utilization as the current capital stock will be obsolete and more efficient capital will available in the near future. This provide an incentive for labor hour in final good production to rise. So, our one sector model with perfect complement able to generate quantitatively realistic business cycles driven solely by agents' changing expectations about future consumption demand.

Figure 3 presents the impulse responses for one time positive investment specific technological shock. When the news about more efficient and cheaper investment goods in future arrives in period one, all except the capital stock increases on impact. Labor hours of both types, investment, output and labor productivity all gradually increase. Consumption falls between period one to four but rises again when the technology is available. Our results from both news shock are persistent with Bresnahan, Brynjolfsson, and Hitt (2002), where they empirically show investment and workplace organization labor are complements to each other and adjustment cost plays an important role to rising demand for both of them together. In our analysis, perfect complements and adjustment cost able to produce comovement among the variables due to an expectation about future productivity.

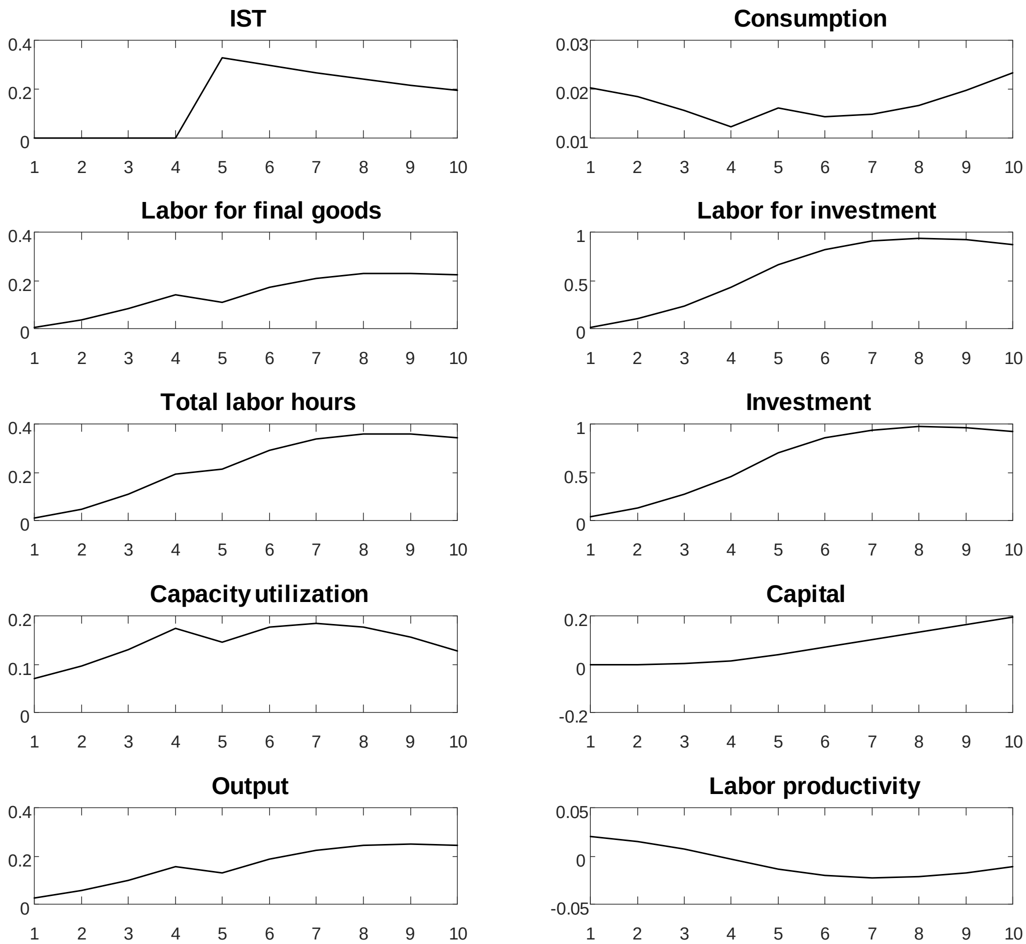

Figure 1: Figure 3: Impulse responses from IST shocks in the baseline model

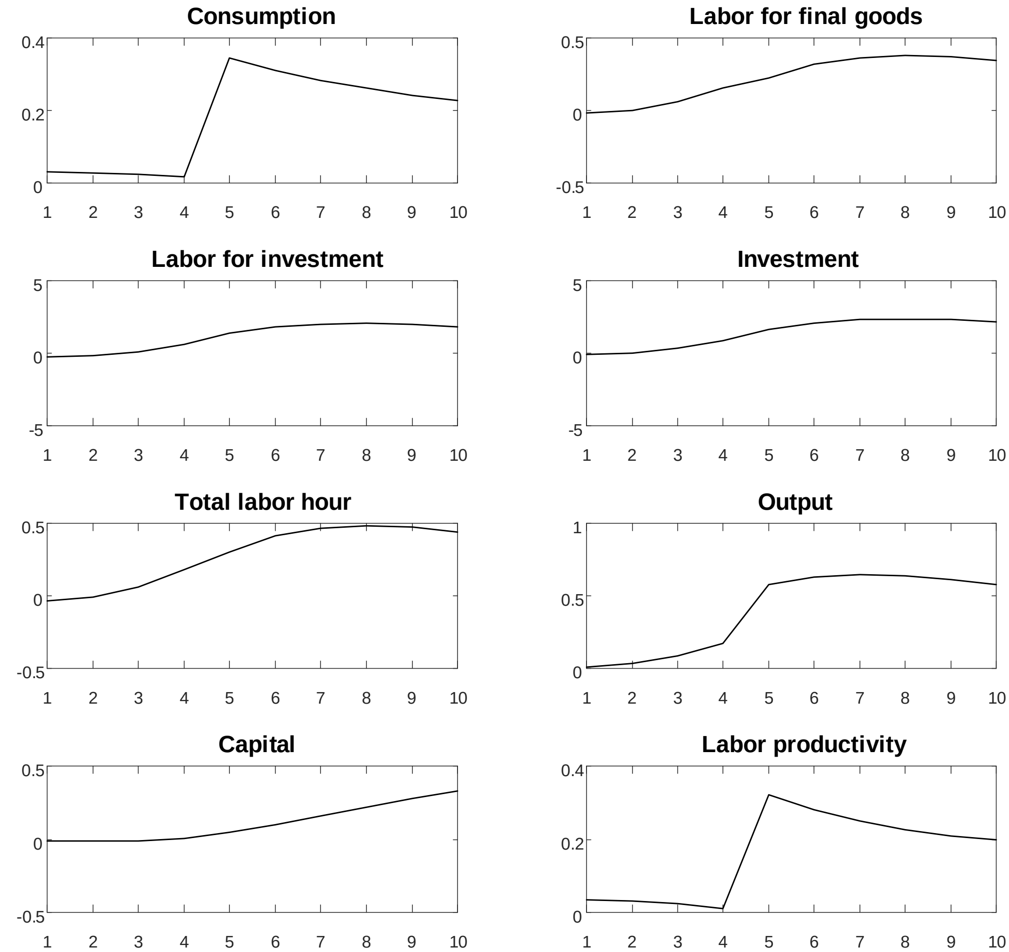

Fig 4: Impulse responses from TFP shocks with high elasticity of substitution

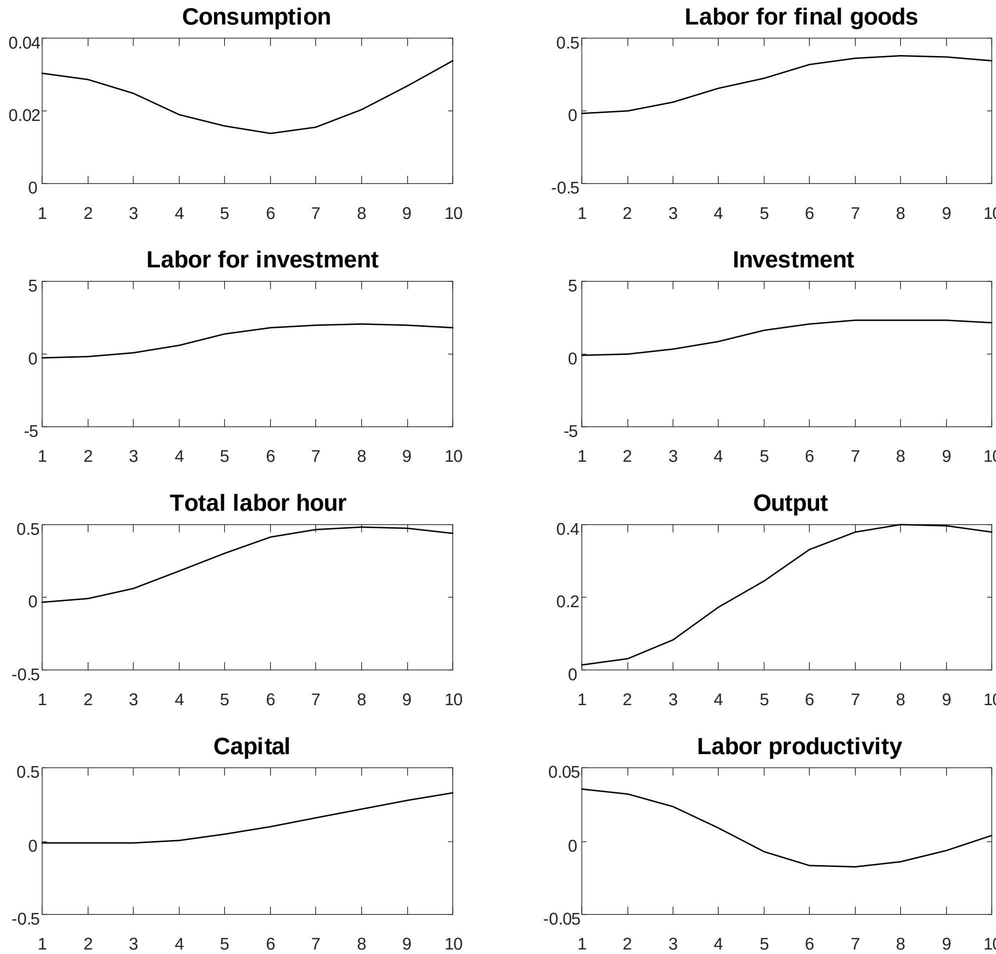

Figure 5: Impulse responses from IST shocks with high elasticity of substitution

Figure 4 and Figure 5 correspond to the impulse responses for TFP and IST news shock when the two inputs of capital production i and n_i, are substitutable. We consider the elasticity parameter $\kappa = 0.5$ in the baseline model. When investment and implementation labor are a substitute for each other, the model does not generate positive movements among the variables in response to anticipated technological changes. We see from the IRFs that labor of both types, total labor, investment, and capital fall immediately. The output do not change but consumption increase on impact. This results shows a notable difference compare to the previous case. It is necessary to have a perfect complementary relationship between the investment and implementation labor input that Bresnahan, Brynjolfsson, and Hitt (2002) empirically observed.

Now we will focus on the relative importance of investment θ_i and θ_n for producing new-driven business cycles. Figure 6 and 7 present impulse responses in the baseline model with θ_n is very low at 0.001 and $\theta_i = 1$ for TFP and IST news shock respectively. In both cases, a low share of implementation labor in the capital production not generate comovement. Intuitively, a boost of labor demand is required to increasing labor productivity and output of the economy to exhibit positive comovements. Hence, the share of labor in capital production needs to high enough to rise in labor demand. We interpret this finding that without moderate importance of labor input in capital production, proper movements among variable is not achievable.

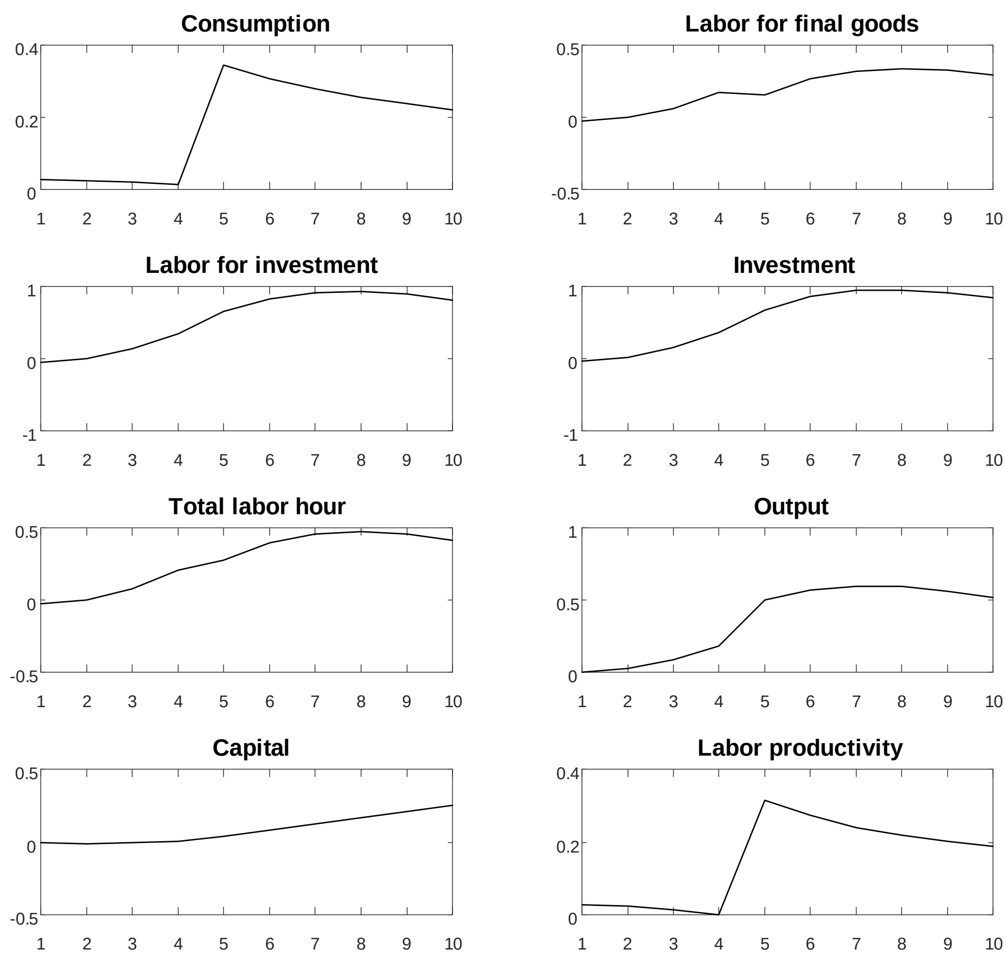

Figure 6: Impulse responses from TFP shocks in the model with $\theta_n = 0.001$

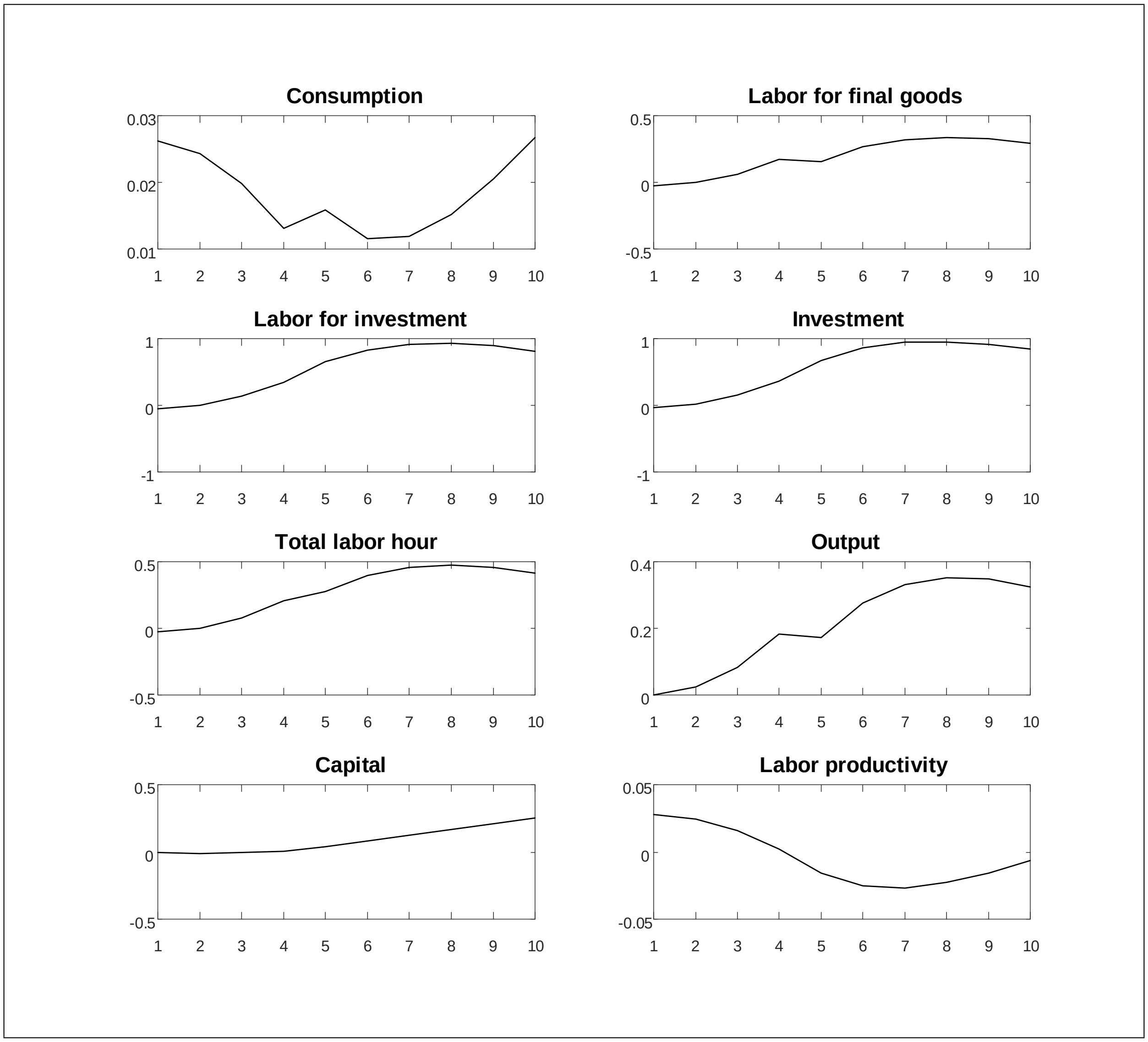

Figure 7: Impulse responses from IST shocks in the model with $\theta_n = 0.001$

In summary, we show that introducing implementation labor in capital production able to produce realistic business cycles in response to a positive expectation about the future technological changes. Unlike standard business cycle theory, where the wealth effect of good news about future productivity increases both current consumption and leisure, thus, decreasing investment and labor hours causing the output to decline, here the positive comovement among the macroeconomic aggregates is coming through different channels. The economic interpretations of our news driven business cycles generated from qualitative analysis IRFs are as follows. First, the responses of news follow directly from the fact that the agent anticipates a high future demand and try to invest early as to make goods available when the needs eventually appear. Investment in capital goods requires labor for implementation. Hence increase in investment directly increase labor demand. Second, following a favorable technological news, agent's incentives to accumulate capital would decrease as it anticipates that existing capital stock would obsolete shortly. The agent would accelerate capacity utilization leading to rapid depreciation of the current capital stock. In one side, the capital stock depreciates, for more utilization, on the other hand, capital increase from rising investment. In our numerical analysis, the capital shock is falling on impact in news shock indicates the first effect dominate the second one. Third, the increased capacity utilization rises the marginal productivity of labor and employment. So far, news increase investment, labor demand and consumption even there is no fundamental changes occur in the economy. Agent feels wealthier due to the expectation about higher future labor productivity,which leads to rising consumption today. They start consuming early even before the realization of news. Increasing marginal productivity of labor and employment also make a positive impact on consumption. Our result is closely consistent with the empirical finding by Bresnahan et al. (2002).

5 Conclusion

The literature related to news-driven business cycles explores several modifications of the baseline real business cycle model that overcomes the static problem. We show that implementation labor provides an additional useful modification of this sort. We begin by arguing that purchasing investment goods does not directly increase the productive capacity of the firm. Workplace reorganization, new management, training and screening of new workers are often required with the changes in the firm. We motivated this from a recent empirical finding that innovation of technology is highly correlated with the changes in firms' workplace organization that increases the labor demand (Bresnahan et al., 2002). We build a real business cycle model that captures these changes in the firm through complementarity between investment and labor input. In essence, a news shock influences the supply of labor used to implement new investment capital as well as labor used in the production of a final good. Labor used for implementation enhances capital accumulation but has no impact on current production. Importantly, then, this labor has no direct effect on the marginal product of labor. This allows more freedom of movement between the marginal rate of substitution and total labor employed. We show that a real business cycle model with our feature, capacity utilization, adjustment cost and separable preferences can yield realistic business cycle due to news shock of technological changes.

We first consider a special case of our model with no adjustment costs which allows us to analytically examine conditions allowing positive comovement between consumption, investment, and total labor hours. Importantly, we show that our new feature gives a boost to the effects of the capacity utilization rate. With this boost, capacity utilization can respond sufficiently to a news shock to allow a general equilibrium increase in the marginal product of labor at the same time that labor employed increases. Absent implementation labor, this cannot occur. Then we apply the numerical approach to solve the dynamic problem for the general case. In particular, we include adjustment costs and considers the impact of implementation labor in our full model. We show that our model with implementation labor, variable capacity utilization, and investment adjustment cost can generate qualitatively realistic aggregate fluctuations driven by news shock to total factor productivity and investment-specific technological change. Additionally, in the existing literature of expectation driven business cycles, the results rely on non-separable preferences. Our analysis does not rely on preference rather a new law of motion of capital that considers labor for implementing the investment. When the economy is subject to anticipated technological shock, the relative importance of implementation labor in capital production is needed to be enhanced for strong labor demand in the economy to exhibit positive comovements among macroeconomic variables.

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6 Appendix

This section shows that sufficient condition for $\frac{\partial c_t}{\partial n_t} > 0$ is $\theta > (\varphi - 1)$.

$$c_t = \frac{x_1\left(\frac{\theta n_t}{\theta+1-\alpha} - \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)^{\frac{\theta\alpha}{\varphi-\alpha}}}{\left(\frac{n_t(1-\alpha)}{(\theta+1-\alpha)} + \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)^{\frac{\alpha(\varphi-1)}{\varphi-\alpha}}}$$

$$\frac{\begin{array}{c}\left(\frac{n_t(1-\alpha)}{(\theta+1-\alpha)} + \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)^{\frac{\alpha(\varphi-1)}{\varphi-\alpha}} x_1 \frac{\theta\alpha}{\varphi-\alpha}\left(\frac{\theta n_t}{\theta+1-\alpha} - \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)^{\frac{\theta\alpha}{\varphi-\alpha}-1} \frac{\theta}{\theta+1-\alpha} \\ -x_1\left(\frac{\theta n_t}{\theta+1-\alpha} - \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)^{\frac{\theta\alpha}{\varphi-\alpha}} \frac{\alpha(\varphi-1)}{\varphi-\alpha}\left(\frac{n_t(1-\alpha)}{(\theta+1-\alpha)} + \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)^{\frac{\alpha(\varphi-1)}{\varphi-\alpha}-1} \frac{(1-\alpha)}{\phi(\theta+1-\alpha)}\end{array}}{\left(\frac{n_t(1-\alpha)}{(\theta+1-\alpha)} + \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)^{\left(\frac{\alpha(\varphi-1)}{\varphi-\alpha}\right)^2}} > 0$$

$$\left(\frac{n_t(1-\alpha)}{(\theta+1-\alpha)} + \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)^{\frac{\alpha(\varphi-1)}{\varphi-\alpha}} x_1 \frac{\theta\alpha}{\varphi-\alpha}\left(\frac{\theta n_t}{\theta+1-\alpha} - \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)^{\frac{\theta\alpha}{\varphi-\alpha}-1} \frac{\theta}{\theta+1-\alpha} \tag{21}$$

$$> \quad x_1\left(\frac{\theta n_t}{\theta+1-\alpha} - \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)^{\frac{\theta\alpha}{\varphi-\alpha}} \frac{\alpha(\varphi-1)}{\varphi-\alpha}\left(\frac{n_t(1-\alpha)}{\phi(\theta+1-\alpha)} + \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)^{\frac{\alpha(\varphi-1)}{\varphi-\alpha}-1} \frac{n_t(1-\alpha)}{(\theta+1-\alpha)} \tag{22}$$

$$\left(\frac{n_t(1-\alpha)}{(\theta+1-\alpha)} + \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)\theta\alpha\theta > \left(\frac{\theta n_t}{\theta+1-\alpha} - \frac{\theta(1-\alpha)}{\phi(\theta+1-\alpha)}\right)\alpha(\varphi-1)(1-\alpha)$$

$$\left(\frac{n_t}{(\theta+1-\alpha)} + \frac{\theta}{\phi(\theta+1-\alpha)}\right)\theta\alpha > \left(\frac{n_t}{\theta+1-\alpha} - \frac{(1-\alpha)}{\phi(\theta+1-\alpha)}\right)\alpha(\varphi-1)$$

$$\left(n_t + \frac{\theta}{\phi}\right)\theta\alpha > \left(n_t - \frac{(1-\alpha)}{\phi}\right)\alpha(\varphi-1)$$

$$(\phi n_t + \theta)\theta > (n_t\phi - (1-\alpha))(\varphi-1)$$

$$\phi n_t(\theta - (\varphi-1)) > -(1-\alpha)(\varphi-1) - \theta^2$$

Since the R.H.S is 0 and $\phi, n_t \geq 0$, this will hold if $(\theta - (\varphi - 1)) > 0$

Find the relationship between $n_{f,t}$ and $n_{i.t.}$

$$c_t^{-\sigma} = \lambda_t$$
$$\phi\left(n_{f,t} + n_{i,t}\right)^\gamma = \lambda_t \left(1-\alpha\right) \frac{y_t}{n_{f,t}}$$
$$\phi\left(n_{f,t} + n_{i,t}\right)^\gamma = \lambda_{k,t} \psi_2 i_t n_{i,t}^{\theta-1}$$
$$\lambda_{k,t}\left(\psi_1 + \frac{\psi_2 n_{i,t}^\theta}{\theta} - \frac{\psi_3}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) + \lambda_{k,t+1}\beta\psi_3 \frac{i_{t+1}^2}{i_t^2}\left(\frac{i_{t+1}}{i_t} - 1\right) = \frac{\lambda_t}{v_t} + \lambda_{k,t}\psi_3 \frac{i_t}{i_{t-1}}\left(\frac{i_t}{i_{t-1}} - 1\right)$$
$$\beta\lambda_{t+1}\frac{y_{t+1}}{k_{t+1}}\alpha + \beta\lambda_{k,t+1}\left(1 - \frac{1}{\varphi}u_{t+1}^\varphi\right) = \lambda_{k,t}$$
$$\lambda_t \alpha y_t = \lambda_{k,t} u_t^\varphi k_t.$$

$$c_t^{-1} = \lambda_t$$
$$\phi = \lambda_t \left(1-\alpha\right)\frac{y_t}{n_{f,t}}$$
$$\phi = \lambda_{k,t}\psi_2 i_t n_{i,t}^{\theta-1}$$
$$\lambda_{k,t}\left(\psi_1 + \frac{\psi_2 n_{i,t}^\theta}{\theta}\right) = \frac{\lambda_t}{v_t}$$
$$\frac{\lambda_t \alpha y_t}{u_t^\varphi k_t} = \lambda_{k,t}.$$

$$n_{f,t} c_t \phi = \left(1-\alpha\right) y_t$$
$$c_t \phi = \frac{\alpha y_t}{u_t^\varphi k_t}\psi_2 i_t n_{i,t}^{\theta-1}$$
$$\phi = \lambda_{k,t}\psi_2 i_t n_{i,t}^{\theta-1}$$
$$\lambda_{k,t}\left(\psi_1 + \frac{\psi_2 n_{i,t}^\theta}{\theta}\right) = \frac{\lambda_t}{v_t}$$
$$= \lambda_{k,t}.$$

$$n_{f,t} c_t \phi = \left(1-\alpha\right) y_t$$
$$c_t k_t u_t^\varphi n_{i,t}^{1-\theta}\phi = \alpha y_t \psi_2 i_t$$
$$\frac{\alpha y_t}{k_t u_t^\varphi}\left(\psi_1 + \frac{\psi_2 n_{i,t}^\theta}{\theta}\right) = \frac{1}{v_t}$$

$$i_t = v_t\left(y_t - c_t\right)$$

$$c_t k_t u_t^{\varphi} n_{i,t}^{1-\theta}\phi = \alpha y_t \psi_2 \left(v_t \left(y_t - c_t\right)\right)$$

$$c_t \frac{n_{f,t}\phi}{1-\alpha} = y_t$$

$$c_t k_t u_t^{\varphi} n_{i,t}^{1-\theta}\phi = \alpha y_t \psi_2 \left(v_t \left(c_t \frac{n_{f,t}\phi}{1-\alpha} - c_t\right)\right)$$

$$k_t u_t^{\varphi} n_{i,t}^{1-\theta}\phi = \alpha y_t \psi_2 \left(v_t \left(\frac{n_{f,t}\phi}{1-\alpha} - 1\right)\right)$$

$$\frac{1}{v_t} k_t u_t^{\varphi} n_{i,t}^{1-\theta}\phi = \alpha y_t \psi_2 \left(\frac{n_{f,t}\phi}{1-\alpha} - 1\right)$$

$$\frac{\alpha y_t}{k_t u_t^{\varphi}} \left(\psi_1 + \frac{\psi_2 n_{i,t}^{\theta}}{\theta}\right) = \frac{1}{v_t}$$

$$\frac{\alpha y_t}{k_t u_t^{\varphi}} \left(\psi_1 + \frac{\psi_2 n_{i,t}^{\theta}}{\theta}\right) k_t u_t^{\varphi} n_{i,t}^{1-\theta}\phi = \alpha y_t \psi_2 \left(\frac{n_{f,t}\phi}{(1-\alpha)} - 1\right)$$

$$\left(\psi_1 + \frac{\psi_2 n_{i,t}^{\theta}}{\theta}\right) n_{i,t}^{1-\theta}\phi = \psi_2 \left(\frac{n_{f,t}\phi}{(1-\alpha)} - 1\right)$$

$$\psi_1 (1-\alpha) n_{i,t}^{1-\theta}\phi + \frac{\psi_2 n_{i,t}^{\theta}}{\theta} (1-\alpha) n_{i,t}^{1-\theta}\phi = \psi_2 n_{f,t}\phi - \psi_2 (1-\alpha)$$

$$\theta\psi_1 (1-\alpha) n_{i,t}^{1-\theta}\phi + \psi_2 n_{i,t}^{\theta} (1-\alpha) n_{i,t}^{1-\theta}\phi = \theta\psi_2 n_{f,t}\phi - \theta\psi_2 (1-\alpha)$$

$$\theta\psi_1 (1-\alpha) n_{i,t}^{1-\theta}\phi + \psi_2 n_{i,t} (1-\alpha)\phi = \theta\psi_2 n_{f,t}\phi - \theta\psi_2 (1-\alpha)$$

$$\begin{aligned}
n_{f,t} &= \frac{\theta\psi_1 (1-\alpha) n_{i,t}^{1-\theta}\phi + \psi_2 n_{i,t} (1-\alpha)\phi + \theta\psi_2 (1-\alpha)}{\theta\psi_2\phi} \\
n_t &= \frac{\theta\psi_1 (1-\alpha) n_{i,t}^{1-\theta}\phi + \psi_2 n_{i,t} (1-\alpha)\phi + \theta\psi_2 (1-\alpha)}{\theta\psi_2\phi} + n_{i,t} \\
n_t &= \frac{\theta\psi_1 (1-\alpha) n_{i,t}^{1-\theta}\phi + (\psi_2 n_{i,t} (1-\alpha)\phi) + \theta\psi_2 (1-\alpha) + \theta\psi_2\phi n_{i,t}}{\theta\psi_2\phi} \\
n_t &= \frac{\theta\psi_1 (1-\alpha) n_{i,t}^{1-\theta}\phi + n_{i,t}\psi_2\phi (1-\alpha+\theta) + \theta\psi_2 (1-\alpha)}{\theta\psi_2\phi} \\
\frac{\partial n_t}{\partial n_{i,t}} &= \frac{(1-\theta)\psi_1 (1-\alpha)}{\psi_2 n_{i,t}^{\theta}} + \frac{1-\alpha+\theta}{\theta}
\end{aligned}$$

Let $\psi_1 = 0$

$$n_{f,t} = \frac{\psi_2 n_{i,t} (1-\alpha)\phi + \theta\psi_2 (1-\alpha)}{\theta\psi_2\phi}$$

$$n_{f,t} = \frac{n_{i,t} (1-\alpha)\phi + \theta (1-\alpha)}{\theta\phi}$$

$$n_{f,t} = \frac{n_{i,t} (1-\alpha)}{\theta} + \frac{(1-\alpha)}{\phi}$$

$$n_t = \frac{n_{i,t}\left(1-\alpha\right)}{\theta} + \frac{\left(1-\alpha\right)}{\phi} + n_{i,t}$$

$$n_t = \frac{n_{i,t}\left(1-\alpha\right)}{\theta} + \frac{\left(1-\alpha\right)}{\phi} + n_{i,t}$$

$$\begin{aligned}
n_{i,t} &= -\frac{n_t + \frac{1}{\phi}\left(\alpha-1\right)}{\frac{1}{\theta}\left(\alpha-1\right)-1} \\
n_{i,t} &= \frac{\phi\theta n_t + \theta\left(\alpha-1\right)}{\phi\left(\left(1-\alpha\right)+\theta\right)} \\
n_{i,t} &= \frac{n_t}{1-\alpha+\theta} - \frac{\theta\left(1-\alpha\right)}{\phi\left(1-\alpha+\theta\right)}
\end{aligned}$$