Help Us to Help You: How Consumer Data Can Alter Quality Races

Christian Trudeau

Department of Economics, University of Windsor
401 Sunset Avenue
Windsor, Ontario N9B 3P4, Canada
trudeauc@uwindsor.ca

Zheng Wang
International School of Economics and Management,
Capital University of Economics and Business
121 Zhangjialukou, Huaxiang Fengtai District,
Beijing, 100070, China
zwang46@163.com

Running Head: Vertical Differentiation and Quality Races

Abstract

Recent technological changes have made it easy for firms to collect data on their consumers, which in turn allow them to improve the efficiency of their R&D. We explore the strategic interaction that occurs when two firms compete in a vertically-differentiated market to acquire this data and invest in R&D. We find that if the initial quality lead is not too large, there exist equilibria where the laggard is able to reverse the lead by being particularly aggressive in acquiring this consumer data. While total welfare is higher when the initial leader maintains its lead, consumers prefer leapfrogging.

1. Introduction

Recent technological changes have made it easy for firms to collect data on their consumers and elicit feedback from them. It seems natural to suppose that investment efficiency is an increasing function of the data collected: more data means a better knowledge of the preferences of consumers or of the aspects of the good/service on which more work is needed. A firm that has more data can thus improve its product in a more cost-efficient manner (saving R&D time or avoiding unnecessary investment).

More precisely, customer feedback can be used to develop the next generation of products and services. For instance, manufacturers are using data obtained from sensors embedded in products. It also applies to software, apps and operating systems developers, who collect data on usage and bugs before implementing improvements. Google, for example, is known for releasing products to the public before they are done in order to get such feedback.[1]

Given the value of this data, we can expect that firms will alter their behavior to acquire this data. Surprisingly, the economic literature is very much silent on this. An exception is a working paper by Baye and Sapi (2014) that looks at the decision to invest in data-collecting devices. We focus on the impact that this consumer data has on price competition, as the firms compete to acquire the data, which in turn will be used to increase the efficiency of R&D in the following period.

In a two-period model where firms amass data and invest in R&D in the first period and compete with the updated quality in the second period, we show that multiple equilibria can occur, including some where the initial laggard in terms of quality can leapfrog its competitor. We get this result even though our model is completely deterministic and gives no advantage to the laggard. This is in opposition with the literature on lead reversals, which include Fudenberg et al. (1983) and Harris and Vickers (1987), who study possible leapfrogging in patent races, and Budd et al. (1993), who build a model to study increasing asymmetry in a dynamic oligopoly where competition depends on effort levels (R&D, advertising, etc.) of firms. Athey and Schmutzler (2001) build a general model of dynamic competition with investment which encompasses many of the previous models. The general conclusions in the above models are that it requires either a large advantage in terms of investment effectiveness for the follower or significant uncertainty to obtain leapfrogging. In general, the leading firm tends to increase its dominance.

The vertical differentiation competition at the heart of our framework is built on Tirole (1988). The high-quality firm earns more profit than its rival (a result that holds for more general frameworks, see Lehmann – Grube (1997)), giving incentives to firms to fight to become that second-period leader. On the other hand, saving the investment cost and differentiating the product generate the incentive for firms to stay behind. It thus becomes a coordination game, in which it might be better to act less aggressively and maximize differentiation of the goods in the second period if the rival firm acts aggressively to become the leader. We are able to solve analytically for all pure-strategy subgame perfect Nash equilibria.

The proposed model is part of the literature on vertical differentiation with endogenous quality initiated by Shaked and Sutton (1982). Our main distinction is that the firms can make their quality improvement technology more efficient by fighting for customers and the data they generate. Common conclusions of vertical differentiation models include different qualities observed in equilibrium and a high-quality advantage. There are various predictions, depending on the exact set-up, if laggards (Grossman and Helpman (1991), Segerstrom (1991)) or leaders (Segerstrom and Zolnierek (1999)) have more incentives to invest in R&D. As a contrast, we show that both are possible, unless the initial quality lead is so large, in which case the leader is the only one to invest.

More generally, the model is part of the large literature on price competition in dynamic oligopolies. Maskin and Tirole (1988) provide a general framework. The many applications of this model include dynamic oligopolies with learning-by-doing (Cabral and Riordan (1994)), network and compatibility or switching costs effects (Katz and Shapiro (1994), Farrell and Klemperer (2007)). As in the above models, our model yields possible pricing below marginal cost.

The paper is structured as follows. We first describe the model set - up in Section 2. We then derive the multiple equilibria in Section 3. Section 4 demonstrates the conditions for each equilibrium and Section 5 examines the welfare consequences. The final section concludes the paper. Proofs are in the appendix.

2. The model

We study a two–period duopoly model. The two firms compete in price over two periods in this vertically differentiated market. We supplement Tirole (1988)'s vertical differentiation model in the following manner: in the first period, firms have the opportunity to invest to improve the quality of their good in the second period. Given that sales in the first period allow the producers to acquire either direct feedback (consumer evaluation and comments) or indirect feedback (data on usage and bugs, ability to do test designs, etc.), the effectiveness of investment is increasing with first-period sales.

More precisely, in period 1, the development phase, p_i, x_i and π_i denote, respectively, the price, quantity and profits of firm i, for $i = a, b$. Let y_i be the investment to improve quality. We suppose that the marginal cost of production is 0.[2] Let v_i be the quality of good i in period 1. It can be interpreted as the reserve price of the agent with the highest valuation for quality.

Let v_a and v_b be given. We suppose that $\Delta v \equiv v_a - v_b \geq 0$; that is, firm a has an initial lead in quality.

In period 2, the maturity phase, p_i', x_i', v_i' and π_i' denote the key variables. In particular, the quality of the product of firm i improves in the following manner: $v_i^{'} = v_i + \sqrt{y_i(1 + x_i)}$. The marginal productivity of the investment is thus decreasing in the investment itself and increasing with the first-period sales. This particular form, in addition to being reasonable, has the advantage of keeping the problem tractable. Let $\Delta v^{'} = |v_a^{'} - v_b^{'}|$. Time is discounted by $\beta \in [0, 1]$. Let $\Pi_i = \pi_i + \beta\pi_i^{'}$ be the present value of profits of firm i.

The price competition in each period is modeled as a classic vertically differentiated model: each consumer k is characterized by a marginal willingness to pay for this value γ_k, uniformly distributed over the support [0, 1], with the population size normalized to 1. Let u_i^k be the utility of consumer k when he buys the product of firm i: $u_i^k = u + \gamma_k v_i - p_i$. A consumer buys one unit or buys nothing. If he buys nothing, his utility is 0. The good is non-durable and the valuation u is high enough so that each agent buys one unit in each period.

The timing of the game is as follows: at the beginning of period 1, firms observe v_a and v_b and then simultaneously choose p_i and y_i. At the beginning of period 2 they observe v_a'and v_b' and then simultaneously choose p_i'.

3. Maturity phase and potential equilibria

The equilibrium concept is subgame perfect Nash-equilibrium, so we proceed by backward induction and start by looking at the second period.

3.1 Maturity phase

We first solve the second period price competition. Suppose that $\Delta v'$ is known. We denote by L the leading firm (in terms of quality) in the maturity phase, with F being the follower. Note that even though we assume that firm a has the lead initially ($v_a \geq v_b$), this order need not hold in the second period. The profits for the leader and the follower in the second stage are $\pi_i' = p_i' x_i'$ for $i = L, F$, with $x_L' = 1 - \frac{p_L' - p_F'}{\Delta v'}$ and $x_F' = \frac{p_L' - p_F'}{\Delta v'}$.[3]

Lemma 1: In the maturity phase (period 2), there exists a unique pure strategy Nash equilibrium $(p_L^*, p_F^*) = (\frac{2\Delta v'}{3}, \frac{\Delta v'}{3})$. It is such that $\pi_L'^* = \frac{4\Delta v'}{9}$ and $\pi_F'^* = \frac{\Delta v'}{9}$.

The proof is easily obtained, given that in the second period, since there are no further investments in quality, the competition is as in a classic vertically differentiated market. Each firm maximizes its own profit with respect to price and simultaneously solving the resulting best response functions yields the above result: the leading firm sells twice as much as the follower, making four times the profits of the follower.

3.2 Development phase: 4 potential types of equilibria

In the first stage, firms maximize the present value of profits $\Pi_i = \pi_i + \beta\pi_i'$. Notice that in the maturity phase the profits of both firms are increasing in the difference in quality, meaning that firms have incentives to vertically differentiate. However, since the leader makes four times as much profits as the follower, the follower might challenge the lead. We thus obtain different types of potential equilibria, depending whether or not firms try to conserve/take the lead. What is clear is that firms will behave differently if they want to be in the lead (high quality if in lead, low quality if not). Further, if a firm tries to be the leader but fails, it cannot be a Nash equilibrium, as it would prefer to change its strategy. The potential equilibria type also depends on corner solutions: since sales are always between 0 and 1, there is a limit to the advantage a firm can obtain with its first period sales.

We first consider equilibria in which firm a keeps the lead. In this case profits are:

$$\Pi_a^C = p_a\left(\frac{\Delta v + p_b - p_a}{\Delta v}\right) + \frac{4\beta}{9}\left(\Delta v + \sqrt{\frac{y_a(2\Delta v + p_b - p_a)}{\Delta v}} - \sqrt{\frac{y_b(\Delta v + p_a - p_b)}{\Delta v}}\right) - y_a \tag{1}$$

$$\Pi_b^C = p_b\left(\frac{p_a - p_b}{\Delta v}\right) + \frac{\beta}{9}\left(\Delta v + \sqrt{\frac{y_a(2\Delta v + p_b - p_a)}{\Delta v}} - \sqrt{\frac{y_b(\Delta v + p_a - p_b)}{\Delta v}}\right) - y_b \tag{2}$$

Given that firm a keeps the lead, it is obvious that it is optimal for firm b to choose $y_b = 0$. We then obtain the following two equilibria types:

- *Lead consolidation with market split (CS):* $0 < x_a < 1$, $0 < x_b < 1$, $y_a > 0$ and $y_b = 0$. Solving the FOCs with respect to p_a, p_b and y_a yields equilibrium values $p_a{}^{CS} = \frac{2\Delta v}{3} - \frac{7\beta^2}{243}$, $p_b{}^{CS} = \frac{\Delta v}{3} - \frac{2\beta^2}{243}$, $y_a{}^{CS} = \frac{20\beta^2}{243} + \frac{20\beta^4}{19683\Delta v}$.
- *Lead consolidation with market capture (CC):* $x_a = 1$, $x_b = 0$, $y_a > 0$ and $y_b = 0$. Solving the FOC with respect to y_a yields equilibrium values $p_a{}^{CC} = p_b{}^{CC} = P^{CC}$, $y_a{}^{CC} = \frac{8\beta^2}{81}$.

We next consider (potential) equilibria where firm b leapfrogs firm a in terms of quality. For this case profits are:

$$\Pi_a^L = p_a\left(\frac{\Delta v + p_b - p_a}{\Delta v}\right) + \frac{\beta}{9}\left(-\Delta v - \sqrt{\frac{y_a(2\Delta v + p_b - p_a)}{\Delta v}} + \sqrt{\frac{y_b(\Delta v + p_a - p_b)}{\Delta v}}\right) - y_a \tag{3}$$

$$\Pi_b^L = p_b\left(\frac{p_a - p_b}{\Delta v}\right) + \frac{4\beta}{9}\left(-\Delta v - \sqrt{\frac{y_a(2\Delta v + p_b - p_a)}{\Delta v}} + \sqrt{\frac{y_b(\Delta v + p_a - p_b)}{\Delta v}}\right) - y_b \tag{4}$$

Given that firm b takes the lead in the second period, it is obvious that it is optimal for firm a to choose $y_a = 0$. We then obtain the following two equilibria types:

- *Leapfrogging with market split (LS):* $0 < x_a < 1$, $0 < x_b < 1$, $y_a = 0$ and $y_b > 0$. Solving the FOCs with respect to p_a, p_b and y_a yields equilibrium values ${p_a}^{LS} = \frac{2\Delta v}{3} - \frac{2\beta^2}{243}$, ${p_b}^{LS} = \frac{\Delta v}{3} - \frac{7\beta^2}{243}$, ${y_b}^{LS} = \frac{16\beta^2}{243} + \frac{20\beta^4}{19683\Delta v}$.
- *Leapfrogging with market capture (LC):* $x_a = 0$, $x_b = 1$, $y_a = 0$ and $y_b > 0$. Solving the FOC with respect to y_b yields equilibrium values ${p_a}^{LC} = P^{LC}$, ${p_b}^{LC} = P^{LC} - \Delta v$, ${y_b}^{LC} = \frac{8\beta^2}{81}$.

4. Equilibria

We can now derive the main result.

Theorem 1: We have the following equilibria, given initial values of Δv and β.

i) If $\Delta v \geq \frac{2\beta}{27}\left(\frac{4}{5} + \frac{\sqrt{144+30\beta}}{15}\right)$, the unique equilibrium is the lead consolidation equilibrium with market split.

ii) If $\frac{5\beta^2}{81} \leq \Delta v < \frac{2\beta}{27}\left(\frac{4}{5} + \frac{\sqrt{144+30\beta}}{15}\right)$, the set of equilibria consists of the lead consolidation equilibrium with market split and the leapfrogging equilibrium with market split.

iii) If $\frac{5\beta^2}{162} \leq \Delta v < \frac{5\beta^2}{81}$, the set of equilibria consists of lead consolidation equilibria with market capture, with $P^{CC} \in [-\frac{4\beta^2}{81} + \Delta v, \frac{\beta^2}{81}]$ and the leapfrogging equilibrium with market split.

iv) If $0 \leq \Delta v < \frac{5\beta^2}{162}$, the set of equilibria consists of lead consolidation equilibria with market capture, with $P^{CC} \in [\ -\frac{4\beta^2}{162} + \Delta v, \frac{\beta^2}{81}]$ and the leapfrogging equilibrium with market capture, with $P^{LC} \in [-\frac{4\beta^2}{81} + 2\Delta v,\ \frac{\beta^2}{81}]$.

Proof is in the appendix. While the proof is space-intensive, it follows a simple procedure. For each type of equilibria, we find conditions for equilibrium by making sure that the resulting sales and quality differences are compatible with the given type, that firms do not have incentives to sell more or less (without changing their investment) and that firms have no incentives to change their strategy with respect to the lead.

Figure 1 summarizes the result of Theorem 1. Thus, if the initial quality lead is large (and/or if firms are impatient), the only equilibrium is one such that firm a consolidates its lead. If $\Delta v < \frac{2\beta}{27}\left(\frac{4}{5} + \frac{\sqrt{144+30\beta}}{15}\right)$, we always have some equilibria in which firm b manages to leapfrog its competitor. Given the relatively small lead, a particularly aggressive pricing strategy in the first period allows firms to gain valuable consumer data, which in turns improves the efficiency of its investment. Faced with an aggressive competitor, firm a will optimally decide to ease up on competition and not invest in improving its quality, in order to become the low-quality alternative in the next period, reaping the benefits of quality differentiation.

This is not the only equilibrium however, as for all values of the parameters, there is also an equilibrium in which firm a consolidates its lead. If Δv is relatively high (or again, if firms are impatient), firms share the market in the first period. If firm a consolidates its lead, it is not worthwhile to engage in cutthroat competition in the first period to attract all consumers, as the cost to acquire them would not be compensated for by the gains in investment efficiency, and subsequently, in the increased quality lead in the second period. In cases where firm b takes the lead, the reason for market sharing is not the same: since there is a (relatively) large advantage for firm a in the first period, acquiring new consumers is especially costly. Firm b needs to offer a discount of Δv to attract all consumers. Even though it would lead to a larger differentiation in the second period, it is not cost-effective to attract more consumers in the first period. For smaller initial leads, these effects are reversed and firms do compete for all consumers in the first period.

Figure 2 describes the resulting quality difference in the second period as a function of the initial quality lead and whether or not firm a keeps the lead. In all cases the quality lead in the second period is larger than in the first. The curves are linear for small values of Δv, as it corresponds to the market capture cases. For larger values of Δv, the curves become non-linear as we move to the market splitting equilibria, where investments are less than maximal. It is not surprising to see that with an equilibrium with lead consolidation the second-period quality lead is increasing in Δv. For the leapfrogging case, it is decreasing in Δv, as not only the size of the leap increases, but the investment stays constant (for market capture equilibria) or declines (for market splitting equilibria). We reach a point (in our example around $\Delta v = 0.124$) where leapfrogging equilibria are no longer feasible.

It is easy to see that if data acquisition had no impact on R&D effectiveness, the equilibrium prices in period 1 would be ${p_a}^* = \frac{2\Delta v}{3}$ and ${p_b}^* = \frac{\Delta v}{3}$. If we consider equilibria with market splitting, it is obvious that the competition to acquire data from consumers pushes the prices down from these levels. However, in the equilibria with market capture, it is not as straightforward. Figure 3 shows the equilibrium prices when we are in the lead consolidation equilibria and with $\beta = 1$. When $\Delta v \leq 0.06$, we have a market capture equilibria, and the region in gray represents all possible equilibrium prices. Otherwise, we have a market splitting equilibria. The dashed line represents the price of firm a when data acquisition has no impact. A first observation is that pricing under marginal cost (here negative prices) is possible if the initial quality lead is small. This is not particularly surprising, as there is a strong gain to differentiate in the second period and a high-level of competition to become the leader. A second and more surprising observation is that for very small values of Δv, the equilibrium price for firm a might be larger than without R&D-improving data acquisition. This can be explained by the fact that while firms have strong incentives to become the leader, their first goal is to differentiate themselves, which might lead one of the firms to be completely non-aggressive in terms of pricing in order to guarantee that the rival will acquire the maximum amount of data and will maximize differentiation in the next period. Similar observations can be made for the leapfrogging equilibria.

5. Welfare consequences

If Δv is large enough, the only equilibrium is the lead consolidation equilibrium. However, for low values of Δv, we have the coexistence of consolidation and leapfrogging equilibria. Which equilibrium arises is an issue of coordination. If the government can intervene to guide coordination (for instance by sending some signals, or by indicating if antitrust authorities will tolerate or not pricing below marginal cost or market capture), which equilibrium should it facilitate?

We examine this question from the perspective of the total welfare criteria. The total welfare follows as the sum of consumer surplus and profits over two periods, where consumer surplus is the multiplication of the average utility and the total number of consumers. Given that a government might place different weights on consumers and firms, we also examine separately the impacts of the equilibrium type on consumer and producer surpluses.

Let I and I' represent, respectively, the agent indifferent between buying from the low and high-quality firms in the first and second period. Agents with values for quality in [0, I] buy from the low-quality firm and those in [I, 1] buy from the high-quality firm. In the lead consolidation case we have that $I = \frac{p_a - p_b}{v_a - v_b}$ and I' = $\frac{(p'_a - p'_b)}{v'_a - v'_b}$.

Let CS and CS' represent, respectively, the consumer surplus in periods 1 and 2. Let ACS = $CS + \beta CS$' be the aggregate consumer surplus and $W = ACS + \Pi_a + \Pi_b$ be total welfare.

In the lead consolidation case we have that

$$CS = (1-I)\left(u + \left(\frac{1+I}{2}\right)v_a - p_a\right) + I(u + \frac{I}{2}v_b - p_b) \tag{5}$$

$$CS' = (1-I')\left(u + \left(\frac{1+I'}{2}\right){v_a}' - {p_a}'\right) + I'(u + \frac{I'}{2}{v_b}' - {p_b}') \tag{6}$$

In the leapfrogging equilibria, firm 2 is the high-quality firm in period 2, and thus I' = $\frac{(p'_a - p'_b)}{v'_a - v'_b}$. CS is as before but

$$CS' = (1-I')\left(u + \left(\frac{1+I'}{2}\right){v_b}' - {p_b}'\right) + I'(u + \frac{I'}{2}{v_a}' - {p_a}') \tag{7}$$

For all our equilibra, we can find the corresponding welfare by replacing the equilibrium prices in the above formulas. In the lead consolidation with market capture equilibria and the leapfrogging with market capture equilibria consumer surplus depends on the equilibrium prices. For simplicity, we assume that in all cases firms coordinate on the highest compatible price, P^{CC} = $P^{LC} = \beta^2/81$.

Theorem 2: For all values of the parameters where we have coexistence of the lead consolidation and leapfrogging equilibria, we have that:

i) the total welfare is higher with the lead consolidation than with the leapfrogging equilibria, regardless of the values of P^{CC} and P^{LC};

ii) the consumer surplus is higher with the leapfrogging than with the lead consolidation equilibria, if $P^{CC} = P^{LC} = \beta^2/81$.

iii) the producer surplus is higher with the consolidation than with the leapfrogging equilibria, if $P^{CC} = P^{LC} = \beta^2/81$.

The proof is in appendix. In the leapfrogging equilibria, the average price level is lower and the quality level is higher than with the lead consolidation. This generates the result that the leapfrogging improves the consumer surplus compared to the case with the lead consolidation. Yet, the resulting lower price and higher investment level hurt profits. The loss in profits dominates the increase in consumer surplus so that total welfare is lower with leapfrogging than with lead consolidation. If a government puts equal weights on consumer and producer surpluses, it will try to encourage the lead consolidation equilibrium. However, if it puts enough weight on the consumer surplus, it will instead push for leapfrogging.

6. Conclusion

We model a two – period duopoly in which two asymmetric firms have the control over price and R&D investments, with the effectiveness of R&D increasing with data acquired from consumers. While we always have equilibria where the initial leader consolidates its lead, we also have equilibria where there is leapfrogging. Our model is deterministic and the follower has no *a priori* advantage over the leader. In fact, in our model, the follower starts with a disadvantage: sales, and thus data acquisition, are more difficult in the first period. The addition of R&D-improving data acquisition to quality races allows us to obtain results that differ considerably from the literature.

From a pure total welfare point of view, lead consolidation is more desirable than leapfrogging, but results in a transfer from consumers to firms. If there is a bias towards consumer surplus then coordination on the leapfrogging equilibria should be encouraged.

Results are obtained in a model that supposes full market coverage: consumers always buy from either firm a or b. If we remove this assumption we lose the ability to solve analytically. However, intuitively, results should not differ: the follower will now make less than four times the profits of the leader in the second period (as it now loses consumers to the outside option), giving firms more incentives to become the leader. While consumers will be more difficult to attract in the first period, aggressive pricing will be more efficient for firm b than firm a, as it is then able to gain consumers who have low-value for quality (who bought nothing) and relatively high-value for quality (who bought from firm a). By opposition a low price for firm a will only allow to gain consumer on one side (relatively low value for quality, who bought from firm b). This asymmetry will allow for leapfrogging.

Appendix 1: Proof of Theorem 1

For the four types of potential equilibria we derive the conditions on Δv and β (and P, if applicable) that make sure that we have no profitable deviations.

1) lead consolidation equilibrium with market split

We need to find conditions such that: i) we have $0 \le x_a{}^{CS} \le 1$; ii) firms a and b have no incentives to deviate by moving to a corner solution; and iii) firm b has no incentives to take the lead.

Substituting $p_a{}^{CS}$ and $p_b{}^{CS}$ into $x_a{}^{CS} = \frac{\Delta v + p_b - p_a}{\Delta v}$ yields $0 \le x_a{}^{CS} = \frac{2}{3} + \frac{5\beta^2}{243\Delta v}\Delta v \le 1$ if $\Delta v \ge 5\beta^2/81$.

It can be shown that neither firm has the incentive to move to a corner solution and firm b has no incentive to take the lead if $\Delta v \ge 5\beta^2/81$.

2) Lead consolidation equilibria with market capture

We need to show that: i) firms a and b have no incentives to deviate by moving away from the corner solution and ii) firm b has no incentives to deviate by trying to take the lead.

It can be shown that i) firm a has no incentive to share the market if $P \le (-4\beta^2/81) - \Delta v$ or $P \ge (-4\beta^2/81) + \Delta v$; firm a has no incentive to sell to nobody if $P \ge -4\beta^2/81$; firm b has no incentive to capture the market if $P \le -\frac{(4-2\sqrt{2})\beta^2}{81} + \Delta v$; firm b has no incentive to share the market if $P \le \beta^2/81$; ii) firm b has no incentive to take the lead and capture the market if $P \le \Delta v + 5\beta\Delta v/9 + (8\sqrt{2} - 4)\beta^2/81$; firm b has no incentive to take the lead and share the market if $P \ge 2\Delta v - 4\beta^2(\sqrt{2} + 1)/81$.

Combining the results above, the lead consolidation equilibrium with market capture is feasible if and only if $(-4\beta^2/81) + \Delta v \le P \le \beta^2/81$ and $\Delta v \le 5\beta^2/81$.

3) leapfrogging equilibrium with market split

We need to show that: i) firm b takes the lead and $0 \leq {x_b}^{LS} \leq 1$; ii) firms a and b have no incentives to deviate by moving to a corner solution; and iii) firms a and b have no incentives to deviate by changing their objective with respect to the lead.

It can be shown that i) $v_a' - v_b' < 0$ if $\Delta v \leq \frac{1}{27}(4\beta + \frac{\beta\sqrt{144+30\beta}}{3})$, $0 \leq {x_b}^{LS} \leq 1$ if $\Delta v \geq 5\beta^2/162$; ii) firm a has no incentive to sell to nobody or to capture the market; firm b has no incentive to sell to nobody or to capture the market; iii) firm b has no incentive to give up the lead and to sell to nobody; firm b has no incentive to give up the lead and to share the market if $\Delta v \leq \frac{2\beta}{27}(\frac{4}{5} + \frac{\sqrt{144+30\beta}}{15})$; firm b has no incentive to give up the lead and to capture the market; firm a has no incentive to take the lead and to capture the market; firm a has no incentive to take the lead and to share the market; firm a has no incentive to take the lead and to sell to nobody.

Combining the results above, the leapfrogging equilibrium with market split is feasible if and only $5\beta^2/162 \leq v \leq \frac{2\beta}{27}(\frac{4}{5} + \frac{\sqrt{144+30\beta}}{15})$.

4) leapfrogging equilibria with market capture

We need to show three results: i) firm b takes the lead, ii) firms a and b have no incentives to deviate by moving away from the corner solution; iii) firms a and b have no incentives to deviate by changing their objective with respect to the lead.

It can be shown that i) firm b takes the lead if $\Delta v \leq 4\beta/9$; ii) firm a has no incentive to share the market if $P \leq \beta^2/81$; firm a has no incentive to capture the market if $P \leq \frac{(4-2\sqrt{2})\beta^2}{81} + \Delta v$; firm b has no incentive to share the market if $P \leq -4\beta^2/81$ or $P \geq (-4\beta^2/81) + 2\Delta v$; firm b has no incentive to sell to nobody if $\Delta v > 2\beta/9$or $P \geq (-4\beta^2/81) + \Delta v$ and $\Delta v \leq 2\beta/9$; iii) firm b has no incentive to give up the lead and to capture the market if $\Delta v \leq 5\beta^2/162$; firm b has no incentive to give up the lead and to share the market; firm *b* has no incentive to give up the lead and to sell to nobody if $P \geq \Delta v + 5\beta\Delta v/9 - 8\beta^2/81$; firm *a* has no incentive to take the lead and to capture the market if $P \leq (8\sqrt{2} - 4)\beta^2/81 + (9 - 5\beta)\Delta v/9$; firm *a* has no incentive to take the lead and to share the market; firm *a* has no incentive to take the lead and sell to nobody if $\Delta v \leq 16\beta/45$.

Combining the results above, the leapfrogging equilibrium with market capture is feasible if and only if $0 \leq \Delta v < 5\beta^2/162$.

Appendix 2: Proof of Theorem 2

We first obtain the profit values for all four equilibria types by replacing for the equilibrium prices found in Theorem 1. We obtain the following values, with $A = \beta^4/118098\Delta v$.

	Π_a	Π_b
lead consolidation with market split	$\frac{4\Delta v(1+\beta)}{9}+\frac{55\beta^2}{729}+50A$	$\frac{\Delta v(1+\beta)}{9}+\frac{23\beta^2}{729}+80A$
lead consolidation with market capture	$\frac{4\beta\Delta v}{9}+\frac{8\beta^2}{81}+P$	$\frac{\beta\Delta v}{9}+\frac{4\beta^2}{81}$
leapfrogging with market split	$\frac{\Delta v(4-\beta)}{9}+\frac{10\beta^2}{729}+80A$	$\frac{\Delta v(1-4\beta)}{9}+\frac{46\beta^2}{729}+50A$
leapfrogging with market capture	$-\frac{\beta\Delta v}{9}+\frac{4\beta^2}{81}$	$-\frac{(9-4\beta)\Delta v}{9}+\frac{8\beta^2}{81}+P$

We then compute the aggregate consumer surplus and total welfare in all cases.

In the lead consolidation with market split (CS) equilibrium, we have

$$ACS^{CS} = ((11v_b - 2v_a)/18 + u)(1 + \beta) - 14\beta^2/729 - 35A$$

$$W^{CS} = ((v_b + 8v_a)/18 + u)(1 + \beta) + 65\beta^2/729 + 95A$$

In the lead consolidation with market capture (CC) equilibrium, we have

$$ACS^{CC} = u(1 + \beta) + v_a/2 - P^{CC} + \beta((11v_b - 2v_a)/18) - 4\beta^2/81$$

$$W^{CC} = u(1 + \beta) + v_a/2 + \beta((v_b + 8v_a)/18) + 8\beta^2/81$$

In the leapfrogging with market split (LS) equilibrium, we have that

$$ACS^{LS} = ((11v_a - 2v_b)/18 + u)(1 + \beta) - 13\beta^2/729 - 35A$$

$$W^{LS} = ((v_a + 8v_b)/18 + u)(1 + \beta) + 43\beta^2/729 + 95A$$

In the leapfrogging with market capture (LC) equilibrium we have that

$$ACS^{LC} = u(1 + \beta) - v_b/2 - P^{LC} + v_a + \beta((11v_a - 2v_b)/18) - 4\beta^2/81$$

$$W^{LC} = u(1 + \beta) + v_b/2 + \beta((v_a - 8v_b)/18) + 8\beta^2/81$$

We now compare lead consolidation and leapfrogging. From Theorem 1, we have three relevant intervals to consider. If $5\beta^2/81 \leq \Delta v < \frac{2\beta}{27}(\frac{4}{5} + \frac{\sqrt{144+30\beta}}{15})$ we need to compare CS and LS. If $5\beta^2/162 \leq \Delta v < 5\beta^2/81$ we need to compare CC and LS. Finally, if $5\beta^2/162 \leq \Delta v < 5\beta^2/81$ we need to compare CC and LC.

i) In interval 1, subtracting W^{LS} from W^{CS} yields: $W^{CS} - W^{LS} = 7\Delta v(1 + \beta)/18 + 22\beta^2/729 > 0$. In interval 3, subtracting W^{LC} from W^{CC} yields: $W^{CC} - W^{LC} = \Delta v(9 + 7\beta)/18 > 0$. In interval 2, subtracting W^{LS} from W^{CC} yields: $W^{CC} - W^{LS} = 4\Delta v/9 + 7\beta\Delta v/18 + 29\beta^2/729 - 95A > 0$ for $5\beta^2/162 < \Delta v < 5\beta^2/81$. Thus, total welfare is always higher with the lead consolidation equilibrium.

ii) In interval 1, subtracting ACS^{CS} from ACS^{LS} yields: $ACS^{LS} - ACS^{CS} = 13\Delta v(1 + \beta)/18 + \beta^2/729 > 0$. In interval 3, subtracting ACS^{CC} from ACS^{LC} yields: $ACS^{LC} - ACS^{CC} = \Delta v(9 + 13\beta)/18 > 0$. In interval 2, substituting $P^{CC} = \beta^2/81$ into $ACS^{LS} - ACS^{CC}$ yields: $\Delta v/9 + 13\beta\Delta v/18 + 14\beta^2/729 - 35A > 0$ for $5\beta^2/162 < \Delta v < 5\beta^2/81$. Thus, consumers prefer the leapfrogging to the lead consolidation equilibrium.

iii) Follows from i) and ii).

References

Athey, S. and A. Schmutzler, 2001, "Investment and market dominance," RAND Journal of Economics, 32, 1—26.

Baye, I. and G. Sapi, 2014, "Targeted pricing, consumer myopia and investment in customer - tracking technology?" Dice Discussion Papers.

Budd, C., C. Harris and J. Vickers, 1993, "A Model of the Evolution of Duopoly: Does the Asymmetry between Firms Tend to Increase or Decrease?" Review of Economic Studies, 60, 543—573.

Cabral, L.M.B. and M. H. Riordan, 1994, "The Learning Curve, Market Dominance, and Predatory Pricing," Econometrica, 62, 1115—1140.

Farrell, J. and P. Klemperer, 2007, Coordination and lock-in: Competition with switching costs and network effects, in: M. Armstrong and R.H. Porter (Eds), Handbook of Industrial Organization, vol. 3. North-Holland, Elsevier.

Fudenberg, D., R. Gilbert, J. Stiglitz and J. Tirole, 1983, "Preemption, leapfrogging and competition in patent races," European Economic Review, 22, 3-31.

Grossman, G.M. and E. Helpman, 1991, "Quality Ladders in the Theory of Growth," Review of Economic Studies, 58, 43-61.

Harris, C., J. Vickers, 1987, "Racing with Uncertainty," Review of Economic Studies, 54, 1—21.

Katz, M.L. and C. Shapiro, 1994, "System Competition and Network Effects," Journal of Economic Perspectives, 8, 93—115.

Lehmann-Grube, U., 1997, "Strategic Choice of Quality When Quality is Costly: The Persistence of the High-Quality Advantage," RAND Journal of Economics, 28, 372-384.

Maskin, E., J. Tirole, 1988, “A Theory of Dynamic Oligopoly, II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles,” Econometrica, 56, 571—599.

Segerstrom, P.S., 1991, “Innovation, Imitation, and Economic Growth,” Journal of Political Economy, 99, 807-27.

Segerstrom, P.S. and J. M. Zolnierek, 1999, “The R&D Incentives of Industry Leaders”, International Economic Review, 40, 745-66.

Shaked, A., J. Sutton, 1982, “Relaxing Price Competition Through Product Differentiation,” Review of Economic Studies, 49, 3—13.

Tirole, J, 1988, The Theory of Industrial Organization, MIT Press.

Figure 1: Types of Equilibria as A Function of Δv and β

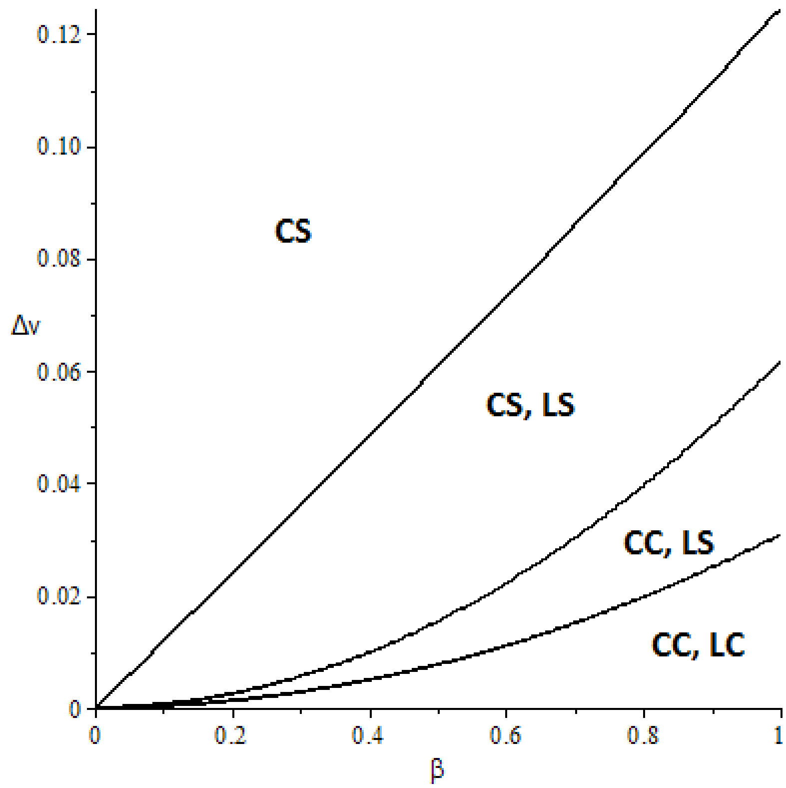


Figure 2: Quality lead in the second period ($\Delta v'$) as of a function of initial quality lead(Δv), with $\beta = 1$

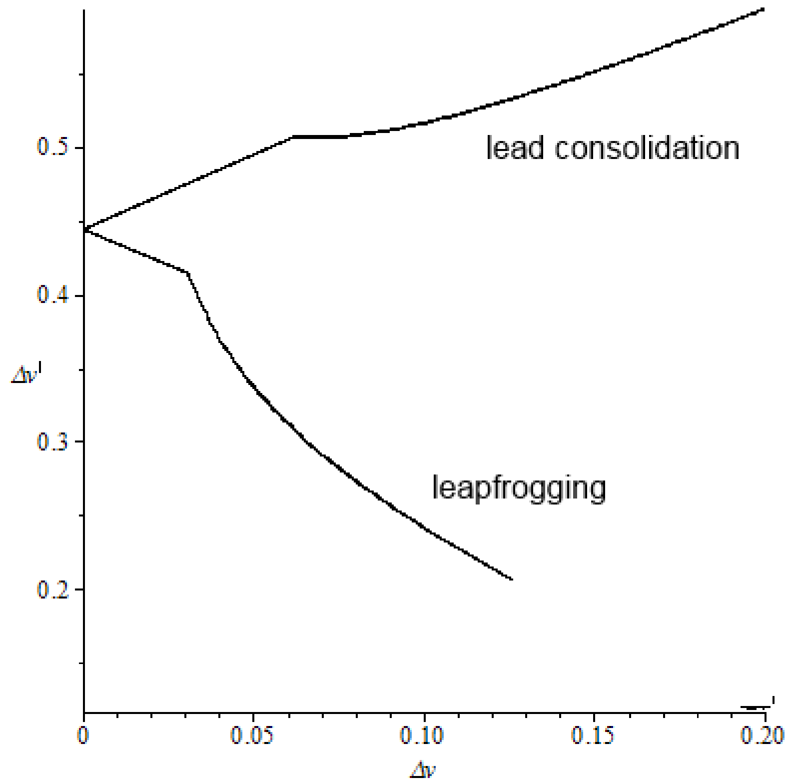
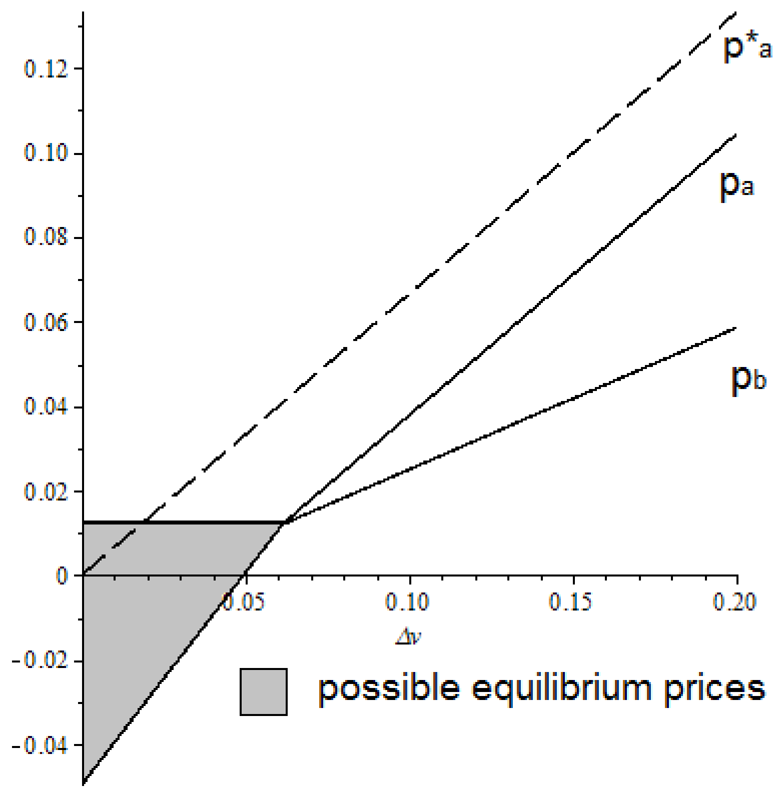


Figure 3: Equilibrium prices in lead consolidation equilibria as of a function of initial quality lead (Δv), with $\beta = 1$.



Endnotes

---

[1] "Small – batch innovation: What the lean Startup model can do for your R&D", Melissa Campeau, Financial Post, October 23, 2014.

[2] Since marginal cost is equal to zero, negative prices allow for pricing under marginal cost. The assumption on marginal cost eliminates variables and allows for a clearer exposition.

[3] If $\Delta v' = 0$, we assume that whoever has the lowest price captures the full market and sells one unit. If both prices are equal, the market is split equally.