The Grandfather of Price Discrimination*

Hyunsoo Kim[†] Brady Vaughan[‡] Aleksandr Yankelevich[§]

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Abstract

We examine firms' motivations for implementing grandfather clauses that allow certain consumers to continue access to a service at a favorable, but no longer available price. We find that when consumers are fully cognizant of their valuations for available product alternatives, firms are typically better off offering all potential consumers the optimal uniform price. However, if grandfathered consumers are made complacent, failing to reevaluate the service over time, grandfather clauses may permit firms to profitably price discriminate between early adopters and new consumers in exchange for forfeiting the right to optimally set prices for early adopters.

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[†]Department of Economics, Michigan State University, 486 W Circle Dr, East Lansing, MI 48824. e-mail address: kimhyu82@msu.edu.

[‡]Soar Technologies Inc., 3600 Green Ct # 600, Ann Arbor, MI 48105. Unaffiliated for research. e-mail address: bvaughan@wustl.edu.

[§]Corresponding author. Brandmeyer Center for Applied Economics, University of Kansas School School of Business, 1654 Naismith Drive, Lawrence, KS 66045. e-mail address: alyankelevich@ku.edu.

1 Introduction

A grandfather clause exempts certain groups of people from a wider change in circumstances. Historically, grandfather clauses were found in a variety of legal contexts.[1] However, as we discuss in greater detail below, grandfather clauses have become common in retail, particularly in online and telecommunications services. In retail, grandfather clauses typically exempt existing subscribers of a service from a price hike or allow them to continue consuming a discontinued service. Thus, grandfather clauses permit firms to discriminate between early adopters and new consumers of their service, albeit at a cost: discrimination prevents the firm from pricing optimally to early adopters.

Grandfather clauses substantially fall into two categories: (i) clauses where grandfathered customers are permitted to continue to pay the same price that they paid in the past, possibly for a product generally viewed as an improvement, whereas new customers are forced to pay a higher price, and (ii) clauses where grandfathered customers end up consuming a distinctly different product from that made available to new consumers. In this manuscript we focus primarily on the profit and welfare implications of the former category with the aim of understanding when businesses might be able to profitably rely on grandfather clauses and whether doing so makes consumers better off. The advantage of focusing on category (i) is that it enables us to hone in on the direct ramifications of offering a grandfather clause rather than on potential consumer switching behavior between products produced by the same firm.[2] Our model of grandfather clauses is motivated by the seminal qualitative choice frameworks of McFadden (1974) and Perloff and Salop (1985). This modeling framework is apt for the analysis of grandfather clauses, which are generally found in product categories

[1] For instance, at the turn of the twentieth century, grandfather clauses were used to circumvent the fifteenth amendment. The grandfather clauses exempted poor whites from poll taxes and literacy requirements used to disenfranchise Southern blacks (Schmidt 1982). Grandfather laws were also occasionally used following state increases in the legal minimum drinking ages to exempt those already permitted to drink from the change in law (Williams et al. 1983).

[2] From a modeling perspective, category (i) only differs from category (ii) insomuch that grandfathered consumers might opt to purchase a product that they value differently (the product made available to new consumers) for a different price from the same firm.

that can reasonably be categorized by oligopolistic competition and discrete choice: for instance, the markets for mobile wireless service, health insurance, and certain on-line content. To make the application of grandfather clauses meaningful within our framework, we allow products and consumer tastes to evolve over time. Thus, a consumer who subscribed to one service in the past might choose a different service today, especially if faced with a higher relative price. In this case, grandfather clauses have bite: they may allow firms to retain existing customers while raising prices for product variants generally viewed as improvements over previous versions.

We find that when consumers are fully cognizant of their valuations for available product alternatives, a firm would prefer to offer all of its potential consumers a uniform price rather than price discriminate via grandfather clauses. In our setting, we suppose that individual consumers' idiosyncratic preferences for products are independent over time. In such a setting, a firm faces the same maximization problem for both, its fully informed existing customers, and potential, but likewise fully informed consumers and should therefore charge both groups of customers the same price. Thus, a grandfather clause, which is by design intended to allow a firm to charge different prices to these two groups of customers would not serve.

This negative result begs the question, "what must a business believe about its customers to want to offer them a grandfather clause?" Although, we consider a number of potential explanations, our primary focus in this manuscript is on an environment in which consumers are complacent with regard to their idiosyncratic preference and alternative price discovery. That is, suppose that because potential consumers are engaged in hundreds or thousands of markets, they do not necessarily make the effort to reconsider their individual specific preferences for products or to price shop, even if general changes in their previously chosen product suggest that idiosyncratic preferences and alternative product prices should have changed as well. However, consumers might be induced to preference and price discovery if their attention to a market is drawn by say a price increase for the service they subscribe to or a general, relative improvement to a competitor's product. We find that in an environment where consumers are complacent, a firm might prefer to offer grandfather clauses in order to prevent preference and price discovery if doing so outweighs the potential gain from charging a higher uniform price to grandfathered customers who would remain with the firm anyway. Even though prevention of preference and price discovery can be detrimental to consumers, consumer surplus goes up if grandfathered customers pay a low enough price for a sufficiently improved product. Yet, even if consumers benefit, total welfare is bound to fall because some consumers don't end up purchasing from the firm that would have left them with a product they value more.

To motivate our analysis, we present various recent examples where grandfather clauses have been used in a retail setting. We suspect that an example that is familiar to most readers is in the U.S. mobile wireless industry, where nationwide service providers have been known to grandfather customers into previously contracted plans that are not made available to new customers. For instance, both AT&T and Verizon Wireless have previously grandfathered existing customers on unlimited data plans.[3] Moreover, in its 2014 Open Internet Order, the U.S. Federal Communications Commission pointed to its concern over Verizon Wireless attempts to limit the speeds of customers on grandfathered unlimited plans in its discussion of exceptions to its "bright line" anti-discrimination rules (see U.S. FCC 2014). Although in the case of these unlimited plans, grandfathered mobile wireless customers end up consuming what is effectively a different product from those marketed to new customers, other customers are simply locked into a lower price for plans that remain on the market. Although some such customers may have initially purchased what is now deemed a slower service, as long as they upgrade their handsets to those capable of operations within the latest technology, grandfathered customers end up with an improved product at a lower price.

[3]See Bea, F. "AT&T Permitting iPhone 5 Buyers to Grandfather in Existing Unlimited Data Plans." *Digital Trends*. September 12, 2012. Retrieved August, 2015. ⟨http://www.digitaltrends.com/mobile/att-letting-iphone-5-owners-keep-their-old-dataplans/#ixzz3XsSqKJNp⟩; Dragani, R. "Verizon Nixes Unlimited Data Grandfather Clause." *E-Commerce Times*. May 17, 2012. Retrieved August, 2015. ⟨http://www.ecommercetimes.com/story/75141.html⟩.

Netflix presents another well publicized example of a grandfather clause where existing consumers retained access to the same service available to new customers at a discounted price. In May 2014, Netflix announced plans to raise prices to subscribers receiving HD-quality service by $1 a month while allowing existing customers to keep their current price for two years.[4] In September 2014, Apple offered certain existing iCloud storage customers the opportunity to keep their current storage plan, which is priced very closely to the lowest paid storage plan available to new customers, but with an additional 5GB of storage.[5] Yet another example of retail grandfather clauses occurs when a retailer begins charging for a service that was previously free. For instance, in 2012 Google ceased offering the free edition of its cloud computing Google Apps software and e-commerce platform Ecwid decreased its product sales limit for new users of its free plan, while permitting prior registered users to retain their existing level of service.[6]

In the U.S., grandfathering in retail has also been the consequence of legislative mandate. The 2010 Patient Protection and Affordable Care Act permitted health insurance providers to continue to offer service to consumers enrolled in certain individual grandfathered plans while discontinuing further enrollment as long as the insurer updated certain provisions of the plans in accordance with the act and notified consumers of these plans that they may not get some rights and protections that new plans satisfying the requirements of the act offer.[7] These grandfathered plans were allowed to persist as long as they were not changed

[4]See Fung, B. "Netflix Prices are Rising Today. But Existing Subscribers Will Get a 2-Year Reprieve." *The Washington Post.* May 9, 2014. Retrieved August, 2015. ⟨https://www.washingtonpost.com/news/the-switch/wp/2014/05/09/netflix-prices-are-rising-today-but-existing-subscribers-will-get-a-2-year-reprieve/⟩.

[5]See Campbell, M. "Apple Offering Existing iCloud Storage Customers Grandfathered Capacities, Cheaper Rates." *AppleInsider.* September 10, 2014. Retrieved August, 2015. ⟨http://appleinsider.com/articles/14/09/10/apple-offering-existing-icloud-storage-customers-grandfathered-capacities-cheaper-rates⟩.

[6]See Google, Google Apps Administrator Help. *Google Apps Free edition (legacy).* December 6, 2012. Retrieved August, 2015. ⟨https://support.google.com/a/answer/2855120?hl=en&ref_topic=6043588⟩; Ecwid, Blog. *Ecwid announces new service plans. No price changes for our existing users.* September 5, 2012. Retrieved August, 2015. ⟨http://www.ecwid.com/blog/ecwid-news/ecwid-announces-new-service-plans-no-price-changes-for-our-existing-users.html⟩.

[7]See Pub.L. 111148, 124 Stat. 119; HealthCare.gov, Health Coverage Rights and Protections. *Grandfathered health insurance plans.* March 23, 2010. Retrieved August, 2015. ⟨https://www.healthcare.gov/health-care-law-protections/grandfathered-plans/⟩. Job-based grandfathered plans could continue to enroll individuals.

in ways that would substantially cut benefits or increase costs to consumers.

To our knowledge, this is the first study that attempts to explicitly analyze the profitability and welfare consequences of using grandfather clauses in retailing. Unlike many other price discrimination strategies commonly used in retail (e.g., sales, coupons, quantity discounts, price-matching guarantees, and add-on pricing[8]) grandfather clauses are best characterized by direct market segmentation—a firm knows precisely who its grandfathered customers will be and can segment the market accordingly.

More precisely, grandfather clauses segment the market based on past purchase history. Previous studies have shown that when it is possible to discriminate among customers according to purchase history, rather than offer existing customers lower prices, firms may use information inherent in past purchase behavior to induce rivals' customers to switch by offering them lower prices instead (Villas-Boas 1999, Fudenberg and Tirole 2000).[9] A related form of pricing sometimes results when consumers face switching costs. When consumers must pay (either implicitly or explicitly) to switch to a rival producer, firms may exercise market power over their existing consumers by charging them more than they paid as new customers—so called "bargains-then-ripoffs" pricing (see for instance Klemperer 1987a, 1987b, 1995; Padilla 1992)—and more than to the existing customers of rival firms—"poaching" (see Chen 1997, Taylor 2003).[10] These models entail vigorous competition for market share through bargains early on followed by ripoffs to customers who would have to pay to switch brands (potentially with inducements offered to rivals' customers).

A closely related literature considers endogenous switching costs via loyalty or reward programs. As in the case with grandfather clauses, and unlike in the literature on exogenous switching costs, loyalty programs allow existing customers to pay lower prices than

[8]The formulation of the price discrimination inherent in these strategies is well described in respectively: Varian (1980), Narasimhan (1984), Dolan (1987), Png and Hirshleifer (1987), and Ellison (2005).

[9]Acquisti and Varian (2005) also examine firms' ability to condition prices on past purchase history via a purchase tracking technology (e.g., HTTP cookies and related devices). As in Fudenberg and Tirole (2000), firms may induce switching behavior, albeit via service personalization instead of through lower prices. Fudenberg and Villas-Boas (2006) survey the literature on price discrimination according to customer recognition.

[10]Farrell and Klemperer (2007) provide a detailed overview of the literature on switching costs.

newcomers. For instance Banerjee and Summers (1987) examine loyalty inducements in a homogenous good setting with sequential pricing and find that loyalty programs can facilitate collusion. In their model, firms benefit from increases in a rival's "loyalty coupon" because the coupon deters the rival from undercutting. In contrast, working in a horizontally differentiated model with simultaneous pricing, Caminal and Matutes (1990) find that precommitments to charge returning customers lower prices lead to a declining price path for all consumers and lower profits than without precommitments.[11]

A driving feature of most exogenous and endogenous switching cost models is forward looking strategic behavior whereby firms use foresight of future prices and market shares in setting prices before the market matures. Instead, in this manuscript, rather than treat grandfather clauses as a strategic device that firms anticipate when setting prices early on, we focus on whether grandfather clauses should be used in a mature market setting. In contrast to much of the literature on exogenous switching costs, Shaffer and Zhang (2000) find that when the customers of a firm are substantially more loyal to that firm than the rival's customers are to that rival, both firms should offer inducements to the rival firm's customers, which entails the rival offering its own customers a lower price than to newcomers. As in Shaffer and Zhang (2000), our model takes past behavior as given and asks how competition today is affected if firms can price-discriminate. However, aside from focusing on a particular form of price discrimination not expressly contemplated by Shaffer and Zhang, our model neither relies on switching costs nor on aggregate differences in preferences or costs across groups of consumers who frequent particular firms.

The remainder of this manuscript is organized as follows. In Section 2 we present a baseline model in which we introduce grandfather clauses in a setting where consumers are perfectly informed about their preferences and the prices for product alternatives. In Section 3 we model grandfather clauses when consumers are complacent and show that

[11] Caminal and Matutes (1990) also examine a framework where firms can reward returning customers using coupons instead of precommitments, but find that coupons do not survive when the choice of precommitment or couponing is endogenized.

in this setting a higher quality firm might wish to grandfather. Section 4 concludes with a discussion of alternative settings in which grandfather clauses might raise profits. The Appendix contains supplemental formal proofs.

2 Baseline Model

2.1 Firms and Consumers

Two firms, labeled 1 and 2, offer differentiated, competing subscription services. Firms face no capacity constraints and have an identical constant cost of 0 of offering one unit of their respective services. There is a unit mass of consumers with idiosyncratic tastes over these services described as follows. Each consumer values consumption of a unit of service i according to some nonstochastic average quality μ_i combined with a stochastic preference parameter $\epsilon_i \in [\underline{\epsilon}_i, \bar{\epsilon}_i]$ that represents the consumer's idiosyncratic preference for brand i.[12]

Consumers freely observe both firms' prices and each consumer subscribes to a single unit of the service that maximizes his utility, or:

$$u_i = \mu_i + \epsilon_i - p_i \tag{1}$$

where p_i is the price of service i and u_i is the consumer's utility. Following Perloff and Salop (1985), we exclude "outside services" from the analysis by assuming that each consumer purchases that service among those offered by firms 1 and 2 that gives him the highest utility regardless of the actual cardinal level of utility.

We assume that individual consumers' idiosyncratic preferences are distributed independently and identically with mean zero within brands and that aggregate preferences for the two brands are distributed independently according to density f_i.[13]

[12] We do not explicitly rule out the possibility that $\underline{\epsilon}_i = -\infty$ or that $\bar{\epsilon}_i = \infty$.

[13] Perloff and Salop (1985) also assume that preferences are symmetric across brands whereas Chen and Riordan (2008), who use a related model to study price-increasing competition, assume that aggregate preferences are symmetric, but not necessarily independent across brands. Chen and Riorden (2008) additionally permit consumers to have an outside option.

For a given consumer, $u_1 \geq u_2$ if and only if $\mu_1 - \mu_2 - p_1 + p_2 + \epsilon_1 \geq \epsilon_2$. Then, letting differentiable function F_i represent the distribution associated with density f_i, the probability that $u_1 \geq u_2$ is given by $F_2(\mu_1 - \mu_2 - p_1 + p_2 + \epsilon_1)$. We can now represent the proportion of consumers who purchase from brand i—alternatively, the quantity of services ordered from firm i—as a function of firm prices and average qualities by:

$$Q_i(p_1,\ p_2;\ \mu_1,\ \mu_2) = \int_{\underline{\epsilon}_i}^{\bar{\epsilon}_i} F_j(\mu_i - \mu_j - p_i + p_j + \epsilon_i) f_i(\epsilon_i) \mathrm{d}\epsilon_i \tag{2}$$

and because there is no outside option, $Q_j(p_1,\ p_2;\ \mu_1,\ \mu_2) = 1 - Q_i(p_1,\ p_2;\ \mu_1,\ \mu_2)$.

For ease of exposition, define $\epsilon \equiv \epsilon_2 - \epsilon_1$ and let $\epsilon \sim F(\epsilon)$ where F is a differentiable distribution function with a density that is symmetric about zero to which we assign additional restrictions as necessary throughout the manuscript. If, for instance, aggregate preferences are distributed identically and independently according to the Type I extreme value distribution, then F becomes the widely used logistic distribution. An example that we will turn to throughout the manuscript will assume that ϵ is distributed uniformly.[14]

Define $\Delta \equiv \mu_1 - \mu_2$. Then, the quantity of services ordered from firm 1 is simply $F(\Delta + p_2 - p_1)$ and firm expected profit functions are given by:

$$\pi_1(p_1;\ p_2,\ \Delta) = p_1 F(\Delta + p_2 - p_1) \tag{3}$$

$$\pi_2(p_2;\ p_1,\ \Delta) = p_2(1 - F(\Delta + p_2 - p_1)) \tag{4}$$

Assumption 1. *Suppose that firms engage in single-period differentiated Bertrand competition. Then there exists a unique interior equilibrium of the duopoly game described above.*

A necessary condition for existence of equilibrium is that both first order conditions are satisfied. Suppose that $\Delta = 0$. Then if a single price equilibrium exists, it is readily shown that the equilibrium price equals, $p = 1/(2f(0))$ where f is the density associated with F. Perloff and Salop (1985) show that when aggregate preferences are distributed symmetrically across brands, a unique single price equilibrium exists and and there are no

[14]Note that in this case, aggregate preferences for individual brands cannot be identically distributed. We impose additional assumptions as necessary (e.g., to calculate welfare).

multi-price equilibria (see Perloff and Salop Proposition 4). However, as seen in the example below, because we allow average quality to differ across firms (so that Δ does not necessarily equal zero), a unique interior equilibrium need not result in a single price.

Example. Suppose that ϵ is distributed uniformly on $[-a,\ a]$. Solving for equilibrium in our baseline model yields prices p_i, quantities q_i, and profits π_i as follows:

$$p_1 = a + \frac{\Delta}{3}, \qquad p_2 = a - \frac{\Delta}{3} \tag{5}$$

$$q_1 = \frac{1}{2a}\left(a + \frac{\Delta}{3}\right), \qquad q_2 = \frac{1}{2a}\left(a - \frac{\Delta}{3}\right) \tag{6}$$

$$\pi_1 = \frac{1}{2a}\left(a + \frac{\Delta}{3}\right)^2, \qquad \pi_2 = \frac{1}{2a}\left(a - \frac{\Delta}{3}\right)^2 \tag{7}$$

Importantly, the prices, quantities, and profits of firm 1 increase and those of firm 2 decrease as firm 1's average quality advantage relative to firm 2 increases.

Even when $\Delta \neq 0$, this framework allows us to place reasonable bounds on prices in a single-period differentiated Bertrand game, which we rely on later in the manuscript. In particular, as we show below, in equilibrium, a firm with higher average quality will not set a lower price than its competitor. Moreover, that price cannot exceed the price of its competitor by more than the difference in average qualities.

Lemma 1. *Suppose that* $\Delta > 0$*. Then* $0 \leq p_1 - p_2 \leq \Delta$*.*

Proof. The first order conditions for firms 1 and 2 respectively are:

$$F(\Delta + p_2 - p_1) - p_1 f(\Delta + p_2 - p_1) = 0 \tag{8}$$

$$1 - F(\Delta + p_2 - p_1) - p_2 f(\Delta + p_2 - p_1) = 0 \tag{9}$$

Solving Equations (8) and (9) for $f(\Delta + p_2 - p_1)$ and rearranging yields:

$$1 = F(\Delta + p_2 - p_1) + \frac{p_2}{p_1 + p_2} \tag{10}$$

Because $F(0) = 1/2$, it follows that if $p_2 > p_1$, then both terms on the right-hand side of Equation (10) are greater than 1/2, a contradiction. If instead $p_1 - p_2 > \Delta$, then both terms on the right-hand side of Equation (10) are less than 1/2, which is likewise a contradiction. □

Inspection of Equation (5) in our uniform distribution example will show that $p_1 - p_2 = 2\Delta/3$. Thus, both bounds hold.

2.2 Grandfather Clauses

We study grandfather clauses using a single-period mature market in which half of all consumers have previously subscribed to firm 1's service and the other half subscribed to that of firm 2. The idea that both firms initially hold half the market may be justified by supposing that prior to market maturity, firms provided services with the same initial quality at the same initial price. We further suppose that the initial price prior to market maturity, p^0, equaled the single-period, single price equilibrium price, $p^0 = 1/(2f(0))$.[15] Thus, if a firm wishes to grandfather its existing consumers, it must offer them a price of p^0 that is different from the price charged to potential new customers. Going forward, we refer to the time prior to market maturity as the initial period and we refer to the stage to be studies as the mature market period.

If neither consumer tastes nor firm quality changes between the initial and mature market periods, then the use of grandfather clauses does not have a real world counterpart in the following sense: grandfathered customers must be made to feel that they are consuming a better product than that available to newcomers or they must be able to obtain a better price than what newcomers would have to pay. Thus, suppose that between the initial and mature market periods, firm 1 announces an average quality improvement of $\nu > 0$. As a result, Δ rises from 0 to ν. Because our focus in this manuscript is on grandfather clauses, we suppose that the quality improvement is exogenous and costless for firm 1.

[15]We note that because we are not solving a two-stage pricing game, equal market shares in this setting imply that the actual (not just expected) realization of idiosyncratic preferences was such that half of consumers initially preferred each service.

For simplicity, we suppose that only firm 1 can offer grandfather clauses. The game proceeds as follows: at the outset of the mature market period, firm 1 decides whether or not it will grandfather its existing consumers and simultaneously chooses a new price while firm 2 simultaneously sets its own price.[16] Neither firm can engage in any other form of price discrimination and grandfathered consumers are free to choose the rival firm's product.[17] Moreover, grandfathered consumers benefit from the average quality improvement, and will choose the new price if that price is lower. After firms set prices, consumers make their purchasing decisions and profits and welfare are realized.

At this juncture, the astute reader may ask why we do not study grandfather clauses in a rational expectations framework in which firms account for the potential competitive effects of grandfather clauses when setting prices in the initial period. As discussed in Section 1, such frameworks are common in the switching cost literature. In the subgame perfect Nash equilibrium of such a game, there is no a priori reason to think that initially symmetric firms would set the same price and end up with the same market share in the initial period if grandfather clauses stipulate that these prices will directly impact profitability in the mature market period. However, mathematical tractability aside,[18] there are two reasons to fix initial prices and market shares regardless of whether grandfather clauses will be used in the mature market period. First, we interpret the time that passes between the initial and mature market periods as sufficiently lengthy to allow for average quality to change and to justify price adjustment. However, without investigating investment in quality in greater detail, it is difficult to determine which of two initially symmetric firms will end up with the relative quality improvement in the mature market period, and consequently, how

[16] The simultaneous move structure of our game is motivated by various of our examples in Section 1, where a mature market firm simultaneously resets its price while grandfathering existing consumers. However, within the setting of our main model in the next section, it will become apparent that the results are robust to a setting where firm 1 informs its rival and consumers of its intent to grandfather prior to setting its price.

[17] So, for instance, we suppose that firm 2 cannot specifically offer firm 1's existing, potentially grandfathered, customers incentives to switch.

[18] In a rational expectations framework, subgame perfection would require us to calculate mature market period prices for any level of initial market shares, and in light of grandfather clauses, it would additionally require prices to be contingent on any initial period price that might be offered to grandfathered consumers.

resulting price adjustments and the application of grandfather clauses in the future should affect prices in the initial period. Second, our primary interest is in informing managers of firms in mature markets who are contemplating implementing a grandfather clause rather than in any gaming (e.g., for the purposes of collusion or exclusion) that grandfather clauses may permit firms to undertake.[19]

In this subsection, we assume that consumers are always cognizant of prices, average qualities, and their idiosyncratic preferences for both brands. We suppose that individuals' idiosyncratic preferences are independent over time—effectively positing that enough time has passed such that the initial period preferences do not convey any idiosyncratic information to the individual when the market has matured. Moreover, we suppose that the ϵ_i are redrawn in the mature market period according to the same distribution as in the initial period—that is, we suppose that individual preferences may have changed, but that aggregate preferences have not.[20]

It turns out that under the assumptions in this section, firm 1's optimal strategy is to offer a uniform price to both its initial period customers and to potential consumers that it could poach from firm 2. As shown in Proposition 1, because this is the case for any potential price that firm 2 may offer, grandfather clauses are not played in equilibrium.

Proposition 1. *Suppose that consumers are aware of firm prices, average qualities, and their idiosyncratic preferences prior to choosing which service to purchase. Then, in equilibrium, firm 1 will choose to offer a uniform price rather than to grandfather its initial period customers by offering them a lower price than to potential consumers.*

Proof. Suppose that firm 2 chooses price p_2. When setting prices, by offering a grandfather

[19]It seems to us sensible that firms would account for say potential consumer switching in the future when setting present prices. However, unlike switching costs, which are estimable in the present, the decision to apply grandfather clauses is contingent on changing firm, consumer and overall market characteristics that appear to us very difficult to predict up front, so that we believe that firms do not necessarily account for the possibility that they may choose to grandfather certain customers in the future when setting their prices in the present.

[20]Thus, our setting is similar to the independent preference setting of Fudenberg and Tirole (2000), who also separately look at fixed preferences. In a future manuscript, we hope to study preferences that are either fixed, or in some sense correlated over time.

clause, firm 1 can price discriminate between its initial period customers and those of firm 2. Given a price p_1, firm 1's initial period customers will remain with firm 1 if $\nu - p_1 + p_2 \geq \epsilon$. Thus, firm 1's expected profit in the mature market period from its initial period customers is given by:

$$\frac{p_1}{2} F(\nu + p_2 - p_1) \tag{11}$$

Because idiosyncratic preferences for firm 2's initial period customers are redrawn from the same distribution, Expression (11) likewise represents firm 1's expected profit from poaching firm 2's initial period customers. Consequently, the value of p_1 that maximizes expected profits from former customers is the same as the value that maximizes expected profits from new customers. As a result, firm 1 will not wish to price discriminate by offering returning customers one price and new customers a higher price.[21] □

When preferences are independent over time, under the assumptions above, knowing whether a potential customer had previously purchased from firm 1 or 2 does not convey any useful information to firm 1 at either the individual consumer level or at any level of consumer aggregation. Because in our setup, firm 1 has no other means of price discrimination at its disposal, it prefers to set a uniform price. In fact, this straightforward result is more broad. Because firm 1's profit maximizing price for the two groups of customers does not depend on initial period market shares or prices, firm 1 would prefer a uniform price even if the initial period equilibrium were asymmetric—that is, when preferences are independent over time, under Assumption 1, uniform pricing turns out to be the outcome of the unique subgame perfect Nash equilibrium of an alternative rational expectations, two-stage pricing game.

[21]More broadly, this proof also rules out price discrimination whereby returning customers are asked to pay a higher price than new customers.

3 Consumer Complacency

Our findings in Proposition 1 suggest that when consumers are perfectly informed about prices charged by firms 1 and 2 as well as regarding their own idiosyncratic preferences, then in the mature market setting being studied here, a firm would not be compelled to grandfather returning customers. This leads us to wonder why various firms facing oligopoly competition in a real world mature market setting would offer grandfather clauses. In this section, we set up a simple model of complacent consumers to offer one potential explanation. In the next section, we contemplate some alternative explanations.

Suppose that consumers are complacent in the following sense: they do not reevaluate their idiosyncratic preferences for the service that they consume, nor seek updated price information from other firms unless one of two things happens: (i) they are faced with a higher price for the service they currently consume or (ii) they discover that a rival service provider offers a higher relative quality alternative.[22] As such, complacency here is not interpreted so much as a cost to uncover additional price and idiosyncratic product match information (as in Wolinsky 1986 or Anderson and Renault 1999), but as a failure to contemplate that this information changed after some period of time.

As in the previous section, we suppose that idiosyncratic preferences are independently and identically distributed in the initial and mature market periods. We also suppose that consumers are perfectly informed of all average quality improvements—that is, all consumers learn ν at the outset of the mature market game. One way to think about this is to suppose that quality improvements that are expected to affect all consumers are broadly advertised by the firms, but that it is more difficult to convey concrete price and individual preference information via advertising.[23]

[22]For a firm i, this happens if μ_j rises relative to μ_i.

[23]Because our core interest is in grandfather clauses, we do not explicitly model the process by which firms convey average quality information to consumers. A standard approach would be to suppose that the cost of advertising is increasing and strictly convex in the proportion of the population that receives an update of average quality (see for instance the price advertising models of Butters 1977 and Robert and Stahl 1993). Alternatively, we could suppose that there is a fixed cost to advertise to the entire population (e.g., Janssen and Non 2008). Our model implicitly presumes that advertising quality is costless.

Assumption 2. *Consumers are complacent in the mature market period.*

Proceeding with our model from Section 2, modified by Assumption 2, we now see that if firm 1 offers to grandfather its initial period consumers, these consumers will not reevaluate their idiosyncratic preferences nor prices for the two service alternatives and remain with firm 1. They will continue to pay $p^0 = 1/(2f(0))$ for firm 1's service, but also benefit from firm 1's general quality improvement ν. Conversely, because firm 1 offers a higher quality product in the mature market period, firm 2's initial period customers will undertake price and idiosyncratic preference discovery (even if they were grandfathered). Thus, under Assumption 2, firm 1 can either choose a uniform price, in which case a price above p^0 will lead both firms to compete for the entire market, or it can grandfather its initial period consumers and restrict competition in the mature market period to firm 2's initial period customers, but sacrifice potential profit gains from raising its price to its initial period customers.

As we show in Proposition 2, under some reasonable assumptions on f, firm 1 will want to offer a grandfather clause as long as ν is not "too high." The intuition is straightforward. If firm 1's relative quality advantage over firm 2 is sufficiently large, firm 1 does not need to worry that its initial period customers will engage in information discovery following a price increase because firm 1's quality advantage will bring most of these customers back. On the other hand, if ν is relatively low, firm 2 remains competitive for firm 1's initial period customers and firm 1 can improve its profit by using a grandfather clause to prevent firm 2 from poaching its customers. Recall that this would not have worked in the previous section—what is crucial is that grandfather clauses may prevent information discovery.

Proposition 2. *Suppose that F is twice continuously differentiable and that f is symmetric about zero and single-peaked. Then under Assumption 2, there exists $\hat{\nu}$ such that for any average quality improvement $\nu < \hat{\nu}$, firm 1 will wish to offer a grandfather clause, and for any $\nu > \hat{\nu}$ it will not.*

Let $\pi_1^*(\nu)$ represent firm 1's equilibrium expected profit when it chooses a uniform price in the mature market period and let $\pi_2^*(\nu)$ represent firm 2's equilibrium expected profit when it believes that firm 1 will play a uniform price (and these beliefs are correct). The key to the proof of Proposition 2 is that under Assumption 2 the firms' expected profit functions when firm 1 chooses to grandfather its initial period customers are simply affine transformations of profits under uniform pricing.[24] In particular, because firms' initial period profits equaled $1/(4f(0))$, firm 1's and 2's respective equilibrium expected profits in the mature market period when firm 1 does grandfather are $1/(4f(0)) + \pi_1^*(\nu)/2$ and $\pi_2^*(\nu)/2$. Thus for firm 1, expected profit when it offers to grandfather exceeds non-grandfathered expected profit if and only if $\pi_1^*(\nu) < 1/(2f(0))$. In the Appendix, we show that $\pi_1^*(\nu)$ is continuously increasing in ν and that there are values of ν low enough to satisfy and high enough to reverse the previous inequality, completing the proof.

In Proposition 2, we placed a number of restrictions on the forms of F and f in order to derive our "cut-off" result. However, as the following example shows, the single-peakedness condition (which we rely on to assure continuity of $\pi_1^*(\nu)$) is not necessary.

Example. Suppose that ϵ is distributed uniformly on $[-a,\ a]$. In this case, $\pi_1^*(\nu)$ is given by replacing Δ in Equation (7) with ν. Similarly, letting $\pi_1^{\mathrm{GFC}}(\nu)$ represent firm 1's expected profit when it chooses to grandfather its initial period customers in the mature market period, we obtain:

$$\pi_1^{\mathrm{GFC}}(\nu) = \frac{a}{2} + \frac{1}{4a}\left(a + \frac{\nu}{3}\right)^2 \tag{12}$$

Firm 1 will want to offer a grandfather clause whenever $\pi_1^{\mathrm{GFC}}(\nu) > \pi_1^*(\nu)$, which simplifies to:

$$\nu < 3a(\sqrt{2} - 1) \tag{13}$$

That is, as in Proposition 2, firm 1 will want to offer a grandfather clause as long as its average quality advantage relative to firm 2 does not exceed $3a(\sqrt{2} - 1)$.

[24] Thus, as suggested in footnote 16, it does not matter if firm 1 informs everyone that it will commit to grandfather before firms set prices or not because under Assumption 2, the prices that solve firms' first order conditions will be the same with or without grandfathering.

Observe that in the example above, the higher a—that is, the greater the variance of the distribution of the difference in idiosyncratic preferences—the greater the range of ν for which firm 1 would want to offer a grandfather clause under consumer complacency. This suggests that grandfather clauses are more likely to prove useful if tastes are more dispersed. In the next proposition, we explore this result in a more general setting.

For any distribution F consider the family of distributions G_α such that $G_\alpha(\epsilon) = F(\alpha\epsilon)$. Observe that G_α is a distribution for any $\alpha \in (0, \infty)$. When $\alpha < 1$, G_α stretches F horizontally, behaving similarly to the original distribution, but with larger variance. Conversely, when $\alpha > 1$, G_α horizontally compresses F.

Proposition 3. *Suppose that F is twice continuously differentiable and that f is symmetric about zero and single-peaked. Then under Assumption 2, if the distribution of the difference in idiosyncratic preferences is given by G_α, for a fixed improvement ν, there exists some $\alpha^*(\nu) > 0$ such that firms will wish to offer a grandfather clause if and only if $\alpha \leq \alpha^*(\nu)$.*

Proposition 3 states that for any $\nu > 0$, there always exists a level of dispersion in idiosyncratic preferences (α low enough) that justifies offering a grandfather clause under Assumption 2. Greater dispersion in tastes in the manner defined above makes ν less effective in retaining customers. Thus, using grandfather clauses to make customers complacent becomes that much more valuable.

Together Propositions 2 and 3 characterize sufficient conditions for a firm to wish to offer a grandfather clause. The firm wishes to grandfather when either the quality improvement it offers is slight enough so as to keep it from having an overwhelming competitive advantage, or if consumer tastes are so dispersed as to make the quality improvement relatively insignificant. Corollary 1 relates these two propositions by telling us that the larger the improvement in quality, the more taste dispersion is needed to justify offering a grandfather clause.

Corollary 1. *$\alpha^*(\nu)$ is decreasing in ν.*

3.1 Welfare

Under Assumption 2, grandfather clauses are unambiguously bad for total welfare. This can be seen without any explicit calculation. Recall that because we have assumed that consumers have no outside option, the price paid for each consumer's chosen service is a pure transfer and may be discounted in total welfare calculations. Thus, only the realizations of idiosyncratic preferences and whether or not a consumer ends up purchasing from the firm that offers the best match (accounting for average quality) matter. When consumers are complacent, grandfather clauses distort optimal matching by keeping firm 1's customers from potentially realizing a better match with firm 2.

As discussed earlier in this section, grandfather clauses are beneficial for firm 1 as long as ν is not too high relative to the level of dispersion in idiosyncratic preferences. Conversely, grandfather clauses are also unambiguously bad for firm 2 because they effectively reduce its market share. Perhaps of greater interest is whether or not grandfather clauses increase consumer surplus. On the one hand, grandfather clauses reduce the average price paid by consumers. On the other hand, they cause some consumers to be matched poorly and to receive a service that they might not enjoy as much relative to the price paid. As shown in a continuation of our uniform distribution example below, the effect is ambiguous and depends on the value of ν relative to the level of dispersion in tastes.

Example. Suppose that ϵ is distributed uniformly on $[-a,\ a]$. In order to calculate consumer surplus, we additionally need to know how ϵ_i is distributed for each individual firm along with firms' initial period average service quality levels. Thus, for the purpose of this example, suppose that ϵ_1 is distributed uniformly on $[-a,\ a]$ whereas ϵ_2 is a point mass at zero and suppose that $a > \nu/3$. As will become evident in Equation (14), the inequality states that there is sufficient taste dispersion relative to the value of ν for some customers to find it worthwhile to frequent firm 2 in the absence of grandfather clauses. Additionally, suppose that initially, $\mu_1 = \mu_2 = \mu$. Then, if firm 1 chooses not to offer a grandfather clause in the mature market period, from Equation (5), we know that consumer surplus is given by:[25]

$$CS = \int_{-\frac{\nu}{3}}^{a} \left[\mu + \nu + \epsilon_1 - \left(a + \frac{\nu}{3}\right)\right] \frac{1}{2a} \mathrm{d}\epsilon_1 + \int_{-a}^{-\frac{\nu}{3}} \left[\mu - \left(a - \frac{\nu}{3}\right)\right] \frac{1}{2a} \mathrm{d}\epsilon_1 \tag{14}$$

where the first integral is for firm 1's consumers and the second is for firm 2's. This expression simplifies to:

$$CS = \mu - \frac{3a}{4} + \frac{\nu}{2} + \frac{\nu^2}{36a} \tag{15}$$

Under Assumption 2, if firm 1 offers a grandfather clause, it retains the half of the market that it possessed in the initial period. Firm 1's initial period customers then pay it a price of a for a product with mature market average quality of $\mu + \nu$. Although these consumers are complacent when firms reset their mature market prices, these consumers ultimately realize idiosyncratic preferences as distributed above. Consumer surplus for the other half of the market is represented by Equation (14). Thus, when firm 1 offers a grandfather clause, consumer surplus is represented by:

$$CS^{\text{GFC}} = \frac{\mu + \nu - a}{2} + \frac{CS}{2} \tag{16}$$

Clearly, $CS^{\text{GFC}} > CS$ if and only if $\mu + \nu - a > CS$, which occurs whenever $\nu > 3a(3 - 2\sqrt{2}) \approx 0.51a$. That is, when ν is sufficiently large relative to a, the price savings to grandfathered customers (which are proportional to ν) are larger than the foregone gains from price and idiosyncratic preference discovery (which are proportional to a). Recall that firm 1 will only wish to offer a grandfather clause when $\nu \leq 3a(\sqrt{2} - 1) \approx 1.24a$. Thus, given some level of idiosyncratic preference dispersion and considering the range of ν under which firm 1 would find it profitable to offer a grandfather clause, consumers are worse off when $\nu \in (0,\ 0.51a)$ and better off when $\nu \in (0.51a,\ 1.24a)$.

[25]Note that the bounds of integration follow because consumers strictly prefer to purchase from firm 1 if and only if $\epsilon_1 > -\nu/3$.

4 Conclusion

In this manuscript we seek to understand firm price discrimination via the use of grandfather clauses. Using a discrete choice setting in which consumer preferences for individual alternatives can vary over time, we found that when individual consumers are perfectly informed, a firm that achieves a relative quality improvement will not find it profitable to grandfather its former customers while charging potential new customers a higher price. Our finding was observed in a setting in which individual preferences were distributed independently (and identically) from one period to the next, but we suspect that this result is more general. For instance, consider a setting in which individual preferences for the same product are positively correlated over time. Then, if a price increase for individuals who consumed a different firm's product in the past is optimal, the price offered to existing customers—who hence value the product more highly in the present—should be no lower than that offered to potential consumers. We hope to formalize this point in a future draft.

Cognizant of the fact that grandfather clauses are present in various retail service settings, we consider an alternative framework in which firms might wish to offer them. Our chosen framework is a "behavioral" model in which grandfather clauses act as a potential barrier to information discovery. In this setting, we find that the higher quality firm will want to grandfather early adopters as long as its relative quality improvement is not so high that the grandfather clause keeps the firm from realizing a large gain in inframarginal profit. Moreover, the range of quality improvements that makes a grandfather clause worthwhile grows with the level of dispersion in idiosyncratic preferences.

Although we rationalize the use of grandfather clauses in a setting with consumer complacency, we believe that alternative models could also serve to explain their recent proliferation. One framework that comes to mind is a model where individual preferences are negatively correlated over time—perhaps because consumers get tired with their initially sought out service. In such a setting, we suspect that the opposite intuition conjectured in the positive correlation setting holds: that is, grandfather clauses permit firms to set a higher price to consumers who are expected to value the service more highly.

An alternative explanation for the use of grandfather clauses is that they may serve as a useful mechanism to mitigate adverse consumer reactions to perceived price unfairness over increases in prices (e.g., see Bolton et al. 2003). When customers view a price increase by their existing firm as unfair, they may be more inclined to try a rival service. Grandfather clauses allow firms to avoid angering former customers using lower prices while offering newcomers (who might direct their anger at a rival firm) a higher price.[26]

Yet a third potential alternative explanation for the prevalence of grandfather clauses is that they permit a firm to maintain a requisite level of market penetration to exploit potential positive network externalities. This explanation appears particularly cogent in cases where grandfathered consumers get to continue to consume a service for free. A related explanation is one where a multi-product firm allows grandfathered consumers to use its (potentially free) service as a loss leader.

Appendix

Proof of Proposition 2.

Proof. This proof proceeds in three steps. First, we show that there exists some positive value of ν for which firm 1 would want to offer a grandfather clause. Second, we show that if firm 1 wants to offer a grandfather clause for some value ν, it must also want to offer a grandfather clause for any average quality improvement lower than that value. Conversely, we show that if firm 1 does not want to offer a grandfather clause for some value of ν, it will not want to offer one for any larger average quality improvement. Finally, we show that there exists a value of ν for which firm 1 does not want to offer a grandfather clause.

[26]We suspect that the Netflix decision to grandfather their existing subscribers in 2014 may have been driven in part by adverse subscriber reactions to an earlier substantial price increase in 2011. See Fung, B. "Netflix Prices are Rising Today. But Existing Subscribers Will Get a 2-Year Reprieve." *The Wall Street Journal*. September 16, 2011. Retrieved October, 2011. ⟨http://www.wsj.com/articles/SB10001424053111904060604576572322651549428/⟩.

Step 1. To show existence of a ν for which firm 1 would want to offer a grandfather clause, we must first show that prices and consequently, firm 1's profit, are continuous in ν. This requires a continuous implicit function that gives the equilibrium prices in terms of ν. First, suppose that firm 1's equilibrium price is increasing in ν when firm 1 does not offer a grandfather clause. This assumption will be confirmed in Step 2. Let

$$H(\nu,\ p_1,\ p_2) := \left(\frac{\partial}{\partial p_1}\left[p_1 F(\nu + p_2 - p_1)\right],\ \frac{\partial}{\partial p_2}\left[p_2(1 - F(\nu + p_2 - p_1))\right]\right) \tag{17}$$

Note that under Assumption 2, $p_1 F(\nu + p_2 - p_1)$ represents either firm 1's expected profit under uniform pricing, or double the non-constant component of expected profit when it offers a grandfather clause. Similarly, $p_2(1 - F(\nu + p_2 - p_1))$ represents either firm 2's expected profit when it believes that firm 1 will set a uniform price, or double that amount when firm 1 is expected to offer a grandfather clause.[27]

Let $J(\nu,\ p_1,\ p_2)$ represent the matrix of the partials of $H(\nu,\ p_1,\ p_2)$:

$$\frac{\partial H(\nu,\ p_1,\ p_2)}{\partial(p_1, p_2)} := J(\nu,\ p_1,\ p_2) = \begin{pmatrix} -2f(\gamma) + p_1 f'(\gamma) & f(\gamma) - p_1 f'(\gamma) \\ f(\gamma) + p_2 f'(\gamma) & -2f(\gamma) - p_2 f'(\gamma) \end{pmatrix} \tag{18}$$

where we define $\gamma \equiv \nu + p_2 - p_1$. In equilibrium, it must be that $H(\nu,\ p_1,\ p_2) = 0$. Thus, we need to find a continuous implicit function $g(\nu) = (p_1,\ p_2)$ such that $H(\nu,\ g(\nu)) = 0$. Such a g exists if $H(\nu,\ p_1,\ p_2)$ is continuously differentiable and $J(\nu,\ p_1,\ p_2)$ is invertible. The first condition is satisfied by assumption. The second follows if the determinant of $J(\nu,\ p_1,\ p_2)$,

$$|J(\nu,\ p_1,\ p_2)| = 3f(\gamma)^2 + (p_2 - p_1) f(\gamma) f'(\gamma) \tag{19}$$

is non-zero. The first term on the right-hand side of Equation (25) is clearly positive. Our assumption that idiosyncratic preferences are independently and identically distributed in the initial and mature market periods together with Assumption 2 imply that Lemma 1 applies. Therefore, $p_2 - p_1 < 0$ and $\gamma > 0$. Then, single-peakedness and symmetry imply that $f'(\gamma) < 0$, such that the second term on the right-hand side of Equation (25) is likewise positive, as is

[27] Thus, the price that satisfies firm 2's first order condition is invariant to its beliefs regarding firm 1's decision to grandfather.

$|J(\nu,\ p_1,\ p_2)|$. Thus, $J(\nu,\ p_1,\ p_2)$ is invertible and the implicit function theorem guarantees the existence of a continuously differentiable g. Therefore, the function that gives firm 1's equilibrium expected profit in terms of ν is likewise continuous. Note that if firm 1 chooses to set a uniform price in the mature market period, its equilibrium expected profit is given by $\pi_1^*(\nu) \equiv g_1(\nu)F(\nu + g_2(\nu) - g_1(\nu))$ where $g(\nu) = (g_1(\nu),\ g_2(\nu))$.

Recall that when $\nu = 0$, firm 1's expected profit in the mature market period under uniform pricing is $1/(4f(0))$ (this was also its realized profit in the initial period). Under Assumption 2, if firm 1 chooses to offer a grandfather clause at some positive ν, its expected profit is $1/(4f(0)) + \pi_1^*(\nu)/2$. This will exceed firm 1's non-grandfathered expected profit if and only if $\pi_1^*(\nu) < 1/(2f(0))$. Continuity of π_1^* implies that for any ε, there exists $\underline{\nu}$ such that if $|\nu| < \underline{\nu}$, $|\pi_1^*(\nu) - 1/(4f(0))| < \varepsilon$. By choosing $\varepsilon < 1/(4f(0))$, we can ensure that for any $\nu < \underline{\nu}$, firm 1's profits will be higher with a grandfather clause than without one.

Step 2. For the proof of this step, it suffices to show that $\pi_1^*(\nu)$ is increasing in ν. This follows immediately if p_1 and γ are increasing in ν.

According to the implicit function theorem the derivative of prices with respect to ν is $-(J(\nu,\ p_1,\ p_2))^{-1}D_\nu H(\nu,\ p_1,\ p_2)$, where $D_\nu H(\nu,\ p_1,\ p_2)$ represents the derivative of $H(\nu,\ p_1,\ p_2)$ with respect to v. After some algebraic manipulation, we find that:

$$\frac{\mathrm{d}p_1}{\mathrm{d}\nu} = \frac{f(\gamma)(f(\gamma) - p_1 f'(\gamma))}{3f(\gamma)^2 + (p_2 - p_1)f(\gamma)f'(\gamma)} \tag{20}$$

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\nu} = \frac{f(\gamma)^2}{3f(\gamma)^2 + (p_2 - p_1)f(\gamma)f'(\gamma)} \tag{21}$$

Note that the denominator in Equations (20) and (21) equals $|J(\nu,\ p_1,\ p_2)|$, which is already established to be positive. Likewise, both numerators are positive. In particular, with regard to Equation (20), as established in the previous step, $f'(\gamma)$ is negative because of Lemma 1, single-peakedness, and symmetry. Therefore, because price is increasing and quantity is non-decreasing in ν, $\pi_1^*(\nu)$ is increasing in ν.

Because $\pi_1^*(\nu)$ is increasing, $\pi_1^*(\hat{\nu}) < 1/(2f(0))$ implies that $\pi_1^*(\nu) < 1/(2f(0))$ whenever

$\nu < \hat{\nu}$. That is, if firm 1 wants to offer a grandfather clause for some $\hat{\nu}$, it will want to offer a grandfather clause for all $\nu < \hat{\nu}$. Conversely, whenever $\pi_1^*(\hat{\nu}) > 1/(2f(0))$, this will also be the case for all $\nu > \hat{\nu}$. Thus, if firm 1 does not want to offer a grandfather clause at $\hat{\nu}$, it will not want to offer one for any $\nu > \hat{\nu}$.

Step 3. To complete the proof we must show that there exists some $\bar{\nu}$ such that $\pi_1^*(\bar{\nu}) > 1/(2f(0))$. Because quantity is bounded above by 1, to guarantee this, we need to show that price is unbounded in ν—or alternatively, that its derivative provided in Equation (20) is bounded away from zero. Using the first order conditions that $f(\gamma) = 1/(p_1 + p_2)$, Equation (20) can be rewritten as:

$$\frac{\mathrm{d}p_1}{\mathrm{d}\nu} = \frac{1}{3 - (p_1^2 - p_2^2)f'(\gamma)} + \frac{-p_1 f'(\gamma)}{3f(\gamma) - (p_1 - p_2)f'(\gamma)} \tag{22}$$

Lemma 1, single-peakedness, and symmetry imply that both terms on the right-hand side are positive. Thus, it suffices to show that the first term is unbounded. Because single-peakedness implies that $f'(\gamma)$ goes to zero as ν approaches infinity, the first term could only go to zero if $(p_1^2 - p_2^2)$ were unbounded. But if that were true, p_1 would be unbounded. Thus, p_1 is unbounded in ν and it must be the case that there exists some $\bar{\nu}$ such that $\pi_1^*(\bar{\nu}) > 1/(2f(0))$. Finally, because $\pi_1^*(\nu)$ is strictly increasing, there exists $\hat{\nu}$ such that $\pi_1^*(\hat{\nu}) = 1/(2f(0))$ and for any average quality improvement $\nu < \hat{\nu}$, firm 1 will wish to offer a grandfather clause, whereas for any $\nu > \hat{\nu}$ it will not. □

Proof of Proposition 3.

Proof. We first show that prices and firm 1's profits are continuous in ν and α. However, because we fix ν for the duration of this proof, going forward, we will suppress ν as an argument in functions. Suppose that the distribution of the difference in idiosyncratic preferences is given by G_α. Additionally, suppose that firm 1's equilibrium price is increasing in ν when firm 1 does not offer a grandfather clause. It is possible to confirm that this assumption does indeed hold in equilibrium following the same methodology used in the proof of Proposition 2 (see Equation (20)). Define the vector

$$H(\alpha,\ p_1,\ p_2) := \left(\frac{\partial}{\partial p_1}\left[p_1 F(\alpha(\nu + p_2 - p_1))\right],\ \frac{\partial}{\partial p_2}\left[p_2(1 - F(\alpha(\nu + p_2 - p_1)))\right]\right) \tag{23}$$

and let $J(\alpha,\ p_1,\ p_2)$ represent the matrix of the partials of $H(\alpha,\ p_1,\ p_2)$:

$$J(\alpha,\ p_1,\ p_2) = \begin{pmatrix} -2\alpha f(\alpha\gamma) + p_1\alpha^2 f'(\alpha\gamma) & \alpha f(\alpha\gamma) - p_1\alpha^2 f'(\alpha\gamma) \\ \alpha f(\alpha\gamma) + p_2\alpha^2 f'(\alpha\gamma) & -2\alpha f(\alpha\gamma) - p_2\alpha^2 f'(\alpha\gamma) \end{pmatrix} \tag{24}$$

where we again define $\gamma \equiv \nu + p_2 - p_1$.

As in the proof of Proposition 2 we need to find a continuous implicit function $g(\alpha) = (p_1,\ p_2)$ such that $H(\alpha,\ g(\alpha)) = 0$. Such a g exists if $H(\alpha,\ g(\alpha))$ is continuously differentiable and $J(\alpha,\ p_1,\ p_2)$ is invertible. The first condition is satisfied by assumption. The second follows if the determinant of $J(\alpha,\ p_1,\ p_2)$,

$$|J(\alpha,\ p_1,\ p_2)| = 3\alpha^2 f(\alpha\gamma)^2 + \alpha^3(p_2 - p_1) f(\alpha\gamma) f'(\alpha\gamma) \tag{25}$$

is non-zero. As in Proposition 2, this follows from Assumption 2, Lemma 1, as well as our single-peakedness and symmetry assumptions. Therefore, the function that gives firm 1's equilibrium expected profit in terms of α and ν is likewise continuous. Note that if firm 1 chooses to set a uniform price in the mature market period, its equilibrium expected profit is given by $\pi_1^*(\alpha) \equiv g_1(\alpha)F(\alpha(\nu + g_2(\alpha) - g_1(\alpha)))$ where $g(\alpha) = (g_1(\alpha),\ g_2(\alpha))$.

Under this new specification, when $\nu = 0$, it is readily shown that firm 1's expected profit in the mature market period under uniform pricing is:

$$\bar{\pi}_1(\alpha) \equiv \frac{1}{4\alpha f(0)} \tag{26}$$

Moreover, under Assumption 2, if firm 1 chooses to offer a grandfather clause at some positive ν, its expected profit is $\bar{\pi}_1(\alpha) + \pi_1^*(\alpha)/2$ so that its profit when it grandfathers exceeds its profit when it doesn't if and only if $\pi_1^*(\alpha) < 2\bar{\pi}_1(\alpha)$. Define:

$$r_\pi(\alpha) \equiv \frac{\pi_1^*(\alpha)}{\bar{\pi}_1(\alpha)} \tag{27}$$

Thus, firm 1 wants (doesn't want) to offer a grandfather clause if $r_\pi(\alpha) < 2$ ($r_\pi(\alpha) >$

2). Then to complete our proof it suffices to show that (i) $r_\pi(\alpha)$ is increasing in α, (ii) $\lim_{\alpha\to 0} r_\pi(\alpha) = 1$, and (iii) $\lim_{\alpha\to\infty} r_\pi(\alpha) = \infty$.

For (i), using the implicit function theorem together with the first order condition $p_1 f(\alpha\gamma) = F(\alpha\gamma)/\alpha$, we find that the derivative of $r_\pi(\alpha)$ with respect to α is:

$$\frac{\partial r_\pi(\alpha)}{\partial \alpha} = \frac{4\nu f(0)F(\alpha\gamma)(2f(\alpha\gamma) - \alpha p_1 f'(\alpha\gamma))}{3f(\alpha\gamma) - \alpha(p_1 - p_2)f'(\alpha\gamma)} \tag{28}$$

In the numerator, ν, $f(0)$, $F(\alpha\gamma)$, α, and p_1 are all positive, whereas $f'(\alpha\gamma)$ is negative (Assumption 2, Lemma 1, single-peakedness and symmetry). Additionally, in the denominator, $f(\alpha\gamma)$ and $p_1 - p_2$ are positive (Lemma 1). Thus, both the numerator and denominator are positive, so that $r_\pi(\alpha)$ is increasing in α.

In proving (ii), we note that as α approaches zero, $\bar{\pi}_1(\alpha)$ increases to infinity, and because $\pi_1^*(\alpha)$ is bounded below by $\bar{\pi}_1(\alpha)$ (this follows because we assume that $\nu > 0$ and because $\pi_1^*(\alpha)$ is increasing in ν—see Step 2 in the proof of Proposition 2) $\pi_1^*(\alpha)$ likewise approaches infinity. Therefore, we can use L'Hôpital's Rule to evaluate the limit of $r_\pi(\alpha)$ as it approaches zero. Again using the implicit function theorem together with firm 1's first order condition, we have:

$$\frac{\partial \pi_1^*(\alpha)}{\partial \alpha} \Big/ \frac{\partial \bar{\pi}_1(\alpha)}{\partial \alpha} = \frac{4\alpha f(0)F(\alpha\gamma)\left[(3p_1 - 2\nu)f(\alpha\gamma) + \alpha p_1 \gamma f'(\alpha\gamma)\right]}{3f(\alpha\gamma) - \alpha(p_1 - p_2)f'(\alpha\gamma)} \tag{29}$$

Using firm 1's first order condition, the numerator in Equation (29) can be written:

$$12f(0)F(\alpha\gamma)^2 - 8\alpha\nu f(0)f(\alpha\gamma)F(\alpha\gamma) + 4\alpha\gamma f(0)f'(\alpha\gamma)f(\alpha\gamma)^{-1}F(\alpha\gamma) \tag{30}$$

Using the fact that $p_1 - p_2 \leq v$ (Lemma 1) and $F(0) = 1/2$, we see that the first term in Expression (30) equals $3f(0)$ in the limit whereas the remaining two terms equal zero. Similarly, the limit of the first term in the denominator of Equation (29) equals $3f(0)$ and that of the second term equals zero, such that $r_\pi(\alpha)$ goes to 1 as α approaches zero.

Finally, for the proof of (iii), assuming that prices are restricted to being non-negative, it suffices to show that firm 1 can always make a profit of at least $\nu/2$. Because $\pi_1^*(\alpha)$ increases in p_2, at worst, when $p_2 = 0$, firm 1 can name a price of ν to earn profit of $\nu F(\alpha \times 0) = \nu/2$.

Therefore, $r_\pi(\alpha) \geq 2\alpha\nu f(0)$ and as α goes to infinity, so does $r_\pi(\alpha)$. □

Proof of Corollary 1.

Proof. Consider $\alpha^*(\nu)$ for some $\nu > 0$. From the proof of Proposition 3, we know that $\alpha^*(\nu)$ is the value of α that satisfies $\pi_1^*(\alpha)/\bar{\pi}_1(\alpha) = 2$. Next, consider some $\nu' > \nu$. Using Step 2 in the proof of Proposition 2, we know that holding α fixed, $\pi_1^*(\alpha)$ rises, but that $\bar{\pi}_1(\alpha)$ remains unchanged. Thus, $r_\pi(\alpha^*(\nu)) > 2$ at ν'. Because $r_\pi(\alpha)$ is increasing in α, $\alpha^*(\nu') < \alpha^*(\nu)$. □

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